**Grade Level/Course:** 5th/6th Grade Math

**Lesson/Unit Plan Name:** Using Prime Factors to find LCM and GCF.

**Rationale/Lesson Abstract:** The objective of this two-part lesson is to give students a clear understanding of LCM and GCF. Students will be able to find the LCM and GCF by comparing prime factors.

**Timeframe:** This lesson is designed to be completed in two one-hour classes.

**Common Core Standard(s):** 6.NS.4
Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).

**Instructional Resources/Materials:**
- Warm-Up (See page 6/7 and 9/10)
- Math Notebook
- Homework (Pages 8 and 11)
- Exit tickets (Page 12)

### Day One: Least Common Multiple

1. **Warm-up.** (See pages 6/7). This can be photocopied and distributed, or written on the board. This warm-up is meant to be both a review of the target knowledge, and an assessment of previous knowledge. Students can work in groups to solve the problems, and teachers should circulate the room to assess what items need the most review.

2. **Review the Warm-up.** Answers can be found after the warm-up page.

3. **Vocabulary Review.** In their math notebooks, students should write down the following vocabulary words and definitions:

   - **Prime Number:** a number that has exactly two different factors: 1 and itself.
   - **Factor:** one of two or more numbers that can be multiplied to form a product. “2, 3, and 5 are all factors of 30.”
   - **Multiple of a number:** the product of a number and any other non-zero number. “8, 16 and 24 are all multiples of 8.”
4. Teach the “Bubble Method” for finding the LCM.

On the board, write: “Find the LCM of 12 and 16.”

a. Find the prime factors of the given numbers.

There are several ways of finding the prime factors of a number. Here are three ways:

<table>
<thead>
<tr>
<th>Start with the smallest prime factor (usually 2 or 3) written on the left, and circle it. Split any remaining factors until only prime factors remain.</th>
<th>Split the top number into any two factors, and work down until all factors are prime and circled.</th>
<th>Use a division ladder, also called “upside down division.” Prime factors are circled on left side of the ladder, composite factors are written below. Continue dividing until the bottom number is a prime.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
<td><img src="image3.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

b. When you are finished, you will have a factor string for each of the numbers:

\[
12 = 2 \cdot 2 \cdot 3 \\
16 = 2 \cdot 2 \cdot 2 \cdot 2
\]

c. Draw a bubble. Write the factors of the first number in the bubble.

![Bubble](image4.png) These are the prime factors of 12.
d. Add any factors to the bubble that would be needed to produce the second number. In this case, we need 2 \cdot 2 \cdot 2 \cdot 2 to produce 16, and we already have 2 \cdot 2 in the bubble. So we add another 2 \cdot 2, so that the bubble has all the factors of 16.

\[2 \cdot 2 \cdot 3 \cdot 2 \cdot 2\]

These are the prime factors of 12 and 16.

e. Multiply all the factors to find the LCM:

\[2 \cdot 2 \cdot 3 \cdot 2 \cdot 2 = 48\]

Therefore, the LCM of 12 and 16 = 48

**You Try!**

a. Find the LCM of 8 and 12.
b. Find the LCM of 6 and 15.
(Answers: 24, 30)

**5. Give one final pair of numbers for students to complete on an exit ticket.** For example: “Find the LCM of 6 and 16.” Choose a pair of numbers from 2 - 25. Pay close attention to how students are combining factor strings in the “factor bubble”. Some students will need to review the “bubble method”, and should be given opportunities for review.

**6. Examine the exit tickets**, and create three piles: one for the students who have the correct answer and show their work correctly, one for the students who have the wrong answer but show some understanding of the method, and one pile for students who are lost and need intervention.
Day Two: Greatest Common Factor

1. **Warm-Up.** (See pages 9/10).

2. **Review Vocabulary:** Prime number, multiple, factor.

   Access prior knowledge by asking students: “Why is 5 a prime number? Why isn’t 1 a prime number? What is a multiple of 9? What are two factors of 24?” Etc.

3. **Add vocabulary:** greatest, common.
   Discuss the meaning of these words with students. In the context of GCF, greatest means “largest” (not “best”), and common means “shared” or “alike”.

4. **Teach the method for using prime factors to find the GCF of 20 and 32.**
   a. Write the two numbers, stacked vertically:
      
      \[
      \begin{array}{l}
      12 \\
      20 \\
      \end{array}
      \]
   
   b. Find the prime factors of each number, using a factor tree if needed.
      Write the factors of each number as a factor string:
      
      \[
      \begin{array}{l}
      12 = 2 \cdot 2 \cdot 3 \\
      20 = 2 \cdot 2 \cdot 5 \\
      \end{array}
      \]
   
   c. Those factors that both numbers share are the common factors. Circle them.
      
      \[
      \begin{array}{l}
      12 = \fbox{2 \cdot 2} \cdot 3 \\
      20 = \fbox{2 \cdot 2} \cdot 5 \\
      \end{array}
      \]
   
   d. In this case, both 12 and 20 have 2 \cdot 2 in common. Multiply the common factors to find the GCF. In the case of 12 and 20, multiply 2 \cdot 2. The GCF is \textbf{4}. 

You Try:

5. **Find the GCF of the following numbers:**

   15 and 40
   20 and 64
   18, 36, and 72
(Answers: 5, 4, 18)

6. **Hand out an exit ticket for each student**, and write a pair of numbers from 2 – 100 on the board. (Example: 36 and 72). Ask students to compare prime factors to find the GCF of the two numbers.

7. **Examine the exit tickets**, and create three piles: one for the students who have the correct answer and show their work correctly, one for the students who have the wrong answer but show some understanding of the method, and one pile for students who are lost and need intervention.
Warm-Up

Name five prime numbers between 0 and 15.

Find all the factors of 36.

Write three multiples of 8.

Find the prime factorization of 12. Use any method you like to find your answers.
**Warm Up**  
**Day One: Answers**

<table>
<thead>
<tr>
<th>Name five prime numbers between 0 and 15.</th>
<th>Find all the factors of 36.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2,3,5,7,11,13</strong></td>
<td><strong>1,2,3,4,6,9,12,18,36</strong></td>
</tr>
</tbody>
</table>

*Students should know that a prime number has exactly 2 different factors: 1 and itself. This is why 1 is not a prime number.*

<table>
<thead>
<tr>
<th>Find the prime factorization of 12. Use any method you like.</th>
<th><strong>2 • 2 • 3</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Review the process of creating factor trees. Students should circle prime numbers at the end of each “branch.”</em></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Write three multiples of 8.</th>
<th>Find all the factors of 36.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8,16,24...</strong></td>
<td><strong>1,2,3,4,6,9,12,18,36</strong></td>
</tr>
</tbody>
</table>

*Students should know that a multiple of a number is equal to or larger than the original number. The original number is always a factor of its multiples.*
Homework!
Least Common Multiple

A. Find the prime factors of each number. Write your answer as a factor string.

1.) 6
2.) 20
3.) 24

6 = __________________
20 = __________________
24 = __________________

B. Use the bubble method to find the LCM of each pair of numbers.

4.) 6 and 20
5.) 20 and 24

4.) LCM = _____________
5.) LCM = _____________
Warm-Up

Find five multiples of 17.

Write the prime factors for the following numbers:

10: __________
9: __________
15: __________
7: __________
21: __________

Explain why 1 is not a prime number.

Is the product a fraction in simplest form? Use prime factors to justify your answer.

\[
\frac{5 \cdot 11}{8 \cdot 12} = \frac{55}{96}
\]
Find five multiples of 17.

17, 34, 51, 68, 85…

*Students often confuse the term “multiple” with the term “factor”. As 17 is a prime number, it only has 2 factors, but an infinite number of multiples.*

<table>
<thead>
<tr>
<th>Write the prime factors for the following numbers:</th>
</tr>
</thead>
<tbody>
<tr>
<td>10: 2 • 5</td>
</tr>
<tr>
<td>9: 3 • 3</td>
</tr>
<tr>
<td>15: 3 • 5</td>
</tr>
<tr>
<td>7: 1 • 7</td>
</tr>
<tr>
<td>21: 3 • 7</td>
</tr>
</tbody>
</table>

Explain why 1 is not a prime number.

*1 is not a prime number because it does not have two different factors.*

Is the product a fraction in simplest form? How can you use prime factors to justify your answer?

\[
\frac{5 \times 11}{8 \times 12} = \frac{55}{96}
\]

*If you expand the numerator and denominator into their prime factors, you have*

\[
\frac{5 \times 11}{2 \times 2 \times 2 \times 2 \times 3}
\]

*As there are no factors shared between the numerator and the denominator, the fraction is in simplest form.*
**Homework!**

**Greatest Common Factor**

Find the prime factors of each number. Then, compare the factor strings to find the GCF of the two given numbers.

<table>
<thead>
<tr>
<th>27</th>
<th>48</th>
<th>36</th>
<th>45</th>
</tr>
</thead>
</table>

The GCF of 27 and 48 is __________.

The GCF of 36 and 45 is __________.

<table>
<thead>
<tr>
<th>80</th>
<th>100</th>
<th>60</th>
<th>72</th>
</tr>
</thead>
</table>

The GCF of 80 and 100 is __________.

The GCF of 60 and 72 is __________.