

## Parallel Lines and a Transversal

*Materials:* Two or three sheets of Patty paper for each student, straightedge, scissors, a sheet of lined paper for making initial parallel lines diagram and taking notes

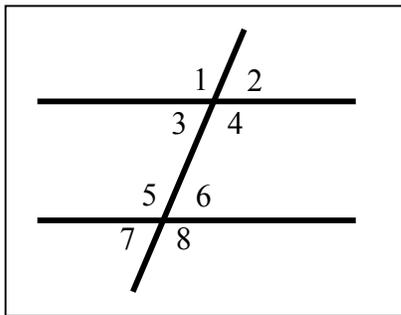
Write the following definition on your note paper:

**Definition:** When two lines are intersected by a third line, the third line is called a **transversal**.

We are going to explore four different kinds of pairs of angles that are formed when two parallel lines are cut by a transversal. For each of these special pairs of angles, the vertex of one of the angles will be on one of the parallel lines, and the vertex of the other angle will be on the other parallel line.

*Set up:*

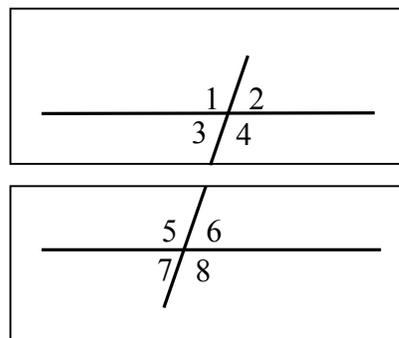
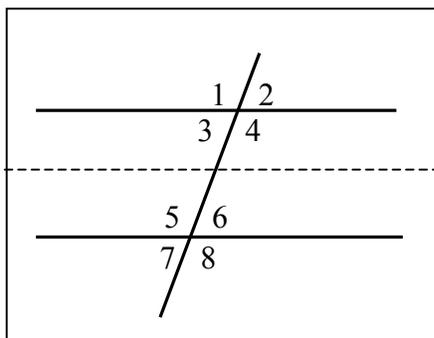
- Using a straightedge, draw two segments, following two of the lines on your binder paper, three lines apart. Make your segments about 3 inches long. You now have two parallel lines on your paper
- Use your straight edge to draw a transversal. It can tilt in any direction.
- Number the angles as follows (go left to right, so that everyone has the same numbering system):



### Corresponding angles:

Using your straightedge, trace your drawing onto one of your pieces of patty paper. Be sure to copy the numbers.

Cut across the middle of the traced picture, between the two lines:



Slide one half on top of the other, matching up the parallel lines and the transversal. Angle 1 should be on top of angle 5.

The pairs of angles that are now on top of each other are called **corresponding angles**.

- 1) Name all four pairs of corresponding angles in the diagram. **(Students should write this on their notes)**

The pairs of corresponding angles are

- $\angle 1$  and  $\angle 5$
- $\angle 2$  and  $\angle 6$
- $\angle 3$  and  $\angle 7$
- $\angle 4$  and  $\angle 8$

- 2) Since each pair of corresponding angles matches up exactly, what can you say about the measures of any pair of corresponding angles formed by parallel lines and a transversal?

[They are equal]

**Write on your note paper:** Corresponding angles formed by parallel lines and a transversal are congruent.

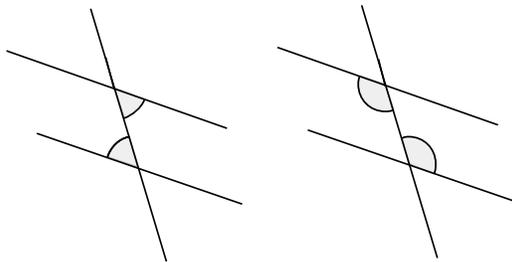
### **Alternate Interior Angles:**

Using a straightedge, trace the original diagram again onto another piece of patty paper. Number the angles the same way they are numbered in the diagram.

The four angles in between the parallel lines are called **interior angles**. Name the four interior angles:

$\angle 3$ ,  $\angle 4$ ,  $\angle 5$ ,  $\angle 6$

(Draw the following diagrams on the board) Each pair of angles shaded in these diagrams are called **Alternate Interior Angles**. Why do you think they are described using the word "alternate"? [They are on alternate sides of the transversal.]



- 3) Name both pairs of Alternate Interior Angles on your diagram. Write them in your notes:

The pairs of alternate interior angles are

- $\angle 3$  and  $\angle 6$
- $\angle 4$  and  $\angle 5$

- 4) Place angle 3 on your patty paper over angle 6 on your binder paper. Are they the same measure? [yes] Check to see if the other pair of alternate interior angles are also congruent. [they are congruent]

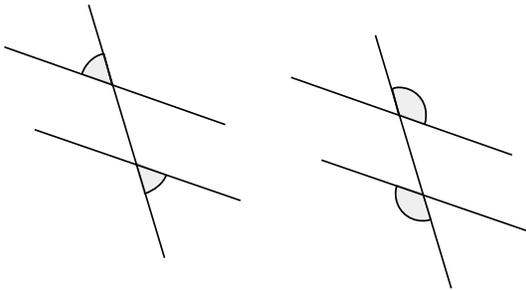
**Write on your note paper:** Alternate interior angles formed by parallel lines and a transversal are congruent.

### Alternate Exterior Angles:

If the four angles in between the parallel lines are called interior angles, what do you think the angles outside of the parallel lines are called? [**exterior angles**] Name the four exterior angles:

$\angle 1, \angle 2, \angle 7, \angle 8$

Each pair of angles shaded in the diagrams below are called **Alternate Exterior Angles**.



- 5) **Think-Pair-Share** Name both pairs of Alternate Exterior Angles on your diagram in your notes, then use your patty paper to see if the angle measures are the same for each pair. Write a sentence in your notes about how the angle measures compare.

Check with a neighbor to see if you have the same results.

The pairs of alternate exterior angles are

$\angle 1$  and  $\angle 8$

$\angle 2$  and  $\angle 7$

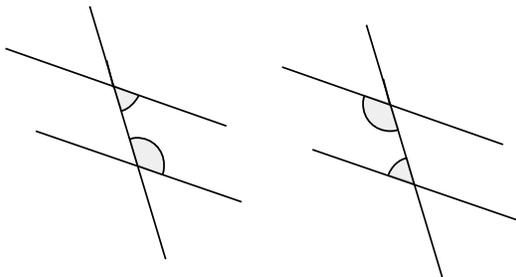
Alternate exterior angles formed by parallel lines and a transversal are congruent.

(Ask for someone to share their answers.)

### Same-side Interior Angles

The pair of interior angles that are on the same side of the transversal are called **Same-side interior** or **Consecutive interior** angles.

Each pair of angles shaded in the diagrams below are same-side interior angles.



- 6) Name both pairs of Same-side Interior Angles on your diagram in your notes, then use your patty paper to see if the angle measures are the same for each pair. Write a sentence in your notes about how the angle measures compare.

The pairs of same-side interior angles are

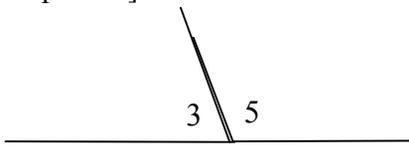
$\angle 3$  and  $\angle 5$

$\angle 4$  and  $\angle 6$

They are not congruent.

We see that same-side interior angles are not congruent like the other pairs we've examined. Let's explore these more:

Trace angle 3 on a piece of patty paper. Then trace angle 5 so that it is adjacent to angle 3. What does it mean for angles to be adjacent? [they have the same vertex, share a side, and do not share any interior points.]



What do you notice about their measures? [The sum of their measure is  $180^\circ$ .]

If the sum of the measures of two angles is  $180^\circ$ , what can we say about the angles? [They are supplementary.]

Check to see if this is also true about  $\angle 4$  and  $\angle 6$ . [It is]

- 7) **Write on your note paper:** Same-side interior angles formed by parallel lines and a transversal are supplementary.

**Check to see if these pairs of angles are congruent for non-parallel lines cut by a transversal:**

Have students draw a set of lines that are not parallel on their papers, and then draw a transversal through them. They can number the angles the same way as in the earlier diagram. Tell them to trace this drawing (they might have room on their patty paper, or might need a third sheet), then check to see if each of the different kinds of pairs of angles are congruent. Help them see that these pairs of angles are only congruent when the lines are parallel.

*Examples:*

- Identify all pairs of the given type.

A) Corresponding

$\angle t$  and  $\angle o$ ,  $\angle w$  and  $\angle a$

$\angle n$  and  $\angle l$ ,  $\angle g$  and  $\angle e$

B) Alternate interior

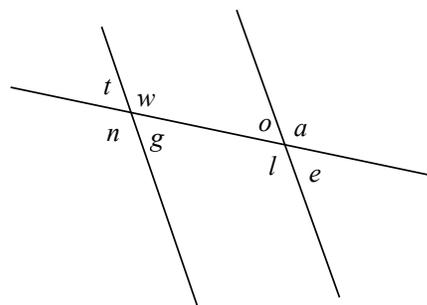
$\angle w$  and  $\angle l$ ,  $\angle g$  and  $\angle o$

C) Alternate exterior

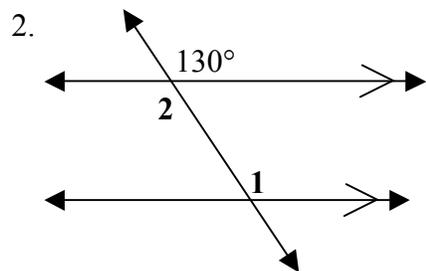
$\angle t$  and  $\angle e$ ,  $\angle n$  and  $\angle a$

D) Same-side interior

$\angle w$  and  $\angle o$ ,  $\angle g$  and  $\angle l$



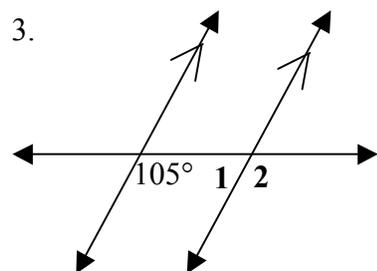
#2 - 4 (a) Find  $m\angle 1$ . Explain your reasoning. (b) Find  $m\angle 2$  two different ways. Explain both methods.



(a)  $\angle 1$  and the given angle are corresponding angles. Since lines are parallel, corresponding angles are congruent, so  $m\angle 1 = 130^\circ$

(b)  $\angle 2$  and the given angle are vertical angles. Since vertical angles are congruent,  $m\angle 2 = 130^\circ$ .

Also,  $\angle 1$  and  $\angle 2$  are alternate interior angles. Since the lines are parallel, alternate interior angles are congruent.

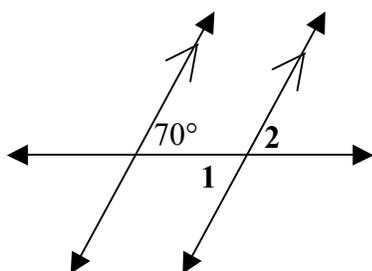


(a)  $\angle 1$  and the given angle are same-side interior angles. Since the lines are parallel, the angles are supplementary so  $m\angle 1 = 180^\circ - 105^\circ = 75^\circ$

(b)  $\angle 1$  and  $\angle 2$  are a linear pair, so they are supplementary. Therefore  $m\angle 2 = 180^\circ - 75^\circ = 105^\circ$

Also, the given angle and  $\angle 2$  are corresponding angles. Since the lines are parallel, they are congruent.

#### 4. YOU TRY



(a)  $\angle 1$  and the given angle are alternate interior angles. Since the lines are parallel, the angles are congruent so  $m\angle 1 = 70^\circ$

(b)  $\angle 1$  and  $\angle 2$  are vertical angles, so they are congruent. Therefore,  $m\angle 2 = 70^\circ$

Also, the given angle and  $\angle 2$  are corresponding angles. Since the lines are parallel, they are congruent.