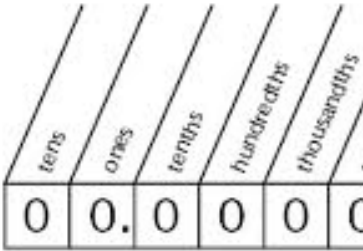


Converting decimals into fractions

Objective: Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number. (SNS 1.2)



Remind students about the names of the place values to the right of the decimal, how to verbally express decimals in word form, and how they are written as fractions.

Decimal	Say (Word Form)	Fraction	Fraction with Powers of 10
0.1	one tenth	$\frac{1}{10}$	$\frac{1}{10^1}$
0.01	one hundredth	$\frac{1}{100}$	$\frac{1}{10^2}$
0.001	one thousandth	$\frac{1}{1000}$	$\frac{1}{10^3}$

ASK: "What do you notice about the number of decimal places and the denominators in each fraction?"

[The number of decimal places is the power of 10 in the denominator: i.e. two decimal places => $10^2 = 100$]

Example 1: (Model with direct instruction)

Express 0.8 as a fraction in simplest form.

Simplify with Prime Factorization

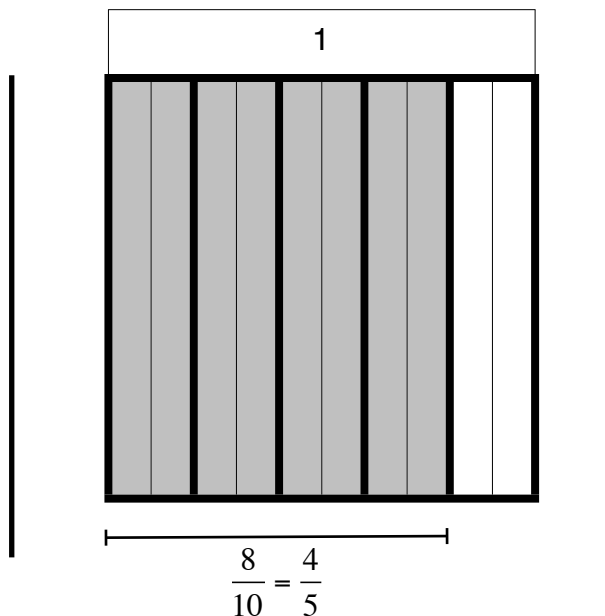
Model with Base-10 Blocks

Simplify with Greatest Common Factor

$$0.8 = \frac{8}{10}$$

$$0.8 = \frac{2 \cdot 2 \cdot 2}{2 \cdot 5}$$

$$0.8 = \frac{4}{5}$$



$$0.8 = \frac{8}{10}$$

$$0.8 = \frac{8}{10} \div \frac{2}{2} \quad \text{GCF} = 2$$

$$0.8 = \frac{4}{5}$$

Example 2: (Model with direct instruction)

Express 0.08 as a fraction in simplest form.

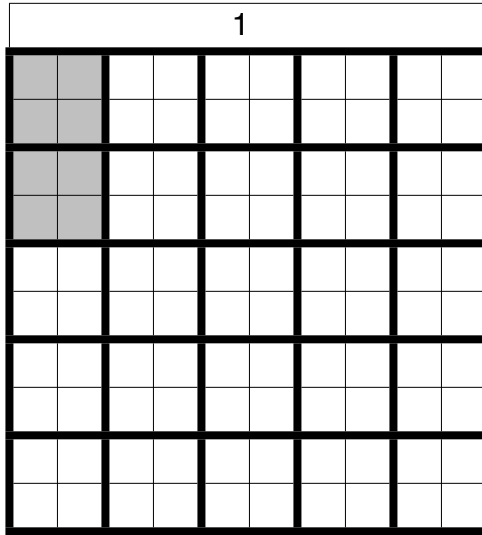
Simplify with Prime Factorization

$$0.08 = \frac{8}{100}$$

$$0.08 = \frac{\cancel{2} \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot 5 \cdot 5}$$

$$0.08 = \frac{2}{25}$$

Model with Base-10 Blocks



$$\frac{8}{100} = \frac{2}{25}$$

Simplify with Greatest Common Factor

$$0.08 = \frac{8}{100}$$

$$0.08 = \frac{8}{100} \div \frac{4}{4} \text{ GCF} = 4$$

$$0.08 = \frac{2}{25}$$

ASK: What do you notice about the two different models for 0.8 and 0.08?
 [Accept any reasonable responses. Sample: 0.08 is much smaller than 0.8.]

Example 3: (You Try!)

Express 0.5 as a fraction in simplest form.

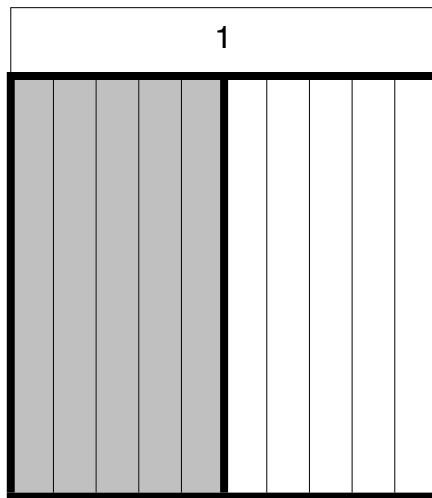
Simplify with Prime Factorization

$$0.5 = \frac{5}{10}$$

$$0.5 = \frac{\cancel{5}}{\cancel{2} \cdot 5}$$

$$0.5 = \frac{1}{2}$$

Model with Base-10 Blocks



$$\frac{5}{10} = \frac{1}{2}$$

Simplify with Greatest Common Factor

$$0.5 = \frac{5}{10}$$

$$0.5 = \frac{5}{10} \div \frac{5}{5} \text{ GCF} = 5$$

$$0.5 = \frac{1}{2}$$

Example 4: (You Try!)

Express 0.28 as a fraction in simplest form.

Simplify with Prime Factorization

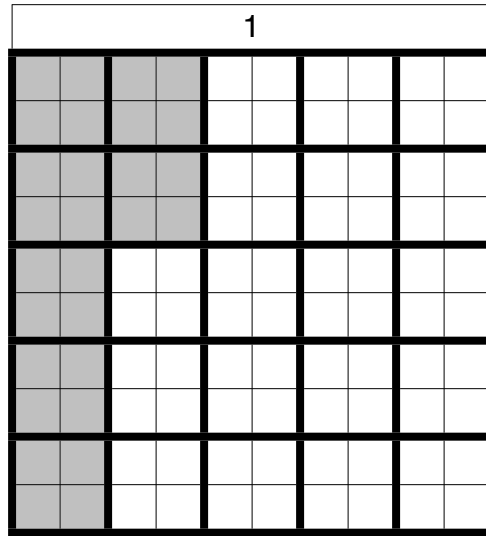
Model with Base-10 Blocks

Simplify with Greatest Common Factor

$$0.28 = \frac{28}{100}$$

$$0.28 = \frac{\cancel{2} \cdot \cancel{2} \cdot 7}{\cancel{2} \cdot \cancel{2} \cdot 5 \cdot 5}$$

$$0.28 = \frac{7}{25}$$



$$0.28 = \frac{28}{100}$$

$$0.28 = \frac{28}{100} \div \frac{4}{4} \text{ GCF} = 4$$

$$0.28 = \frac{7}{25}$$

$$\frac{28}{100} = \frac{7}{25}$$

CST Released Test Questions: (Once students understand the conceptual model, move away from it and use only as needed to scaffold the concept.)

What is the decimal 0.4 written as a fraction?

What is the decimal 0.48 written as a fraction?

Simplify with Prime Factorization

Simplify with Greatest Common Factor

Simplify with Prime Factorization

Simplify with Greatest Common Factor

$$0.4 = \frac{4}{10}$$

$$0.4 = \frac{\cancel{2} \cdot 2}{\cancel{2} \cdot 5}$$

$$0.4 = \frac{2}{5}$$

GCF = 2

$$0.4 = \frac{4}{10}$$

$$0.4 = \frac{4}{10} \div \frac{2}{2}$$

$$0.4 = \frac{2}{5}$$

$$0.48 = \frac{48}{100}$$

$$0.48 = \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 3}{\cancel{2} \cdot \cancel{2} \cdot 5 \cdot 5}$$

$$0.48 = \frac{12}{25}$$

GCF = 4

$$0.48 = \frac{48}{100}$$

$$0.48 = \frac{48}{100} \div \frac{4}{4}$$

$$0.48 = \frac{12}{25}$$

Converting Fractions to Decimals

When the denominator is a 10, 100, or 1,000:

$$0.7 = \frac{7}{10}$$

$$0.81 = \frac{81}{100}$$

$$0.009 = \frac{9}{1000}$$

$$0.407 = \frac{407}{1000}$$

Remind students that when we convert decimals to fractions, the number of decimal places reveals the value of the denominator. The same is true when converting fractions to decimals.

The denominator informs us how many decimal places our number will contain. $1000 = 10^3$, therefore our answer has three (3) decimal places.

Therefore:

$$\frac{8}{10} = 0.8$$

$$\frac{25}{100} = 0.25$$

$$\frac{276}{1000} = 0.276$$

$$\frac{17}{10} = 1.7$$

Making equivalent fractions:

$$\frac{2}{5} = \frac{2 \cdot 2}{5 \cdot 2}$$

$$= \frac{4}{10}$$

$$= 0.4$$

$\frac{2}{2} = 1$, therefore according to the **Identity Property of Multiplication** the value of $\frac{2}{5}$ remains the same.

Other examples:

$$\frac{3}{50} = \frac{3 \cdot 2}{50 \cdot 2}$$

$$= \frac{6}{100}$$

$$= 0.06$$

$$\frac{6}{25} = \frac{6 \cdot 4}{25 \cdot 4}$$

$$= \frac{24}{100}$$

$$= 0.24$$

$$\frac{5}{4} = \frac{5 \cdot 25}{4 \cdot 25}$$

$$= \frac{125}{100}$$

$$= 1.25$$

$$\frac{7}{250} = \frac{7 \cdot 4}{250 \cdot 4}$$

$$= \frac{28}{1000}$$

$$= 0.028$$

CST Released Test Question:

What decimal is equal to $\frac{3}{5}$?

- A) 0.30
- B) 0.35
- C) 0.60
- D) 1.67

Solution:

$$\frac{3}{5} = \frac{3 \cdot 20}{5 \cdot 20}$$

$$= \frac{60}{100}$$

$$= 0.60$$

Using division:

$$\frac{5}{8} = 5 \div 8$$

$$\begin{array}{r} 0.625 \\ 8 \overline{)5.000} \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

The Secret to Dividing with Decimals
Adding zeros after (or to the right) of a decimal point does not change a number's value!

So, $\frac{5}{8} = 0.625$

Other examples:

$$\frac{15}{9} = \frac{3 \cdot 5}{3 \cdot 3} = 5 \div 3$$

$$\begin{array}{r} 1.\overline{66} \\ 3 \overline{)5.00} \\ \underline{-3} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

So, $\frac{15}{9} = 1.\overline{66}$

$$\begin{aligned} 2\frac{2}{5} &= 2 + \frac{2}{5} \\ &= 2 + (2 \div 5) \\ &= 2 + 0.4 \\ &= 2.4 \end{aligned}$$

$$\begin{array}{r} 0.4 \\ 5 \overline{)2.0} \\ \underline{-20} \\ 0 \end{array}$$

So, $2\frac{2}{5} = 2.4$

Converting Decimals and Fractions into Percents

Decimal-to-percent conversions are made by moving the decimal point two places to the right.

$$0.15 = 15\%$$

$$2.97 = 297\%$$

$$0.0043 = 0.43\%$$

The Math Behind Moving the Decimal
 Per cent means "per hundred" in Latin. So, when we multiply a number by 100, the decimal moves two places to the right and we get our percent.

Note: 0.43% is less than 1% and *should not be confused* with 43% which is 0.43 as a decimal!

Converting **fractions-to-percents** is actually a two step process. First convert the fraction into a decimal, then convert the equivalent decimal into a percent as demonstrated in the above procedure.

$$\frac{1}{4} = 1 \div 4 \longrightarrow \begin{array}{r} 0.25 \\ 4 \overline{)1.00} \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array} \longrightarrow \frac{1}{4} = 0.25 = 25\%$$

$0.25 \cdot 100 = 25$

$$\frac{3}{2} = 3 \div 2 \longrightarrow \begin{array}{r} 1.5 \\ 2 \overline{)3.0} \\ \underline{-2} \\ 10 \\ \underline{-10} \\ 0 \end{array} \longrightarrow \frac{3}{2} = 1.5 = 150\%$$

$1.5 \cdot 100 = 150$

CST Released Test Question:

A company donated 200 books to a local library. If 70 of them were fiction, what percent of the donated books are fiction?

- A) 35%
- B) 40%
- C) 60%
- D) 65%

Solution:

$$\begin{aligned} \frac{\text{fiction}}{\text{all}} &= \frac{70}{200} \\ &= \frac{7 \cdot 10}{20 \cdot 10} \\ &= \frac{7}{20} \\ &= 7 \div 20 \end{aligned}$$

$$\begin{array}{r} 0.35 \\ 20 \overline{)7.00} \\ \underline{-60} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

So, $\frac{7}{20}$, $0.35=35\%$.

A is the correct answer.