Multiplying Fractions Parents' Guide

Teaching Mathematics That Makes Sense

Multiplying Fractions

As with anything in life, there are often many ways to do something. Some ways are more efficient than others. This is true of mathematics in general and specifically with multiplying fractions.

Often students are taught to multiply fractions by simply "multiply across" – multiply the **numerators** and then multiply the **denominators**. Many textbooks provide a "general rule" for multiplying fractions:

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{a \bullet c}{b \bullet d}$$

Unfortunately this is just a rule and although it works it is most often the least efficient way to multiply fractions and get the simplified answer. Look what happens in this example:

$$\frac{2}{3} \bullet \frac{9}{8}$$

$$= \frac{2 \bullet 9}{3 \bullet 8}$$ multiplying numerators & multiplying denominators

$$= \frac{18}{24}$$ now you must simplify the fraction

$$= \frac{18 \div 6}{24 \div 6}$$ There are better ways to simplify fractions – see the Simplifying Fractions Parents' Guide.

$$= \frac{3}{4}$$

Although this works it is not very efficient. You multiplied the numerators and denominators together which made the numbers bigger and then you had to turn around and make the numbers smaller to simplify the fraction.

Another way students are often shown how to multiply fractions is to do what is frequently called "cross cancel." This is a short cut which is fine to use if students understand the short cut. Otherwise students will often use the short cut incorrectly.

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Multiplying Fractions by "Cross Canceling"

Here is how cross canceling is often shown:

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \bullet \frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{8}}}$$

$$= \frac{3}{4}$$

Several problems when using this technique are:

- Often students think that you can only cancel if the numbers are across from each other.
- Students try to use this technique with adding fractions and it doesn't work with adding fractions.
- Students don't understand what is meant by "canceling" and they often cancel everything because it is easier to cancel something than to deal with it appropriately.

It is fine to show the short cut once students understand the mathematics. If students are taught how to multiply fractions by decomposing the numbers as they multiply, it makes the work easier and more efficient, students will be able to better understand cross canceling, and they will easily transfer their knowledge to more difficult problems and to algebraic fractions.

Multiplying Fractions by Decomposition

$$\frac{2}{3} \bullet \frac{9}{8}$$

$$= \frac{2 \bullet 3 \bullet 3}{3 \bullet 2 \bullet 2 \bullet 2}$$

multiplying numerators & multiplying denominators by using decomposition as you multiply

$$= \frac{2 \bullet \quad 3 \bullet 3}{2 \bullet 2 \bullet 2 \bullet 3}$$

using the commutative property of multiplication to line up common factors – this step is not necessary but a good way to show it at first

$$= \frac{\boxed{2} \bullet \quad 3 \bullet \boxed{3}}{\boxed{2} \bullet 2 \bullet 2 \bullet \boxed{3}}$$

using the multiplicative identity to "take the ones out" leaves the simplified fraction – the answer.

$$= \frac{3}{4}$$

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A Few More Examples

$$\frac{4}{3}\bullet\frac{9}{8}$$

$$=\frac{2\bullet 2\bullet 3\bullet 3}{3\bullet 2\bullet 2\bullet 2}$$

$$=\frac{2\bullet 2\bullet 3\bullet 3}{2\bullet 2\bullet 2\bullet 3}$$

$$=\frac{3}{2}$$

$$\frac{4}{3}\bullet\frac{9}{8}$$

$$=\frac{\cancel{2}\bullet\cancel{2}\bullet\cancel{3}\bullet 3}{\cancel{3}\bullet\cancel{2}\bullet\cancel{2}\bullet 2}$$

$$=\frac{3}{2}$$

Rather than rewriting the factors in order to "take out" the big ones, we usually just cross off the factors that are common in both the numerator and denominator. In mathematics, we call these common factors "equivalent forms of 1."

$$\frac{2}{3}\bullet\frac{5}{8}\bullet\frac{9}{10}$$

If you multiplied numerators and multiplied denominators you would get $\frac{90}{240}$ then you would have to simplify that fraction.

$$=\frac{2\bullet 5\bullet 3\bullet 3}{3\bullet 2\bullet 4\bullet 2\bullet 5}$$

$$=\frac{\cancel{2}\bullet\cancel{5}\bullet\cancel{3}\bullet 3}{\cancel{3}\bullet\cancel{2}\bullet 4\bullet 2\bullet\cancel{5}}$$

Now using the multiplicative identity you can "take out" your equivalent forms of one to simplify the fraction.

$$=\frac{3}{8}$$

This technique (decomposition) works for algebraic fractions too.

$$\frac{2x^2}{3y^2}\bullet\frac{9y^3}{8x^5}$$

$$=\frac{\cancel{2}\bullet\cancel{x}\bullet\cancel{x}}{\cancel{3}\bullet\cancel{y}\bullet\cancel{y}}\bullet\frac{\cancel{3}\bullet 3\bullet\cancel{y}\bullet\cancel{y}\bullet y}{\cancel{2}\bullet 4\bullet\cancel{x}\bullet\cancel{x}\bullet x\bullet x\bullet x}$$

$$=\frac{3y}{4x^3}$$