Grade Level/Course: Math 8

Lesson/Unit Plan Name: Solving Linear Equations with One Solution, No Solutions and Infinitely Many Solutions.

Rationale/Lesson Abstract: The objective of this lesson is to give students an understanding of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Students will be able to write an equation that can be used to determine the number of solutions of the form $x=a$, $a=a$, or $a=b$.

Timeframe: 2-3 60 minute periods

Common Core Standard(s):

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x=a$, $a=a$, or $a=b$ results (where a and b are different numbers).

Instructional Resources/Materials:

Paper, Pencil, Warm-up (p.18-19), Index Cards (p.9), Graphic Organizer (p.2-8) and Answer Keys (p.9-17)

Activity/Lesson:

Solving Linear Equations with one solution, no solutions and infinitely many solutions.

	One Solution $x = a$	No Solutions $a = b$	Infinitely Many Solutions $x = x$
Equation	$7x - 3 = 5x + 5$	$7x - 3 = 7x + 5$	$7x - 3 = -3 + 7x$
Use properties of equality **"Solve"**			
Check your solution			
End Results			

YOU TRY!!!

	One Solution $a = a$	**No Solutions** $a = b$	**Infinitely Many Solutions** $x = x$
Equation	$3(y + 2) = 30$	$5(2 + c) = 45 + 5c$	$2(a - 2) = 2a - 4$
Use properties of equality **"Solve"**			
Check your solution			
End Results			

Identify linear equation solutions without solving

	One Solution $x = a$	**No Solutions** $a = b$	**Infinitely Many Solutions** $x = x$
Equation	$7x - 3 = 5x + 5$	$7x - 3 = 7x + 5$	$7x - 3 = -3 + 7x$
What can you say about the <u>coefficients of the terms with x</u> in the equation?			
What can you say about the <u>constants on both sides</u> of the equation?			
In general,			

Quick Write

What does it mean to have ***one solution*** in a linear equation?

What does it mean to have ***no solutions*** in a linear equation?

What does it mean to have ***infinitely many solutions*** in a linear equation?

Write an equation with ***one solution***, ***no solutions*** ***and infinitely many solutions***.

	One Solution $x = a$	**No Solutions** $a = b$	**Infinitely Many Solutions** $x = x$
Begin with	**Solution: Variable = Constant**	**False Statement: Constant = Different Constant**	**True Statement: Constant = Constant**
Use properties of equality	add, subtract, multiply or divide by the same **number** on both sides	add, subtract, multiply or divide by the same **variable** on both sides	add, subtract, multiply or divide by the same **variable** on both sides
Make another change using the distributive property	add, subtract, multiply or divide by the **another number** on both sides	add, subtract, multiply or divide by the same **number** on both sides	add, subtract, multiply or divide by the same **number** on both sides
End Results			

SPECIAL CASE

$x = 0$ is a ***one solution*** answer

	One Solution $x = a$	**One Solution** $x = a$	**One Solution** $x = a$
Equation	$2x + 12 = x + 12$	You Try! $3y - 7 = y - 7$	**True Statement: Variable = Zero**
Use properties of equality **"Solve"**			add, subtract, multiply or divide by the same **variable** on both sides
Check your solution			add, subtract, multiply or divide by the same **number** on both sides
End Results			

Note: If the coefficients of x are different and the value of the constants are the same, the ONLY SOLUTION IS $x = 0$.

Quick Write

What did you learn about linear equations today?

Activity:

Step 1: Write Linear Equations on Index cards or posters.

Step 2: Have students place each linear equation in the correct category of one solution, no solutions and infinitely many solutions.

One Solution	No Solutions	Infinitely many solutions
$3x + 4 = 8x - 9$	$6x + 5 = 8 + 6x$	$10x - 4 = -4 + 10x$
$-4x - 5 = 6 - 11x$	$12 - 15x = -2 - 15x$	$-2x + 5 = -2x + 5$
$9 + \frac{1}{2}x = 5x - 1$	$\frac{5}{4}x - 1 = 1 + \frac{5}{4}x$	$7 + 9x = 9x + 7$

Option #2 Activity: Cut out boxes on this page and mix up answers for students

One Solution	No Solutions	Infinitely many solutions
$3x + 4 = 8x - 9$	$6x + 5 = 8 + 6x$	$10x - 4 = -4 + 10x$
$-4x - 5 = 6 - 11x$	$12 - 15 = -2 - 15x$	$-2x + 5 = -2x + 5$
$9 + \frac{1}{2}x = 5x - 1$	$\frac{5}{4}x - 1 = 1 + \frac{5}{4}x$	$7 + 9x = 9x + 7$

Solving Linear Equations with one solution, no solutions and infinitely many solutions.

	One Solution $x = a$	**No Solutions** $a = b$	**Infinitely Many Solutions** $x = x$
Equation	$7x - 3 = 5x + 5$	$7x - 3 = 7x + 5$	$7x - 3 = -3 + 7x$
Use properties of equality **"Solve"**	$7x - 3 = 5x + 5$ $-5x + 2x - 3 = -5x + 5$ $2x - 3 = 5$ $2x - 3 = 5 - 3 + 3$ $2x = 8$ $x + x = 4 + 4$ $\boxed{x = 4}$	$7x - 3 = 7x + 5$ $-7x - 3 = -7x + 5$ $\boxed{-3 = 5}$	$7x - 3 = -3 + 7x$ $7x - 3 = -3 + 7x$ $7x = 7x$ $\boxed{x = x}$
Check your solution	$7x - 3 = 5x + 5$ $7(4) - 3 = 28 - 3 = (25)$ | $5(4) + 5 = 20 + 5 = (25)$ ☑	$7x - 3 = 7x + 5$ $7(-3) - 3 = -21 - 3 = (-24)$ | $7(-3) + 5 = -21 + 5 = (-16)$ False!	$7x - 3 = -3 + 7x$ $7(7) - 3 = 49 - 3 = (46)$ | $-3 + 7(7) = -3 + 49 = (46)$ ☑
End Results	The statement $x = 4$ is true. ∴ The equation has one solution. The value of the variable is 4.	The statement $-3 = 5$ is false. ∴ The equation has no solutions. There are no values of the variable that will make the equation true.	The statement $x = x$ is always true. ∴ The equation has infinitely many solutions. Any value of the variable is true.

YOU TRY!!!

	One Solution $x = a$	**No Solutions** $a = b$	**Infinitely Many Solutions** $x = x$
Equation	$3(y+2) = 30$	$5(2+c) = 45 + 5c$	$2(a-2) = 2a - 4$
Use properties of equality **"Solve"**	$3(y+2) = 30$ $\frac{3(y+2)}{3} = \frac{30}{3}$ $y + 2 = 10$ $y + \cancel{2} = 8 + \cancel{2}$ $\boxed{y = 8}$	$5(2+c) = 45 + 5c$ $5(2) + 5(c) = 45 + 5c$ $10 + 5c = 45 + 5c$ $10 + \cancel{5c} = 45 + \cancel{5c}$ $\boxed{10 = 45}$	$2(a-2) = 2a - 4$ $2a - 4 = 2a - 4$ $\cancel{2a} - 4 = \cancel{2a} - 4$ $\boxed{-4 = -4}$
Check your solution	$3(y+2) = 30$ $3(8+2) \mid = 30$ $= 3(8) + 3(2) \mid = 30$ $= 24 + 6 \mid = 30$ $= (30) \mid = (30)$ ☑	$5(2+c) = 45 + 5c$ $5(2+10) \mid 45 +$ $5(10)$ $= 5(12) \mid = 45 + 50$ $= (60) \mid = (95)$ False	$2(a-2) = 2a - 4$ $2(-4-2) \mid 2(-4) - 4$ $= 2(-4) + 2(-2) \mid = -8 - 4$ $= -8 - 4 \mid = -8 - 4$ $= (-12) \mid = (-12)$ ☑
End Results	The statement $y = 8$ is true. ∴ The equation has one solution. The value of the variable is 8.	The statement $10 = 45$ is false. ∴ The equation has no solutions. There are no values of the variable that will make the equation true.	The statement $-4 = -4$ is always true. ∴ The equation has infinitely many solutions. Any value of the variable is true.

Identify linear equation solutions without solving

	One Solution $x = a$	**No Solutions** $a = b$	**Infinitely Many Solutions** $x = x$
Equation	$7x - 3 = 5x + 5$	$7x - 3 = 7x + 5$	$7x - 3 = -3 + 7x$
What can you say about the <u>coefficients of the terms with x</u> in the equation?	$7x$ *and* $5x$ The coefficients are 7 and 5. The coefficients are **<u>different</u>**.	$7x$ *and* $7x$ The coefficients are 7 and 7. The coefficients are the **<u>same</u>**.	$7x$ *and* $7x$ The coefficients are 7 and 7. The coefficients are the **<u>same.</u>**
What can you say about the <u>constants on both sides</u> of the equation?	-3 *and* 5 The constants are **<u>different.</u>**	-3 *and* 5 The constants are **<u>different</u>**.	-3 *and* -3 The constants are the **<u>same.</u>**
In general,	$ax + b = cx + d$ $x = a$ ∴ The equation has one solution.	$x + a = x + b$ $a = b$ ∴ The equation has no solutions.	$x + a = x + a$ $x = x$ ∴ The equation has infinitely many solutions.

Quick Write

What does it mean to have ***one solution*** in a linear equation?

Possible answer: When you solve a linear equation and get one solution, it means the value of the variable is the only unique solution. You can check this unique solution by substituting it into the original equation. You will get a true statement. An example would be $y = 8$.

What does it mean to have ***no solutions*** in a linear equation?

Possible answer: When you solve a linear equation and get no solutions, it means there are no values of x in which you can substitute into the equation and make a true statement. The statement will always be false. An example would be 10=45.

What does it mean to have ***infinitely many solutions*** in a linear equation?

Possible answer: When you solve a linear equation and get infinitely many solutions, it means you can substitute any value of x into the equation and make a always true statement. An example would be $-4 = -4$ or $x = x$.

Write an equation with ***one solution***, ***no solutions and infinitely many solutions***.

	One Solution $x = a$	**No Solutions** $a = b$	**Infinitely Many Solutions** $x = x$
Begin with	**Solution:** **Variable = Constant** $y = 8$	**False Statement:** **Constant = Different Constant** $10 = 45$	**True Statement:** **Constant = Constant** $-4 = -4$
Use properties of equality	add, subtract, multiply or divide by the same **number** on both sides $y = 8$ $y + 2 = 8 + 2$ $y + 2 = 10$	add, subtract, multiply or divide by the same **variable** on both sides $10 = 45$ $10 + 5c = 45 + 5c$	add, subtract, multiply or divide by the same **variable** on both sides $-4 = -4$ $-4 + 2a = -4 + 2a$
Make another change using the distributive property	add, subtract, multiply or divide by the **another number** on both sides $y = 8$ $y + 2 = 8 + 2$ $y + 2 = 10$ $3(y + 2) = 3(10)$	add, subtract, multiply or divide by the same **number** on both sides $10 = 45$ $10 + 5c = 45 + 5c$ $5(2 + c) = 45 + 5c$	add, subtract, multiply or divide by the same **number** on both sides $-4 = -4$ $-4 + 2a = -4 + 2a$ $2a - 4 = 2a - 4$ $2(a - 2) = 2a - 4$
End Results	$3(y + 2) = 30$	$5(2 + c) = 45 + 5c$	$2(a - 2) = 2a - 4$

SPECIAL CASE

$x = 0$ is a ***one solution*** answer

	One Solution $x = a$	**One Solution** $x = a$	**One Solution** $x = a$
Equation	$2x + 12 = x + 12$	You Try! $3y - 7 = y - 7$	**True Statement: Variable = Zero** $x = 0$
Use properties of equality **"Solve"**	$2x + 12 = x + 12$ $2x + \cancel{12} = x + \cancel{12}$ $2x = x$ $x + \cancel{x} = \cancel{x}$ $\boxed{x = 0}$	$3y - 7 = y - 7$ $3y \cancel{- 7} = y \cancel{- 7}$ $3y = y$ $y + y + \cancel{y} = \cancel{y}$ $y + y = 0$ $y + y = 0 + 0$ $\boxed{y = 0}$	add, subtract, multiply or divide <u>by the same</u> **<u>variable</u>** <u>on both sides</u> $x = 0$ $x + x = x + 0$ $2x = x$
Check your solution	$2x + 12 = x + 12$ $2(0) + 12 \mid 0 + 12$ $= 0 + 12 \mid = (12)$ $= (12)$ ☑	$3y - 7 = y - 7$ $3(0) - 7 \mid 0 - 7$ $= 0 - 7 \mid = (-7)$ $= (-7)$ ☑	add, subtract, multiply or divide <u>by the same</u> **<u>number</u>** <u>on both sides</u> $x = 0$ $x + x = x + 0$ $2x = x$ $2x + 12 = x + 12$
End Results	The statement $x = 0$ is true.∴ The equation has one solution. The value of the variable is 0.	The statement $x = 0$ is true.∴ The equation has one solution. The value of the variable is 0.	$2x + 12 = x + 12$

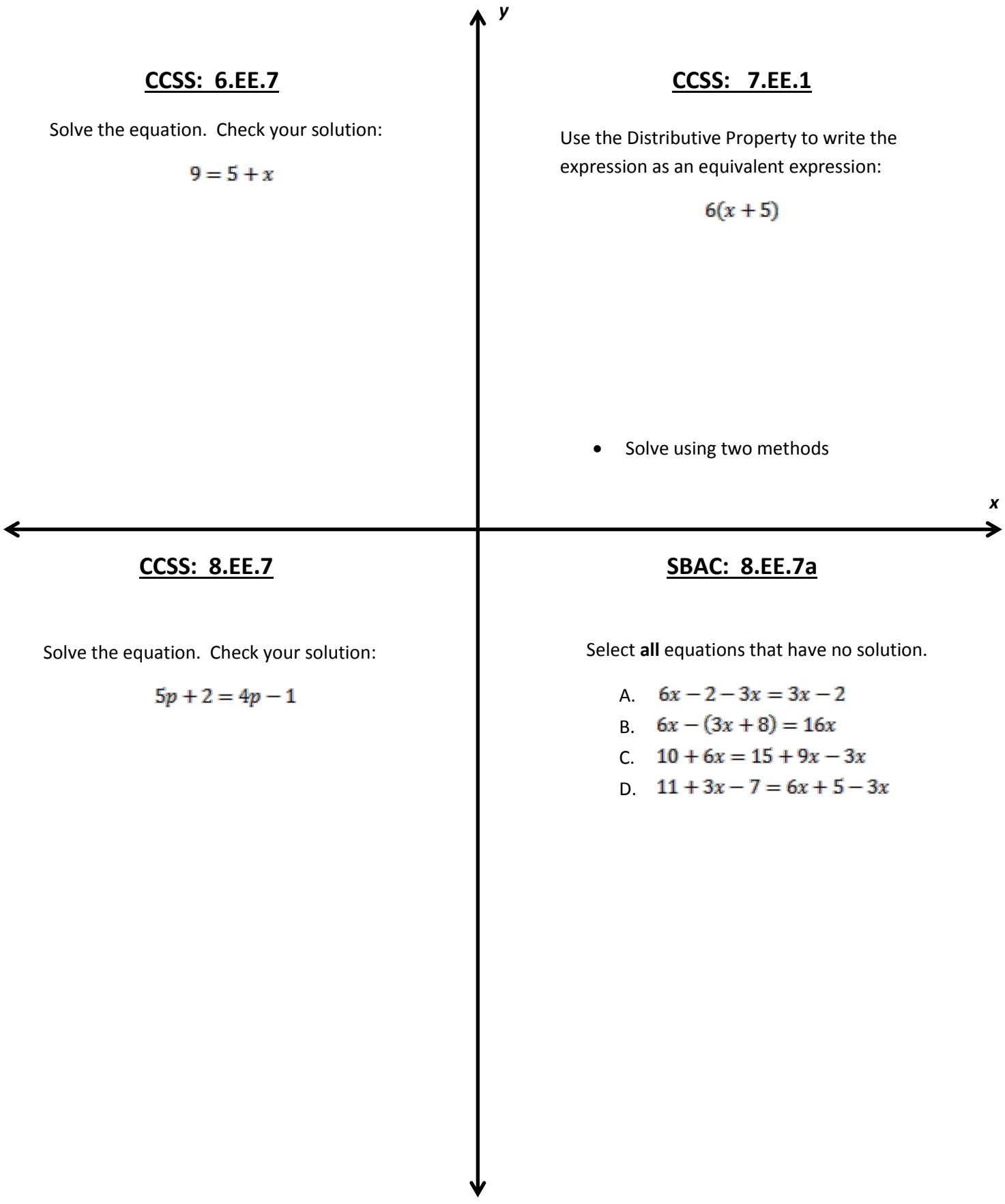
Note: If the coefficients of x are different and the value of the constants are the same, the ONLY SOLUTION IS $x = 0$.

Quick Write

What did you learn about linear equations today?

Answers may vary

Warm-Up



CCSS: 6.EE.7

Solve the equation. Check your solution:

$9 = 5 + x$

CCSS: 7.EE.1

Use the Distributive Property to write the expression as an equivalent expression:

$6(x + 5)$

- Solve using two methods

CCSS: 8.EE.7

Solve the equation. Check your solution:

$5p + 2 = 4p - 1$

SBAC: 8.EE.7a

Select **all** equations that have no solution.

A. $6x - 2 - 3x = 3x - 2$
B. $6x - (3x + 8) = 16x$
C. $10 + 6x = 15 + 9x - 3x$
D. $11 + 3x - 7 = 6x + 5 - 3x$

Warm-Up: Answer Key

CCSS: 6.EE.7

Solve the equation. Check your solution:

$9 = 5 + x$

[x=4]

CCSS: 7.EE.1

Use the Distributive Property to write the expression as an equivalent expression:

$6(x + 5)$

[6x+30]

- Solve using two methods

CCSS: 8.EE.7

Solve the equation. Check your solution:

$5p + 2 = 4p - 1$

[p=-3]

SBAC: 8.EE.7a

Select **all** equations that have no solution.

A. $6x - 2 - 3x = 3x - 2$
B. $6x - (3x + 8) = 16x$
C. $10 + 6x = 15 + 9x - 3x$
D. $11 + 3x - 7 = 6x + 5 - 3x$

[C and D]