**Grade Level/Course:** 7th grade Mathematics

**Lesson/Unit Plan Name:** Complex Fractions

**Rationale/Lesson Abstract:** For the first time, 7th graders are being asked to work with complex fractions.

**Timeframe:** Introduction to complex fractions 2 days (depending on amount of time in a period, and the level of comfort with division of fractions). Other uses of complex fractions could follow as 1-2 days.

**Common Core Standard(s): 7NS33 and 7RP1**

CCSS.Math.Content.7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

CCSS.Math.Content.7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour*.

**Instructional Resources/Materials:**
Student handouts, document camera if available to show student work

Activity/Lesson:

Have you ever seen something that looked really good, but then tasted it and it was terrible? Or have you ever seen something that looked nasty, like some of the health food drinks, then tasted it and it was really good?

Complex fractions look hard, but they are just a division problem involving fractions. Give students time to copy the following.
Notes: A complex fraction is a fraction that has a fraction as its numerator, denominator, or both. With a complex fraction, you merely have a division of fractions problem written vertically rather than horizontally.
After they have finished copying, have the class do a choral read of the three sentences. You can then, with equity sticks or other random methods, call on two-three individual students to repeat what is stated, but in their own words.

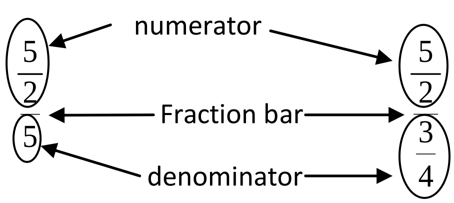
Show the example, and then have students quickly do the two you-trys on a white board at their seats, or in their notes.

| Example: | You try1: | You try2: |
|---|---|---|
| $\frac{\frac{5}{2}}{\frac{3}{4}}$ vertically presented | $\frac{\frac{2}{3}}{\frac{3}{4}}$ vertically presented | ? vertically presented |
| $=\frac{5}{2}\div\frac{3}{4}$ horizontally presented | $=$ ? horizontally presented | $=\frac{4}{7}\div\frac{7}{12}$ horizontally presented |

Ask: What in the past have you seen presented both in vertical and horizontal form?

$3+4$ vs. $\begin{array}{r} 3 \\ +4 \\ \hline \end{array}$ $3\times 4$ vs. $\begin{array}{r} 3 \\ \times 4 \\ \hline \end{array}$ So now you know something else that can be presented in either vertical or horizontal form, division of fractions.

Notes: The following are two more examples of complex fractions.




Say "The first example is read as five-halves divided by 5. It asks how many units of 5 fit into $\frac{5}{2}$ ?"

Repeat, but have students read with you. Ask, what is the question? Have students in choral response answer.
Have them put what you just had them read in their notes.

Say "The second example is read five-halves divided by three-fourths. It asks how many $\frac{3}{4}$'s fit into $\frac{5}{2}$ ?"

Repeat, but have students read with you. Ask, what is the question? Have students in choral response answer.
Have them put what you just had them read in their notes.

Partner talk: With a partner, one person reads the first problem, identifies the numerator, and identifies the denominator. He/she also identifies the question being asked. The second person does the same for the second expression.
Expression 1 can be put on the board, or the overhead, and Partner A does as instructed. Repeat with Expression 2 and Partner B.

Expression 1: $\dfrac{3}{\frac{2}{5}}$ Expression 2: $\dfrac{\frac{2}{7}}{\frac{5}{9}}$

Ask: How many mathematical symbols do we have to show addition? [*One, the plus sign*] How many ways do we have to show subtraction? [*One, the minus sign.*] How many ways do we have to show multiplication? [*There are several. The x, the dot, no sign, parenthesis*.]
Ask, how many ways do we have to show division*?*[*Most students will identify the two ways, and some may realize the fraction bar is also a division sign.*]

Notes: Division is shown by $\overline{)\quad}$ symbol, by the $\div$ symbol, and by the fraction bar.

How could I write eight divided by 11 in three ways? $11\overline{)8}$, $8 \div 11$, and $\dfrac{8}{11}$.

You do1: Write five divided by two in three ways.

You do2: Write two-fifths divided by one-half in three ways.

Ask: Do we use the $\overline{)\quad}$ symbol for fractions? *[Not usually, unless we write the fractions as decimals.]*
When we write division of fractions, we will either use the $\div$sign, or write a complex fraction. So, for division of fractions, we will just use two ways.

Write $\dfrac{3}{4} \div \dfrac{1}{2}$ on the board or the overhead. Write $\dfrac{\frac{3}{4}}{\frac{1}{2}}$ side by side to the horizontal presentation.

Partner talk: Partner A states three things that are the same about the problem. Partner B, states three things that are different about the problem.
Have some of the students share out. Possible responses:
*Same problem, different look. Both ask the same question, how many halves fit into three-fourths. One uses the division sign, one uses the fraction bar to indicate the operation of division. The first is written horizontally, the second vertically.*
Ask: Is the question the same in either presentation? [Yes.] What is the question? Choral response [*How many halves fit into* $\dfrac{3}{4}$ *?*]
Which is the larger number? The numerator or denominator? So do you expect an answer greater than 1 or less than 1? You may do an example with integers, so the students can revisit the concept of dividing a larger number by a smaller one vs. dividing a smaller number by a larger one.

Now, let's look at the "mechanics" of simplifying a complex fraction.

Three methods to simplify the complex fraction are shown, <u>all using the first step as transforming the vertical presentation (the complex fraction) into a horizontal writing of the division problem</u>. These can be shown on the overhead allowing students to discuss the three methods with partner talk.

**Multiplying by The reciprocal**

$$\frac{\frac{3}{4}}{\frac{1}{2}}$$
$$=\frac{3}{4}\div\frac{1}{2}$$
$$=\frac{3}{4}\cdot\frac{2}{1}$$
$$=\frac{3\cdot 2}{2\cdot 2}$$
$$=\frac{3}{2}$$
$$=1\frac{1}{2}$$

**Dividing straight across the numerator and denominator**

$$\frac{\frac{3}{4}}{\frac{1}{2}}$$
$$=\frac{3}{4}\div\frac{1}{2}$$
$$=\frac{3\div 1}{4\div 2}$$
$$=\frac{3}{2}$$
$$=1\frac{1}{2}$$

**Establishing a common denominator then dividing the numerators and denominators**

$$\frac{\frac{3}{4}}{\frac{1}{2}}$$
$$=\frac{3}{4}\div\frac{1}{2}$$
$$=\frac{3}{4}\div\frac{1}{2}\left(\frac{2}{2}\right)$$
$$=\frac{3}{4}\div\frac{2}{4}$$
$$=\frac{3\div 2}{4\div 4}$$
$$=\frac{\frac{3}{2}}{1}$$
$$=\frac{3}{2}$$

Debrief each method. For this problem, which method was easier? [This is student preference with no "right" or "wrong" answer.]

State and have students write in their notes:

We do <u>not</u> have to change the vertical form to a horizontal form in order to perform the division. We can simply work with the complex fraction and the concept of the reciprocal. Here is a method working with the problem as a complex fraction. Show method 1. (Method 2 is a preview of where students will go in upper level math.)

**Method 1**

Multiplying by a form of 1, $\left(\frac{\frac{2}{1}}{\frac{2}{1}}\right)$ in this case.

$$\frac{\frac{3}{4}\cdot\left(\frac{2}{1}\right)}{\frac{1}{2}\cdot\left(\frac{2}{1}\right)}$$
$$=\frac{\frac{3}{4}\cdot\left(\frac{2}{1}\right)}{1}$$
$$=\frac{3\cdot 2}{2\cdot 2}$$
$$=\frac{3}{2}$$

**Method 2** (This is a method that will be expected in algebra2 and pre-calculus)

Multiplying by a form of 1, $\left(\frac{2}{2}\right)$ in this case

$$\frac{\frac{3}{4}}{\frac{1}{2}}\left(\frac{2}{2}\right)$$
$$=\frac{3\cdot 2}{2\cdot 2}$$
$$=\frac{3}{2}$$

Show and have students compare the horizontal and the vertical methods for the same problem:

**Horizontal Method**
Multiplying by the the reciprocal

$$\frac{\frac{3}{4}}{\frac{1}{2}}$$
$$=\frac{3}{4}\div\frac{1}{2}$$
$$=\frac{3}{4}\cdot\frac{2}{1}$$
$$=\frac{3\cdot 2}{2\cdot 2}$$
$$=\frac{3}{2}$$

**Vertical Method**
Multiplying by the reciprocal

$$\frac{\frac{3}{4}}{\frac{1}{2}}\cdot\frac{\left(\frac{2}{1}\right)}{\left(\frac{2}{1}\right)}$$
$$=\frac{\frac{3}{4}\cdot\left(\frac{2}{1}\right)}{1}$$
$$=\frac{3\cdot 2}{2\cdot 2}$$
$$=\frac{3}{2}$$

**Vertical Method**
Multiplying by a form of 1.

$$\frac{\frac{3}{4}}{\frac{1}{2}}\left(\frac{2}{2}\right)$$
$$=\frac{3\cdot 2}{2\cdot 2}$$
$$=\frac{3}{2}$$

**Horizontal Method**
Getting a common denominator

$$\frac{\frac{3}{4}}{\frac{1}{2}}$$
$$=\frac{3}{4}\div\frac{1}{2}$$
$$=\frac{3}{4}\div\frac{1}{2}\left(\frac{2}{2}\right)$$
$$=\frac{3}{4}\div\frac{2}{4}$$
$$=\frac{3\div 2}{4\div 4}$$
$$=\frac{\frac{3}{2}}{1}$$
$$=\frac{3}{2}$$

Allow student discussion. Go through each step and identify what is happening. Students could indicate which the easier method is for them from the above.

*Comment: If the answer is an improper fraction, students certainly may transform the improper fraction into a mixed number. Later, in algebra and above, students will be encouraged to leave the number as an improper fraction rather than express it as a mixed number. The important piece here is to understand that an improper fraction (or any fraction) is in simplest form if there are no factors in common between the numerator and the denominator other than 1.*

$\frac{3}{2}$ *is simplified. Transforming to a mixed number is <u>not</u> simplifying. It is merely expressing the improper fraction in an alternate form.*

We do: Students should write the example in their notes.

Example: $\dfrac{\frac{5}{18}}{\frac{1}{3}}$  Ask: What is the question? [*How many* $\frac{1}{3}$ *'s are in* $\frac{5}{18}$ *?*]

Ask: Do we expect an answer greater than one or less than one? [*Let kids predict.*] Why? Which is larger, the numerator of the complex fraction or the denominator of the complex fraction? [The *denominator, so a quotient < 1 is expected.*]

Let's work with the division problem in <u>vertical form</u>. (You may need to discuss how to check a division problem.)

$$\frac{\frac{5}{18}}{\frac{1}{3}} \bullet \frac{\left(\frac{3}{1}\right)}{\left(\frac{3}{1}\right)}$$
$$= \frac{5 \cdot 3}{6 \cdot 3}$$
$$= \frac{5}{6}$$

So, $\frac{5}{6}$ of $\frac{1}{3}$ fits into $\frac{5}{18}$. Check your answer:

$$\frac{5}{6} \cdot \frac{1}{3}$$
$$= \frac{5 \cdot 1}{6 \cdot 3}$$
$$= \frac{5}{18}$$

**You try**: Simplify: $\dfrac{\frac{9}{20}}{\frac{3}{5}}$  Interpret your answer. Check your answer.

$$\frac{\frac{9}{20}}{\frac{3}{5}}$$
$$= \frac{\frac{9}{20}}{\frac{3}{5}} \cdot \frac{\left(\frac{5}{3}\right)}{\left(\frac{5}{3}\right)}$$
$$= \frac{3 \cdot 3 \cdot 5}{3 \cdot 4 \cdot 5}$$
$$= \frac{3}{4}$$

So three-fourths of three-fifths fits into $\frac{9}{20}$.  Check:

$$\frac{3}{5} \cdot \frac{3}{4}$$
$$= \frac{3 \cdot 3}{5 \cdot 3}$$
$$= \frac{9}{20}$$

Depending on the time, you could have students show their work on the document camera, and do another You try. The following page is a student sheet for more practice that could be done in class and turned into the teacher, or go home for homework.

**Practice**: Simplify each expression, once from horizontal form and once from vertical form. Check your answers. Describe each step to your partner correctly using as much of the mathematical language listed below as you can. Give yourself a point each time you correctly use a vocabulary word.

| Written Expression | Prediction | Horizontal form | Vertical Form | Check |
|---|---|---|---|---|
| Eight-ninths divided by three-fourths | The quotient will be<br>greater than 1<br>Or<br>less than 1<br>Circle your prediction.<br>Why do you think that? | | | |
| Twelve fifths divided by one-half | The quotient will be<br>greater than 1<br>Or<br>less than 1<br>Circle your prediction.<br>Why do you think that? | | | |
| Two-thirds divided by four-fifths | The quotient will be<br>greater than 1<br>Or<br>less than 1<br>Circle your prediction.<br>Why do you think that? | | | |

| | | | |
|---|---|---|---|
| complex fraction | divisor | inverse operation | quotient |
| Denominator | equivalent | multiplication | reciprocal |
| Dividend | horizontal form | numerator | unit |
| Division | improper fraction | proper fraction | vertical form |

What are some other uses for complex fractions in $7^{th}$ grade?
Extend the previous lesson to include mixed fractions.

Order of operations and complex fractions. Any practice with fractions and order of operations may be presented in complex fraction form.

If you had the expression $\frac{3+5}{4}$, you could write it as $(3+5)\div 4$. The fraction bar is both a grouping symbol and a division symbol. What is the expression simplified?

$$\frac{3+5}{4} = \frac{8}{4} = 2$$

## You do:

Translate and simplify the phrase "the sum of 5 plus 8 divided by 7".
Check and compare translations and simplification.
Translate and simplify the phrase "the sum of 5 plus 9 divided by the sum of 5 plus 2".
Check and compare translations and simplification.
Translate and simplify the phrase "three-fourths divided by one-half".
Check and compare translations and simplification.

Practice problems:
Students can write each problem in horizontal form, or can work with the expression in vertical form, but obviously, they must follow order of operations. Examples:

$$\frac{\frac{2}{5}+\frac{1}{5}}{\frac{3}{7}-\frac{1}{7}} \qquad \frac{\left(\frac{2}{5}\right)\left(\frac{12}{5}\right)}{3} \qquad \frac{\left(\frac{2}{5}\right)\left(\frac{12}{5}\right)-\frac{2}{3}}{\frac{5}{6}+\frac{2}{3}}$$

The Use of the Complex Fraction when comparing fractions.
Often, you are asked to put the correct sign into the blank to compare the fractions. Normally, you find a common denominator for comparison. Can you use complex fractions to compare fractions, and if yes, how?

$\frac{5}{9} \bigcirc \frac{4}{7}$ Fill in the blank with the appropriate symbol $<, >, \leq,$ or $\geq$

If the quotient is > 1, is the denominator larger or smaller than the numerator?

If the quotient = 1, then what do you know about the numerator and denominator?

If the quotient is < 1, is the denominator larger or smaller than the numerator?

$$\frac{\frac{5}{9}\cdot\frac{7}{4}}{\frac{4}{7}\cdot\frac{7}{4}} = \frac{5\cdot 7}{9\cdot 4} = \frac{35}{36}$$

The quotient < 1, so the denominator must be larger than the numerator since not the whole denominator (divisor) could fit into the numerator (dividend).

## The use of complex fractions as a ratio:

What is the definition of a complex fraction? Look in your notes. Do a choral response read of the definition.

Another way to use a complex fraction is as a ratio of two fractions.

Finding the unit rate:

Example: There is $\frac{1}{4}$ tbs. of salt to every $\frac{2}{3}$ cup of water in a recipe.

If you take the above example and simplify $\frac{\frac{1}{4}}{\frac{2}{3}} \frac{\text{tbs. salt}}{\text{cup of water}}$, would you get how much salt you need for 1 cup of water? Work with the ratio in vertical form. Multiplying by the reciprocal will get you 1 cup of water in the denominator. So, you do get a unit rate.

Example: You walk half a mile in one-tenth of an hour. How many miles did you walk in an hour?

$$\frac{\frac{1}{2}}{\frac{1}{10}} \frac{\text{mile}}{\text{hour}}$$

$$\frac{\frac{1}{2} \cdot \frac{10}{1}}{\frac{1}{10} \cdot \frac{10}{1}} \frac{\text{mile}}{\text{hour}}$$

$$= \frac{10}{2}$$

$$= 5 \text{ miles in } 1 \text{ hour}$$

If you phrased the question, "What is your rate in miles per hour?" then you would need to consider the units in a different way.

Increasing or decreasing a recipe:

If you were to make the recipe from above four times as large, how much water and how much salt would you add?

$$\frac{\frac{2}{3} \cdot \frac{4}{1}}{\frac{1}{4} \cdot \frac{4}{1}} \frac{\text{water}}{\text{salt}}$$

$$= \frac{\frac{8}{3}}{\frac{4}{4}} \frac{\text{water}}{\text{salt}}$$

So, you would use $\frac{8}{3}$ of a cup of water, and use 1 tbs. of salt

If you were to cut the recipe in half, how much water and how much salt would you add?

$$\frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{1}{4}\cdot\frac{1}{2}} \quad \frac{\text{water}}{\text{salt}}$$

$$=\frac{\frac{1}{3}}{\frac{1}{8}} \quad \frac{\text{water}}{\text{salt}}$$

| |
|---|
| So, you would use $\frac{1}{3}$ of a cup of water, and use $\frac{1}{8}$ tbs. of salt |

You try: You are working with a recipe that calls for $2\frac{1}{3}$ cups of chicken stock and $\frac{1}{2}$ tsp. of pepper to serve 4 people. How much of each ingredient will you need if you plan to serve 8 people?

You try: You are working with a recipe that calls for $2\frac{1}{3}$ cup of chicken stock and $\frac{1}{2}$ tsp. of pepper to serve 4 people. How much of each ingredient will you need if you plan to serve 6 people?

Certainly, you could also present the complex fraction with variables if students are ready.

Write $(a \div b) \div (c \div d)$ as a complex fraction. $\frac{\frac{a}{b}}{\frac{c}{d}}$ How would you simply the expression?

Debriefing the warm-ups:

Quadrant I: Students should understand inverse operations, and the fact that we check our answers using inverse operations. We can transform a division problem into a multiplication, just as we can write a subtraction problem as an addition one. Teaching students to check their answer is a critical part of successful performance.

Quadrant II: Identifying the reciprocal is different from understanding the importance of being able to have a reciprocal for all numbers (except zero). After identifying the reciprocal, ask "how do you know?". [*The product of a number and its reciprocal is one.*] Ask "so what, why is that important"? Students should understand that without the reciprocal and equivalent forms of 1, we could never solve any equation. Follow with asking which is the opposite of $\frac{12}{7}$? How do you know? [*The sum of a number and its opposite is zero.*] "Why is this important?" Students should understand that without the opposite and equivalent forms of 0, we could never solve any equation.

Quadrant III: Have students identify the question being asked, how many two-thirds are in seven-thirds. Which is the larger number? What do we expect for a quotient, a number larger than 1 or less than 1? Why? Show at least two ways to perform the algorithm (multiplication by the reciprocal, and division straight across the numerators and straight across the denominators).

Quadrant IV: Ask for diagrams of the situation. You could draw out $\frac{7}{3}$ on a number line or in a bar model, then have a cut out of $\frac{2}{3}$ that you lie directly on the model to show that you have 3 sets of $\frac{2}{3}$ and then half of a set that can fit in to the $\frac{7}{3}$. So, you do have three and one-half two-thirds in seven-thirds. Being able to tell an appropriate story about an algorithm is a solid indication of mathematical understanding.

# Warm-Up

## SBAC:

What is the reciprocal of $\frac{12}{7}$?

a. $-\frac{12}{7}$ b. $\frac{7}{12}$ c. $-\frac{7}{12}$ d. $0$ e. $1$

How do you know your answer is correct?

## Review: Grade 7

Solve for *x*:

a. $10 \div x = 2$

Could you use this equation, $2x = 10$, to solve for *x*? Explain.

## Current: Grade 7

Simplify: $\frac{7}{3} \div \frac{2}{3}$ and check your answer

Challenge. Solve for x. (Hint: Refer to Quadrant I)

$\frac{12}{7} \div \frac{x}{5} = \frac{10}{7}$

## Other: Grade 7

Would the algorithm in Quadrant III, $\frac{7}{3} \div \frac{2}{3}$, be used to solve the following word problem?

You have two and one-third of a foot of ribbon from which to cut bows. If each bow is to be two-thirds of a foot long, how many bows can you make from the ribbon you have?

Solve the problem. Interpret your answer.