

Grade Level/Course: Grades 3-6

Lesson/Unit Plan Name: Side-By-Side Division Algorithms

Rationale/Lesson Abstract: All in one place, a lesson that spans teaching techniques for long division with five algorithms: Guess and Check (Partial Quotients), Stacking Method, Place Value Long Division, Traditional Long Division, and Short Division.

Timeframe: *Will depend on grade level and pace at which algorithms are being taught.*

Common Core Standards:

3.OA.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

4.NBT.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

6.NS.2: Fluently divide multi-digit numbers using the standard algorithm.

Instructional Resources/Materials:

Student notebooks

Teacher Notes:

This lesson includes five algorithms for division which move from the more conceptual to the more rote. Depending on their grade level and their students' knowledge, teachers may choose not to teach all of the algorithms.

The first example that is shown works out nicely and neatly. The second example is worked through all five times focusing on the areas that are traditionally harder for students to understand. There is also an example shown where the quotient has a remainder.

This lesson is written using 4-digit dividends and 1-digit divisors. Teachers should introduce the algorithms using grade-level appropriate dividends and divisors.

Division should be represented all three ways interchangeably so that students internalize that each representation means the same thing:

$$6 \div 3 \quad 3 \overline{)6} \quad \frac{6}{3}$$

Questions to ask:

- ***How many groups of 3 are in 6?***
- ***How many 3's are in 6?***
- ***What number multiplied by 3 gets us closest to 6?***
- **Try to stay away from *How many 3's are in 6?* as that is not mathematical language.**

Lesson:

Partial Quotients (Guess and Check)

Start by telling the story. *Three students are walking in the woods and come upon a pot of gold. In that pot are 3639 gold coins that the students decide to share equally among themselves. How many will each student get?*

They could split them up one by one until they all had the same amount, but that wouldn't be time efficient. Could they divvy them up more efficiently by giving out more than one at a time? Encourage students to start with nice round numbers, an even 10, 100, or 1000 that might work.

$$\begin{array}{r} 3 \overline{) 3639} \\ \underline{-3000} \\ 639 \end{array} \quad \begin{array}{l} 1000 \\ \\ \end{array}$$

If we gave each of the 3 students, 1000 gold coins, then we have given out 3 x 1000, or 3000 coins.

If we started with 3639 coins in our pot and we gave away 3000 total, how can we find out how many gold coins are still left for us to divide out? We subtract.

How many coins are left in the pot? 639. Do we have more coins to divide out?

$$\begin{array}{r} 3 \overline{) 3639} \\ \underline{-3000} \\ 639 \\ \underline{-300} \\ 339 \\ \underline{-300} \\ 39 \\ \underline{-30} \\ 9 \\ \underline{-9} \\ 0 \end{array} \quad \begin{array}{l} 1000 \\ \\ 100 \\ 100 \\ 10 \\ 10 \\ 3 \end{array}$$

Continue asking the same questions about dividing, multiplying, and subtracting **each time**.

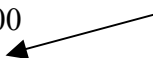
There is no one right way to solve this problem. Student guesses will oftentimes begin with smaller numbers which they feel comfortable multiplying. Teachers should encourage students to be efficient, but not require one particular way to get to the answer.

Each step should be debriefed to ensure student clarity. *What does this 39 mean? What does this 0 mean?*

$$\begin{array}{r} 1000 \\ 100 \\ 100 \\ 10 \\ + 3 \\ \hline \end{array}$$

To find out the total quotient for our original problem, we need to add together our partial quotients. *If we gave each student 1000 gold coins, and then 100, 100, 10, and 3 gold coins, how can we find out how many gold coins each student got?*

$$\begin{array}{r} 3 \overline{) 3639} \\ \end{array}$$



Common mistake: Students will often write the number by which they are divvying out next to the dividend (or partial dividend). It is helpful to have them write it next to where they will write the partial quotient. This makes it easier to identify where that partial quotient came from.

Lesson, continued:

Stacking Algorithm

This method builds on the idea of estimating that students did using Partial Quotients, but now they are incorporating more place value. Continuing to use the same context (i.e., telling the same story) will help students maintain a connection to the concept of division. *Three students are walking in the woods and come upon a pot of gold. In that pot are 3639 gold coins that the students decide to share equally among themselves. How many will each student get?*

Continue to repeat the same questions in the same ways. Only with this method, you will always begin with the largest place value that you have in your dividend.

← With the Stacking Algorithm, students have to leave several lines blank above where they write the problem.

$$\begin{array}{r} 1000 \\ 3 \overline{) 3639} \\ \underline{-3000} \\ 639 \end{array}$$

When we look at how many thousands are in our dividend (3), the question becomes: *3 times how many thousands gets us closest to 3000?* The answer is that 3 times 1 thousand gets us closest to 3000. We write that 1000 (our partial quotient) where we usually write the quotient, being sure to write thousands in the thousands place, hundreds in the hundreds place, etc.

$$\begin{array}{r} 200 \\ 1000 \\ 3 \overline{) 3639} \\ \underline{-3000} \\ 639 \\ \underline{-600} \\ 39 \end{array}$$

Now we are done with the thousands place and moving on to the hundreds place. The question becomes: *3 times how many hundreds gets us closest to 600?* The answer is that 3 times 2 hundreds gets us closest to 600. We stack that partial quotient above our 1000, lining up our place values.

Lesson, continued:

$$\begin{array}{r} 3 \\ 10 \\ 200 \\ 1000 \\ 3 \overline{) 3639} \\ \underline{-3000} \\ 639 \\ \underline{-600} \\ 39 \\ \underline{-30} \\ 9 \\ \underline{-9} \\ 0 \end{array}$$

We continue the process for **each** place value that we have.

$$3 \overline{) \begin{array}{r} 1213 \\ 3639 \end{array}}$$

The stacked partial quotients are in expanded form and simply have to be added together to find the quotient.

Lesson, continued:

Long Division with Zeroes (Place Value Long Division)

This method works just like the Traditional Algorithm, only students can keep track of their place value. It eliminates the "bring down" step which is not mathematical. Continuing to use the same context (i.e., telling the same story) will help students maintain a connection to the concept of division. *Three students are walking in the woods and come upon a pot of gold. In that pot are 3639 gold coins that the students decide to share equally among themselves. How many will each student get?*

Continue to repeat the same questions in the same ways. Only with this method, you will abbreviate each step and only write each digit of the quotient in its place.

$$\begin{array}{r} 1 \\ 3 \overline{) 3639} \\ \underline{-3000} \\ 639 \end{array}$$

When doing this long division, we look at each place one at a time. If we cover up everything other than the thousands place, our question is: *3 times what number gets us closest to 3?* The answer is 1. But that isn't really a 3, it's a 3000, so our answer is actually 1000. Therefore we will write the 1 in the thousands place.

We continue to ask the same questions we always have to proceed to the next step: *If we have divided out 1000 to each, how much have we divided out altogether? How much is left for us to divvy out?*

$$\begin{array}{r} 1213 \\ 3 \overline{) 3639} \\ \underline{-3000} \\ 639 \\ \underline{-600} \\ 39 \\ \underline{-30} \\ 9 \\ \underline{-9} \\ 0 \end{array}$$

Now we have 639 left to divide out. We have finished with the 1000's place so we are moving on to the 100's place, still looking at each place one at a time. If we cover up everything up to the hundreds place, our question is: *3 times what number gets us closest to 6?* The answer is 2. But that isn't really a 6, it's a 600, so our answer is actually 200. Therefore we will write the 2 in the hundreds place.

Following this pattern, we ask the same questions for the 10's place and the 1's place.

Lesson, continued:

Traditional Algorithm for Long Division

Even with the Traditional Algorithm, it is helpful to continue to use the same context (i.e., telling the same story) to help students maintain a connection to the concept of division. *Three students are walking in the woods and come upon a pot of gold. In that pot are 3639 gold coins that the students decide to share equally among themselves. How many will each student get?*

Continue using the same questions you have been using. That helps students to make sense of the "rules" (divide, multiply, subtract, bring down). This time, we will abbreviate each place value entirely. This algorithm simply becomes a shortcut.

$$\begin{array}{r} 1213 \\ 3 \overline{) 3639} \\ \underline{-3} \\ 06 \\ \underline{-6} \\ 03 \\ \underline{-3} \\ 09 \\ \underline{-9} \\ 0 \end{array}$$

Lesson, continued:

Short Division

This is a complete shortcut for Long Division. Students should demonstrate mastery of the Traditional Algorithm before attempting the shortcut. **Note:** This particular problem is not the best example of the power of Short Division. The second problem below is a better example.

$$\begin{array}{r} 1\ 2\ 1\ 3 \\ 3 \overline{) 3\ 6\ 3\ 9} \end{array}$$

For each place, we ask the same questions, only this time we do the work in our head and abbreviate it.

Starting in the thousands place, we ask *How many groups of 3 are in 3?* We write the answer 1 in the thousands place and then mentally multiply $1 \times 3 = 3$. We mentally subtract 3 from 3 and get 0. Without any hundreds left over, we know that we can move on to the next place value.

Moving on the hundreds place, we now have 6 hundreds. *How many groups of 3 are in 6?* We write the answer 2 in the hundreds place and then mentally multiply $2 \times 3 = 6$. We mentally subtract 6 from 6 and get 0. Without any tens left over, we know that we can move on to the next place value.

Moving on the tens place, we now have 3 tens. *How many groups of 3 are in 3?* We write the answer 1 in the tens place and then mentally multiply $1 \times 3 = 3$. We mentally subtract 3 from 3 and get 0. Without any ones left over, we know that we can move on to the next place value.

Finally in the ones place, we now have 9 ones. *How many groups of 3 are in 9?* We write the answer 3 in the ones place and then mentally multiply $3 \times 3 = 9$. We mentally subtract 9 from 9 and get 0. That lets us know that we have no remainder.

$$\begin{array}{r} 2\ 4\ 7\ 2 \\ 3 \overline{) 7\ 14\ 21\ 6} \end{array}$$

Starting in the thousands place, we ask *How many groups of 3 are in 7?* We write the answer 2 in the thousands place and then mentally multiply $2 \times 3 = 6$. We mentally subtract 6 from 7 and get 1. That means we have 1 hundred (or 10 tens) left to divide out, so that 1 gets written in the hundreds place in front of the 4 tens we already had.

Moving on the hundreds place, we now have 14 hundreds. *How many groups of 3 are in 14?* We write the answer 4 in the hundreds place and then mentally multiply $4 \times 3 = 12$. We mentally subtract 12 from 14 and get 2. That 2 gets written in front of the digit in the tens place of our dividend, the 1.

Moving on the tens place, we now have 21 tens. *How many groups of 3 are in 21?* We write the answer 7 in the tens place and then mentally multiply $7 \times 3 = 21$. We know that $21 - 21 = 0$, so we will not have any tens left over and can move on to the ones place.

Finally in the ones place, we now have 6 ones. *How many groups of 3 are in 6?* We write the answer 2 in the ones place and then mentally multiply $2 \times 3 = 6$. We know that $6 - 6 = 0$ and therefore we have no remainder.

Lesson, continued:

Partial Quotients (Guess and Check), Example 2

Again, there is no "correct" way to solve this problem. Students will stay within guesses where they are comfortable. Encourage students to start with nice round numbers, an even 10, 100, or 1000 that might work. A couple of possible solutions are shown below.

Continue asking the same questions about dividing, multiplying, and subtracting each time. Each step should be debriefed to ensure student clarity. *How much is still left in our pot? Why do we need to subtract?*

$$\begin{array}{r}
 621 \\
 5 \overline{) 3105} \\
 \underline{- 500} \quad 100 \\
 2605 \\
 \underline{- 500} \quad 100 \\
 2105 \\
 \underline{- 500} \quad 100 \\
 1605 \\
 \underline{- 500} \quad 100 \\
 1105 \\
 \underline{- 500} \quad 100 \\
 605 \\
 \underline{- 500} \quad 100 \\
 105 \\
 \underline{- 100} \quad 20 \\
 5 \\
 \underline{- 5} \quad 1 \\
 0
 \end{array}$$

$$\begin{array}{r}
 621 \\
 5 \overline{) 3105} \\
 \underline{- 1000} \quad 200 \\
 2105 \\
 \underline{- 1000} \quad 200 \\
 1105 \\
 \underline{- 1000} \quad 200 \\
 105 \\
 \underline{- 100} \quad 20 \\
 5 \\
 \underline{- 5} \quad 1 \\
 0
 \end{array}$$

$$\begin{array}{r}
 200 \\
 200 \\
 200 \\
 20 \\
 + 1 \\
 \hline
 621
 \end{array}$$

$$\begin{array}{r}
 100 \\
 100 \\
 100 \\
 100 \\
 100 \\
 100 \\
 100 \\
 20 \\
 + 1 \\
 \hline
 621
 \end{array}$$

Lesson, continued:

Stacking Algorithm, Example 2

$$\begin{array}{r} 600 \\ 5 \overline{) 3105} \\ \underline{-3000} \\ 105 \end{array}$$

We usually look at this problem, start with looking at how many thousands are in our dividend (3), and ask the question: *How many 5's are in 3000?* Obviously, there are many 5's in 3000, but we cannot multiply 5×1000 without divvying out 5000, which is more than what we started with. Therefore, our quotient doesn't have any thousands in it.

We then move on to our hundreds place and see that what we have in 31 hundreds. So we can ask *How many 5's are in 31 hundreds?*

$$\begin{array}{r} 1 \\ 20 = 621 \\ 600 \\ 5 \overline{) 3105} \\ \underline{-3000} \\ 105 \\ \underline{-100} \\ 5 \\ \underline{-5} \\ 0 \end{array}$$

Lesson, continued:

Long Division with Zeroes (Place Value Long Division) , Example 2

$$\begin{array}{r} 06 \\ 5 \overline{) 3105} \\ \underline{-3000} \\ 105 \end{array}$$

When doing this long division, we look at each place one at a time. If we cover up everything other than the thousands place, our question is: *How many 5's are in 3000?* Obviously, there are many 5's in 3000, but we cannot multiply 5×1000 without divvying out 5000, which is more than what we started with. Therefore, our quotient doesn't have any thousands in it. Some students will want to write a 0 in the thousands place, which is fine.

We then move on to our hundreds place and see that what we have in 31 hundreds. So we can ask *How many 5's are in 31 hundreds?*

$$\begin{array}{r} 621 \\ 5 \overline{) 3105} \\ \underline{-3000} \\ 105 \\ \underline{-100} \\ 5 \\ \underline{-5} \\ 0 \end{array}$$

Now we have 105 left to divide out. We have finished with the 1000's and the 100's place so we are moving on to the 10's place, still looking at each place one at a time. If we cover up everything other than the tens place, our question is: *5 times what number gets us closest to 10?* The answer is 20, so we write a 2 in the tens place.

Lesson, continued:

Traditional Algorithm for Long Division, Example 2

$$\begin{array}{r} 621 \\ 5 \overline{) 3105} \\ \underline{-30} \\ 10 \\ \underline{-10} \\ 05 \\ \underline{-5} \\ 0 \end{array}$$

Many students will look at this problem and immediately know that there are no complete groups of 5 in 3, and they will move on to looking at how many groups of 5 are in 31. Some students will need to write a 0 in the thousands place of the quotient to remind themselves until they develop more fluency with the algorithm.

Lesson, continued:

Short Division, Example 2

This is a complete shortcut for Long Division. Students should demonstrate mastery of the Traditional Algorithm before attempting the shortcut.

$$\begin{array}{r} 6 \ 2 \ 1 \\ 5 \overline{) 3 \ 1 \ 10 \ 05} \end{array}$$

For each place, we ask the same questions, only this time we do the work in our head and abbreviate it.

Starting in the thousands place, we ask *How many groups of 5 are in 3?* Since there isn't a complete group of 5 in 3, we move on to the hundreds place.

We now have 31 hundreds. *How many groups of 5 are in 31?* We write the answer 6 in the hundreds place and then mentally multiply $6 \times 5 = 30$. We mentally subtract 30 from 31 and get 1. That 1 gets written in front of the digit in the tens place of our dividend, the 0.

Moving on the tens place, we now have 10 tens. *How many groups of 5 are in 10?* We write the answer 2 in the tens place and then mentally multiply $2 \times 5 = 10$. We mentally subtract 10 from 10 and get 0. That 0 gets written in front of the digit in the ones place of our dividend, the 5.

Finally in the ones place, we now have 05 (or just 5) ones. *How many groups of 5 are in 5?* We write the answer 1 in the ones place and then mentally multiply $1 \times 5 = 5$. We mentally subtract 5 from 5 and get 0. That lets us know that we have no remainder.

Lesson, continued:

Partial Quotients (Guess and Check), Example 3

Again, there is no "correct" way to solve this problem. Students will stay within guesses where they are comfortable. Encourage students to start with nice round numbers, an even 10, 100, or 1000 that might work. A couple of possible solutions are shown below.

Continue asking the same questions about dividing, multiplying, and subtracting each time. Each step should be debriefed to ensure student clarity. *How much is still left in our pot? Why do we need to subtract?*

$$\begin{array}{r}
 2455 \overset{r}{3} \\
 4 \overline{) 9823} \\
 \underline{-4000} \quad 1000 \\
 5823 \\
 \underline{-4000} \quad 1000 \\
 1823 \\
 \underline{-400} \quad 100 \\
 1423 \\
 \underline{-400} \quad 100 \\
 1023 \\
 \underline{-400} \quad 100 \\
 623 \\
 \underline{-400} \quad 100 \\
 223 \\
 \underline{-200} \quad 50 \\
 23 \\
 \underline{20} \quad 5 \\
 \boxed{3}
 \end{array}$$

$$\begin{array}{r}
 2455 \overset{r}{3} \\
 4 \overline{) 9823} \\
 \underline{-8000} \quad 2000 \\
 1823 \\
 \underline{-1600} \quad 400 \\
 223 \\
 \underline{-80} \quad 20 \\
 143 \\
 \underline{-120} \quad 30 \\
 23 \\
 \underline{-20} \quad 5 \\
 \boxed{3}
 \end{array}$$

$$\begin{array}{r}
 2000 \\
 400 \\
 20 \\
 30 \\
 + 5 \\
 \hline
 2455
 \end{array}$$

$$\begin{array}{r}
 1000 \\
 1000 \\
 100 \\
 100 \\
 100 \\
 100 \\
 100 \\
 50 \\
 + 5 \\
 \hline
 2455
 \end{array}$$

While this lesson does not focus on interpreting remainders, please note that there are several ways that a student could interpret this remainder of 3. Within the context of the story of the pot of gold, there are now 3 gold coins left in the pot, but 4 students. There would be 3 coins left over. But if we were talking about apples instead of coins, the apples could be cut up and each student would receive an additional three-fourths of an apple. Remainders can also be divided into the decimal place values at the upper grades, making the answer to this problem 2455.75.

Lesson, continued:

Stacking Algorithm, Example 3

$$\begin{array}{r} 5 \\ 50 = 2455r3 \\ 400 \\ \hline 2000 \\ 4 \overline{) 9823} \\ \underline{-8000} \\ 1823 \\ \underline{-1623} \\ 223 \\ \underline{200} \\ 23 \\ \underline{-20} \\ \textcircled{3} \end{array}$$

Long Division with Zeroes
(Place Value Long Division) , Example 3

$$\begin{array}{r} 2455r3 \\ 4 \overline{) 9823} \\ \underline{-8000} \\ 1823 \\ \underline{-1623} \\ 223 \\ \underline{-200} \\ 23 \\ \underline{-20} \\ \textcircled{3} \end{array}$$

Lesson, continued:

Traditional Algorithm for Long Division,
Example 3

Short Division, Example 3

$$\begin{array}{r} 2455r3 \\ 4 \overline{) 9823} \\ \underline{-8} \\ 18 \\ \underline{-16} \\ 22 \\ \underline{-20} \\ 23 \\ \underline{-20} \\ \textcircled{3} \end{array}$$

$$4 \overline{) \begin{array}{cccc} 2 & 4 & 5 & 5 \\ 9 & 18 & 22 & 23 \end{array} r3}$$