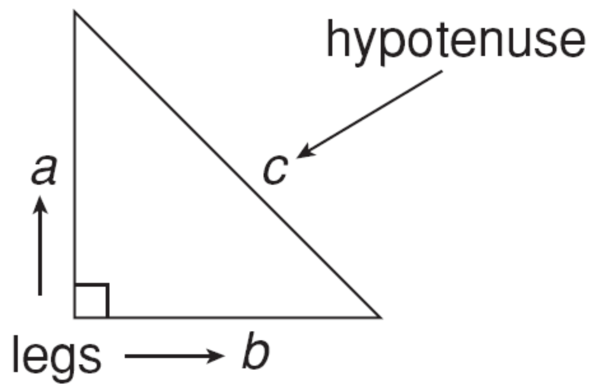
The Pythagorean Theorem & Its Converse

The formula for the Pythagorean Theorem is $a^2+b^2=c^2$. This lesson will demonstrate the proof help students understand the theorem and its converse.

Grade Level:

7th Pre-Algebra
8th Algebra I/Geometry



Strategies:

Choral Response
You Tries
Active Participation
Student Talk

Standards:

7NS2.4 - Use the inverse relationship between raising to a power and extracting the root of a perfect square integer.

7MG3.3 – Know and understand the Pythagorean Theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean Theorem by direct measurement.

The Pythagorean Theorem:

The Pythagorean Theorem is a relation among the three sides of a ***right triangle***. It states that in any right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse. In other words, $a^2+b^2=c^2$.

The Converse of the Pythagorean Theorem:

The **converse** of the theorem is also true. For any triangle with sides *a*, *b*, *c*, if $a^2+b^2=c^2$, then the triangle is a right angled triangle.

Proof of the Pythagorean Theorem:

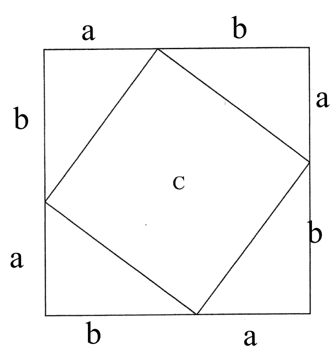
Using Algebra, we can show that $a^2+b^2=c^2$;

$$(a+b)(a+b)=c^2+2ab$$

$$a^2+2ab+b^2=c^2+2ab$$

$$a^2+2ab-2ab+b^2=c^2+2ab+2ab$$

$$a^2+b^2=c^2$$



Date October 22, 2011

Warm-Up

CST/CAHSEE: 7NS2.4

28

$\sqrt{225} =$

A 15

B 25

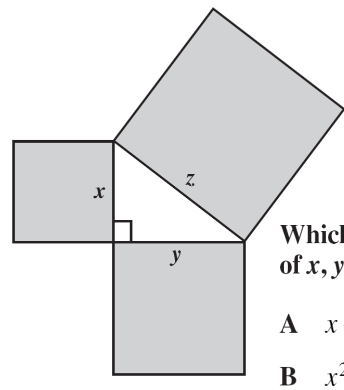
C 35

D 45

Square the remaining 3 distracters. Explain any patterns you discover.

Review: CAHSEE 7MG3.3

149. In the drawing below, the figure formed by the squares with sides that are labeled x, y, and z is a right triangle.



Which equation is true for all values of x, y, and z?

A $x + y = z$

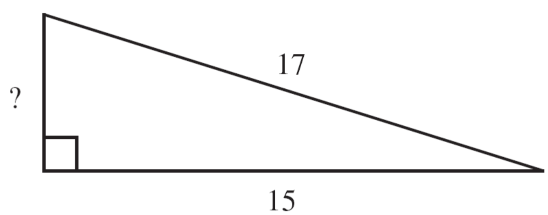
B $x^2 + y^2 = z^2$

C $x^2 \bullet y^2 = z^2$

D $\frac{1}{2}xy = z$

Current: CST 7MG3.3

86 What is the length of the missing side of this triangle?



A 2

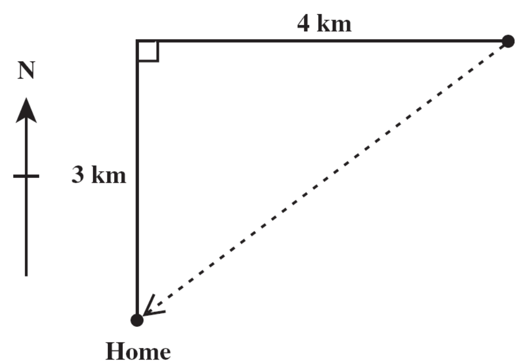
B 8

C $\sqrt{2}$

D $\sqrt{514}$

Which two distracters could be eliminated immediately and why?

Other: CAHSEE 7MG3.3



147. The club members hiked 3 kilometers north and 4 kilometers east, but then went directly home as shown by the dotted line. How far did they travel to get home?

A 4 km

B 5 km

C 6 km

D 7 km

Eliminate one distracter prior to selecting your answer.

Standards: 7NS2.4, 7MG3.3

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The Pythagorean Theorem
&
Its Converse

Example 1: Pythagorean Theorem Proof Activity

You Try 1: Pythagorean Theorem Proof Activity

Example 2: Finding the Missing Hypotenuse

You Try 2: Finding the Missing Hypotenuse

Example 3: Finding the Missing Leg

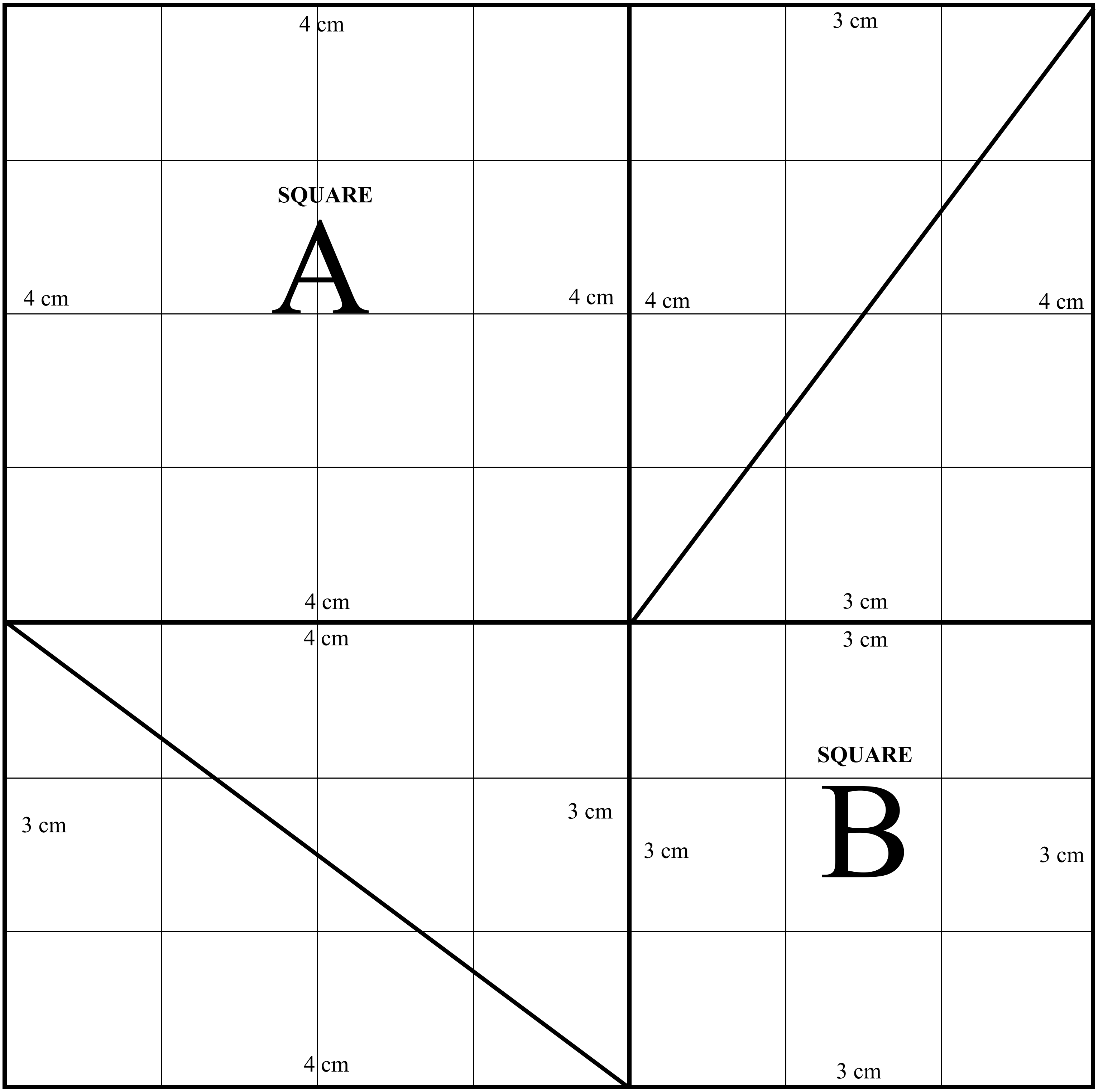
You Try 3: Finding the Missing Leg

Example 4: Pythagorean Triples

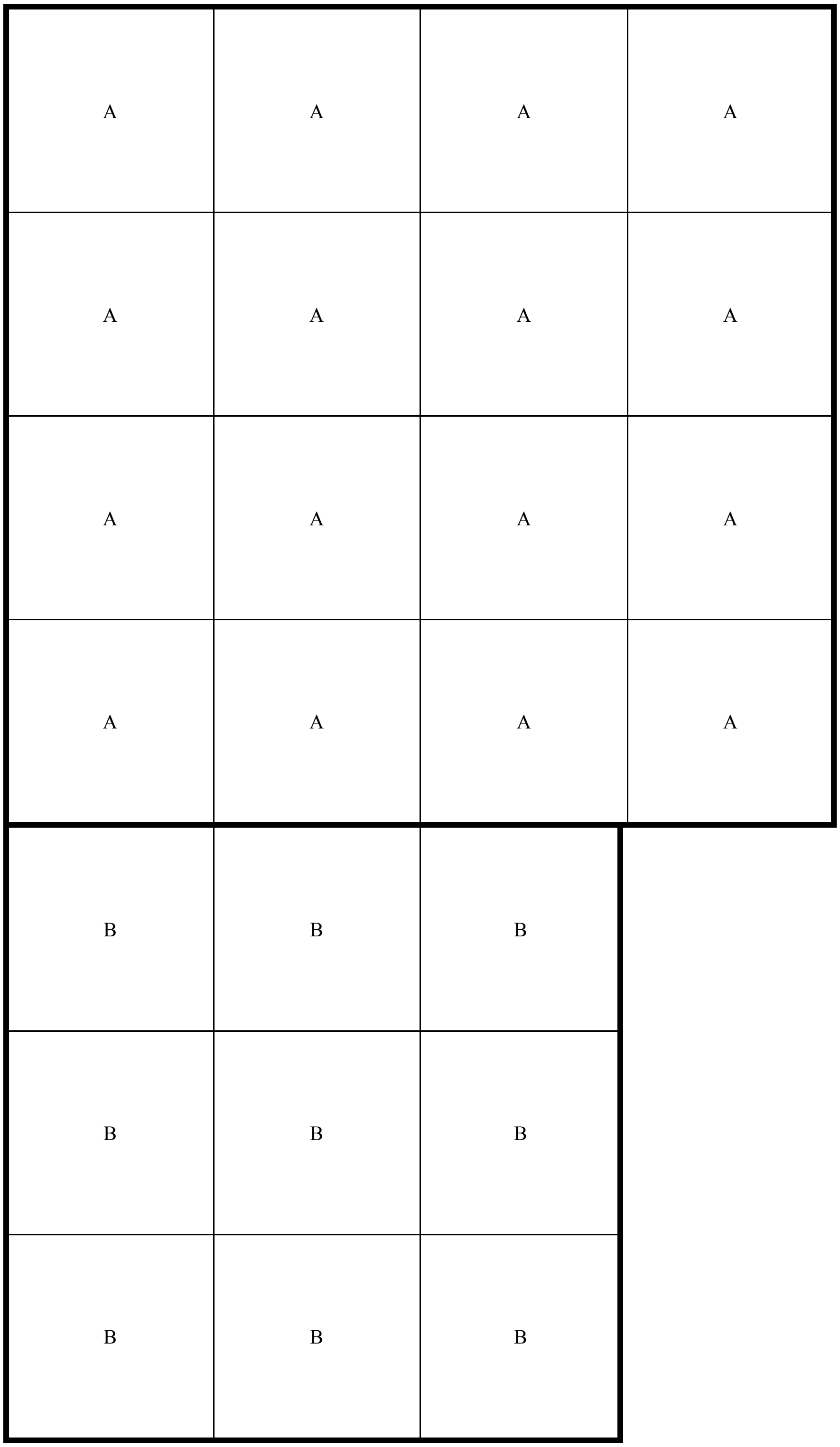
You Try 4a: Pythagorean Triple

You Try 4b: Pythagorean Triple

Pythagorean Theorem Proof
Example 1



Pythagorean Theorem Proof
Example 1



Directions for Example 1

1. Use the Pythagorean Theorem Proof worksheet to record your work.

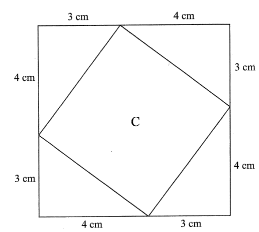
2. Cut out square A, square B and the 4 right triangles. Color square A yellow, square B pink and the triangles green.

3. Label each of the four triangles with a right angle symbol.

4. Label each of the four triangles with *leg, leg,* and *hypotenuse*.

5. Label each triangle as follows; side length 4, *a*; side length 3, *b* and the hypotenuse, *c*.

6. Arrange the four right triangles to form the diagram below. Glue onto separate piece of paper.



7. Outline square C in orange and the larger square in blue.

8. What is the area of square A?

9. What is the area of square B?

10. What is the total area for square A and square B?

11. Choose one green triangle. Glue square A and B onto the appropriate sides of the triangle.

12. Color the additional square A and square B orange. Using the orange squares, fill in the missing square C.

13. What is the Area of square C?

14. Does square A + square B = square C?

15. What is the length of the hypotenuse?

16. What is the area of the four green triangles?

17. What is the area of the blue square?

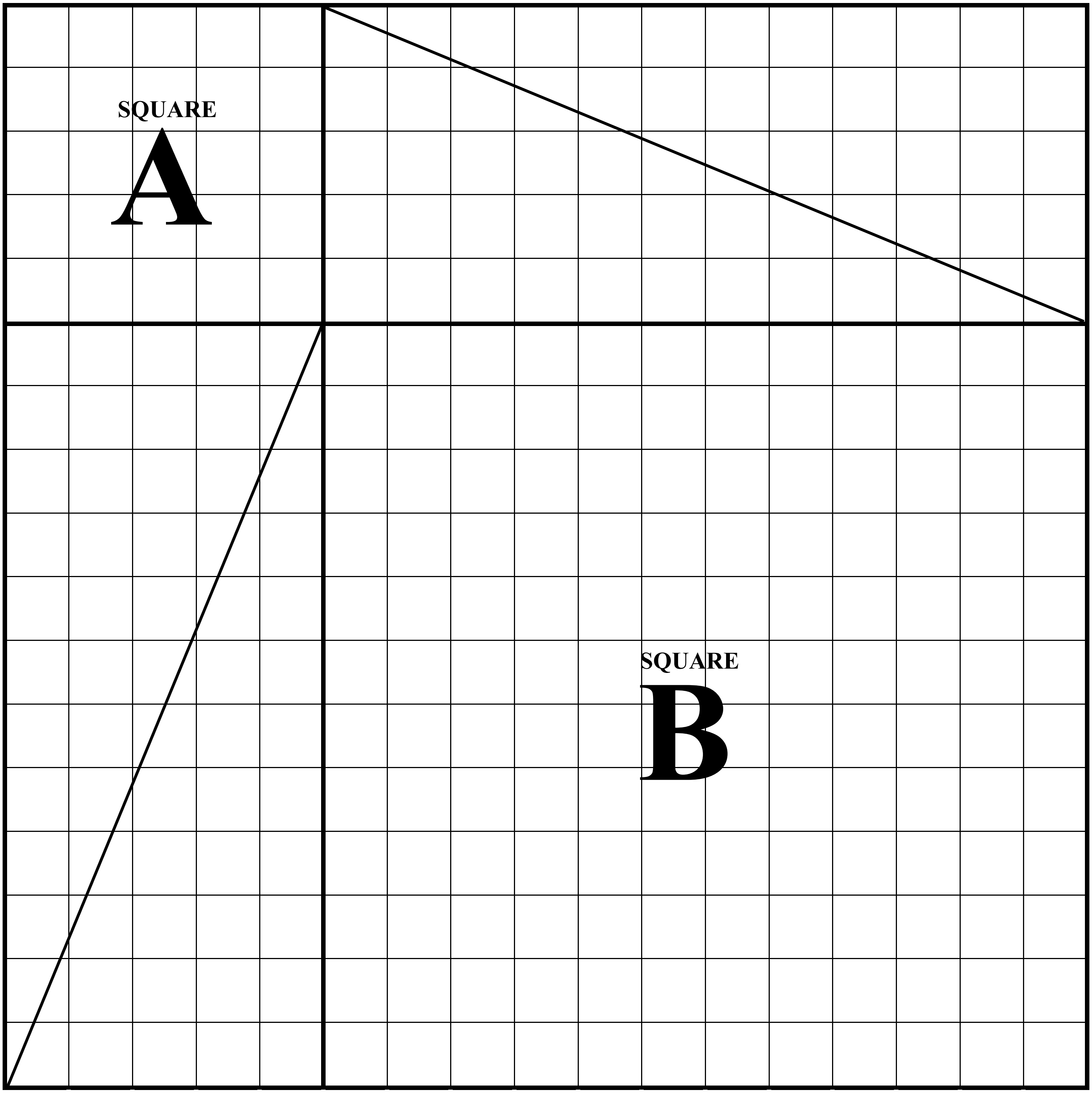
Pythagorean Theorem Proof Worksheet
Example 1

Square A	Square B	Square C	Green Triangles	Blue Square

Square A + Square B	Square C + Green Triangles

Pythagorean Theorem *Find the missing hypotenuse.*	The Converse of Pythagorean Theorem *Is the triangle a right angled triangle?*	Pythagorean Theorem Proof *Prove Pythagorean Theorem true.*

Pythagorean Theorem Proof
You Try 1



Pythagorean Theorem Proof
You Try 1

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A	A	A	A	A
A	A	A	A	A

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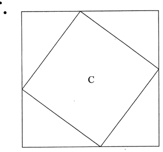
Directions for You Try 1:

1. Use the Pythagorean Theorem Proof worksheet to record your work.
2. Cut out square A, square B and the 4 right triangles. Color square A yellow, square B pink and the triangles green.
3. Label each of the four triangles with a right angle symbol.
4. Label each of the four triangles with *leg, leg,* and *hypotenuse*.
5. Label each triangle as follows; side length 5*, a*; side length 12*, b* and the hypotenuse, *c*.
6. Arrange the four right triangles to form the diagram below. Glue onto a separate piece of paper.



7. Outline square C in orange and the larger square in blue.
8. What is the area of square A?
9. What is the area of square B?
10. What is the total area for square A and square B?
11. Choose one green triangle. Glue square A and square B onto the appropriate sides of the triangle.
12. Color the additional square A and square B orange. Using the orange squares, fill in the missing square C.
13. What is the Area of square C?
14. Does square A + square B = square C?
15. What is the length of the hypotenuse?
16. What is the area of the four green triangles?
17. What is the area of the blue square?

Pythagorean Theorem Proof Worksheet
You Try 1

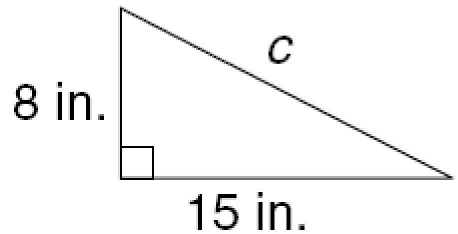
Square A	Square B	Square C	Green Triangles	Blue Square

Square A + Square B	Square C + Green Triangles

Pythagorean Theorem *Find the missing hypotenuse.*	**The Converse of Pythagorean Theorem** *Is the triangle a right angled triangle?*	**Pythagorean Theorem Proof** *Prove Pythagorean Theorem true.*

Pythagorean Theorem
Finding the Missing Hypotenuse
Example 2

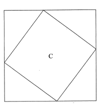
Find the missing length in the right triangle.



Directions for Example 2

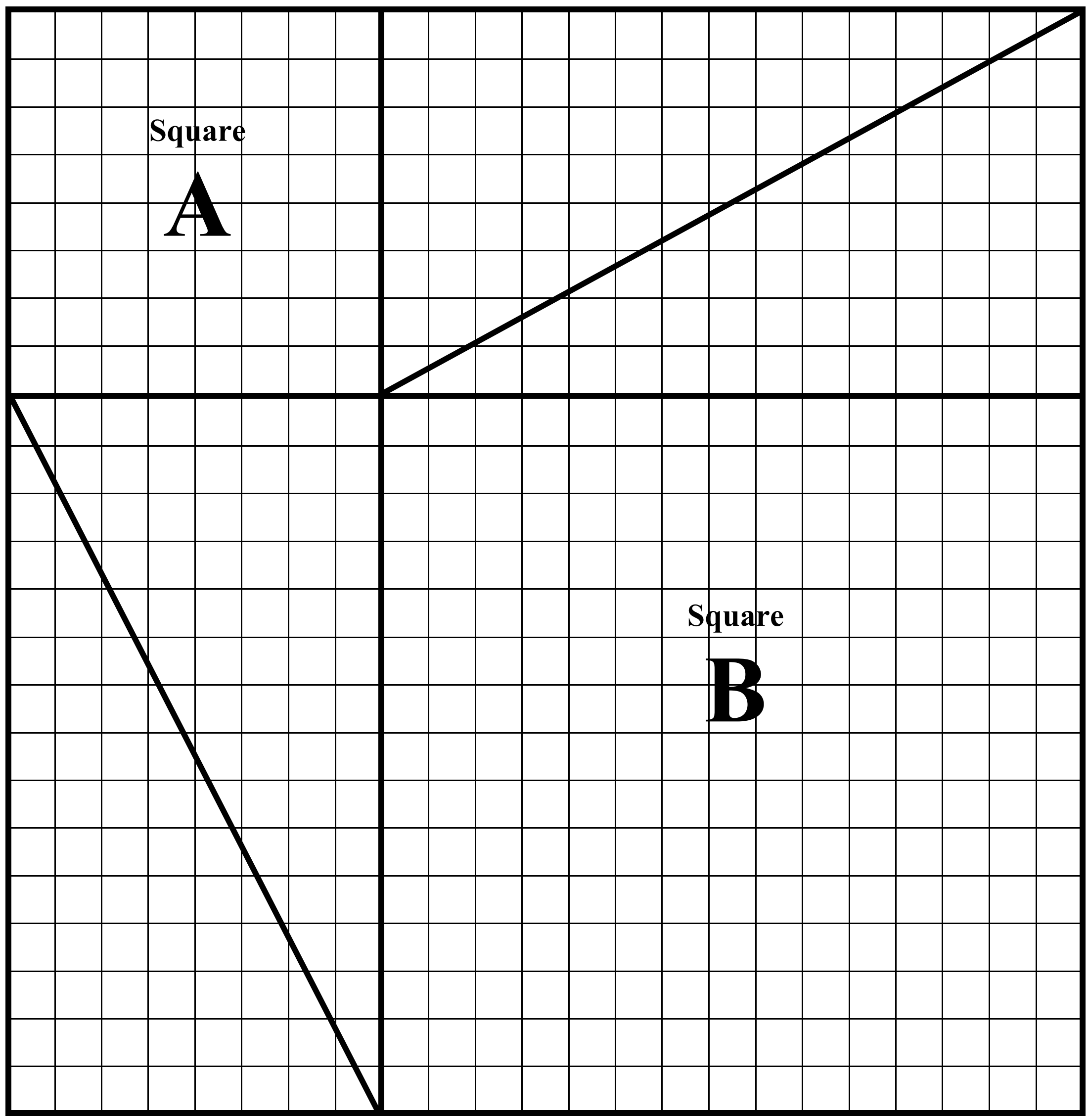
Use the Pythagorean Theorem Proof worksheet to record your work.

1. Cut out square A, square B and the 4 right triangles. Color square A yellow, square B pink and the triangles green.
2. Label each of the four triangles with a right angle symbol.
3. Label each of the four triangles with *leg, leg,* and *hypotenuse*.
4. Label each triangle as follows; side length 8, *a*; side length 15, *b* and the hypotenuse, *c*.
5. Arrange the four right triangles to form the diagram below. Glue onto a separate piece of paper.



6. Outline square C in orange and the larger square in blue.
7. What is the area of square A?
8. What is the area of square B?
9. What is the total area for square A and square B?
10. Choose one green triangle. Glue square A and square B onto the appropriate sides of the triangle.
11. Color the additional square A and square B orange. Using the orange squares, fill in the missing square C.
12. What is the Area of square C?
13. Does square A + square B = square C?
14. What is the length of the hypotenuse?
15. What is the area of the four green triangles?
16. What is the area of the blue square?

Pythagorean Theorem Proof

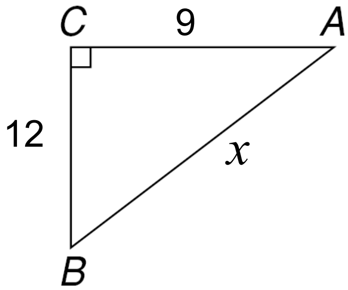


Pythagorean Theorem
Finding the Missing Hypotenuse Worksheet
Example 2

Square A	Square B	Square C	Green Triangles	Blue Square

Square A + Square B	Square C + Green Triangles

Pythagorean Theorem *Find the missing hypotenuse.*	**The Converse of Pythagorean Theorem** *Is the triangle a right angled triangle?*	**Pythagorean Theorem Proof** *Prove Pythagorean Theorem true.*



Pythagorean Theorem
Finding the Missing Hypotenuse Worksheet
You Try 2

Find the missing length in the right triangle.
Use a piece of graph paper and draw the proof.
Follow directions from Example 2.

Square A	Square B	Square C	Green Triangles	Blue Square

Square A + Square B	Square C + Green Triangles

Pythagorean Theorem *Find the missing hypotenuse.*	**The Converse of Pythagorean Theorem** *Is the triangle a right angled triangle?*	**Pythagorean Theorem Proof** *Prove Pythagorean Theorem true.*

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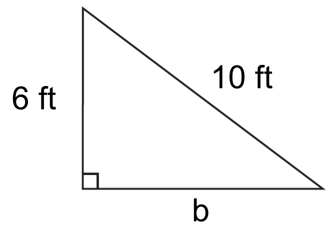
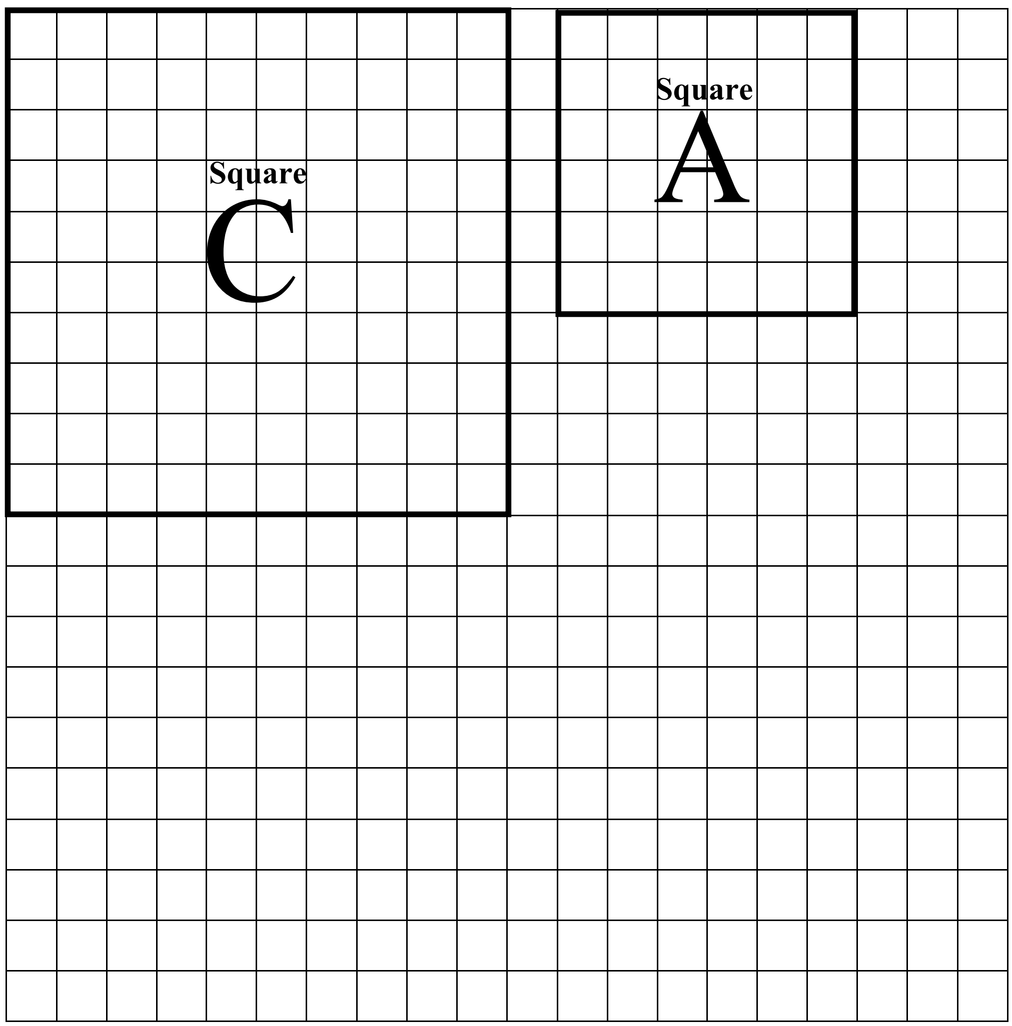
Pythagorean Theorem
Finding the Missing Leg Measure
Example 3

Find the missing length in the right triangle.



Directions for Example 3

1. Use attached worksheet to record your work.
2. Color square A yellow and square C orange. Cut square A and C out.
3. Label the side lengths of square A.
4. What is the area of square A?
5. Label the side lengths of square C.
6. What is the area of square C?
7. What is the difference of the area of square C and square A?
8. Draw a third ***square*** on the remaining graph paper using the ***difference*** from question 4.
9. Color the third square pink and label it square B.
10. Label the side lengths of square B.
11. What is the area of square B?
12. Glue squares A, B and C onto the right triangle.
13. Color the triangle green. Label the sides with their lengths.

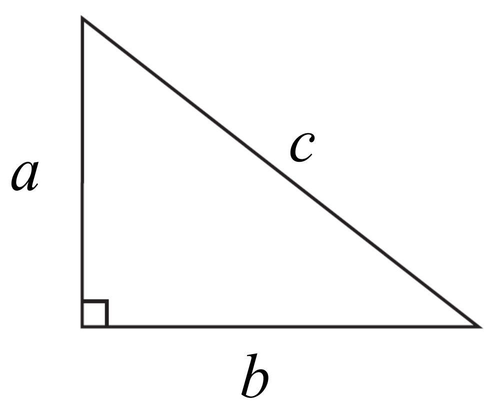


Pythagorean Theorem
Finding the Missing Leg Measure Worksheet
Example 3

Square C	Square A	Square B	Side Length of Square B

Square C – Square A

Method 1
Squares on the Sides of a Triangle

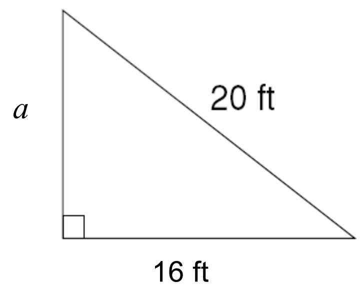


Method 2
Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Pythagorean Theorem
Finding the Missing Leg Measure
You Try 3

Find the missing length in the right triangle.



Directions for You Try 3

1. Use the Finding the Missing Leg Measure worksheet to record your work.
2. Color square A, yellow and square C, orange. Cut square A and square C out.
3. Label the side lengths of square A.
4. What is the area of square A?
5. Label the side lengths of square C.
6. What is the area of square C?
7. What is the difference of the area of square C and square A?
8. Draw a third ***square*** on the remaining graph paper using the ***difference***.
9. Color the third square pink and label it square B.
10. Label the side lengths of square B.
11. What is the area of square B?
12. Arrange square A, B and C to form a right triangle. Glue onto worksheet under Method 1.
13. Outline the right triangle green.

**Use a piece of graph paper and draw square B and square C based on their side lengths.*

Pythagorean Theorem
Finding the Missing Leg Measure Worksheet
You Try 3

Square C	Square A	Square B	Side Length of Square B

Square C − Square A

Method 1
Squares on the Sides of a Triangle

Method 2
Pythagorean Theorem

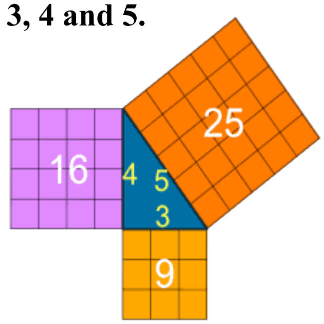
$$a^2 + b^2 = c^2$$

Pythagorean Theorem
The Pythagorean Theorem Triples
Example 4

A *Pythagorean triple* is an ordered triple (a, b, c) of three positive integers such that $a^2 + b^2 = c^2$.

The smallest Pythagorean Triple is 3, 4 and 5.

$a = 3,\ b = 4,\ \text{and } c = 5$



$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

$\therefore$ It is true. The Pythagorean Triple of 3, 4 and 5 makes a Right Angled Triangle

Here is a list of the first four Pythagorean Triples and their multiples:

**Students should memorize the first 4 Pythagorean Triples.*

Multiples	**(3,4,5)**	**(5,12,13)**	**(7,24,25)**	**(8,15,17)**
×2	(6,8,10)	(10,24,26)	(14,48,50)	(16,30,34)
×3	(9,12,15)	(15,36,39)	(21,72,75)	(24,45,51)
×4	(12,16,20)	(20,48,52)	(28,96,100)	(32,60,68)
×5	(15,20,25)	(25,60,65)	(35,120,125)	(40,75,85)

You Try 4

Tell whether the given side lengths are a Pythagorean Triple.

You Try 4a

$a = 9, b = 11, c = 14$

You Try 4b

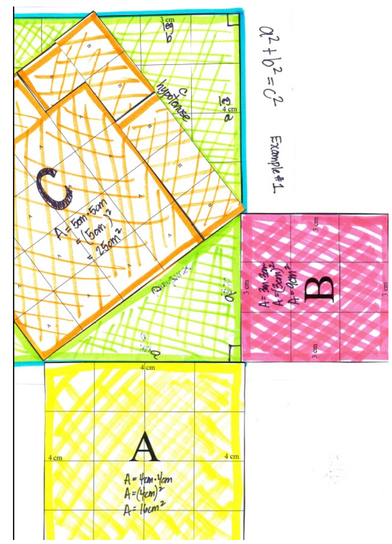
$a = 12, b = 35, c = 37$

Solutions

EX 1)

Square A	Square B	Square C	Green Triangles	Blue Square
$A = 4cm \bullet 4cm$ $= 16cm^2$	$A = 3cm \bullet 3cm$ $= 9cm^2$	$A = 5cm \bullet 5cm$ $= 25cm^2$	$A = 4\left(\frac{a \bullet b}{2}\right)$ $= 4\left(\frac{4 \bullet 3}{2}\right)$ $= 4(6)$ $= 24cm^2$	$A = 7cm \bullet 7cm$ $= 49cm^2$

Square A + Square B	Square C + Green Triangles
$16cm^2 + 9cm^2$ $= 25cm^2$	$25cm^2 + 24cm^2$ $= 49cm^2$

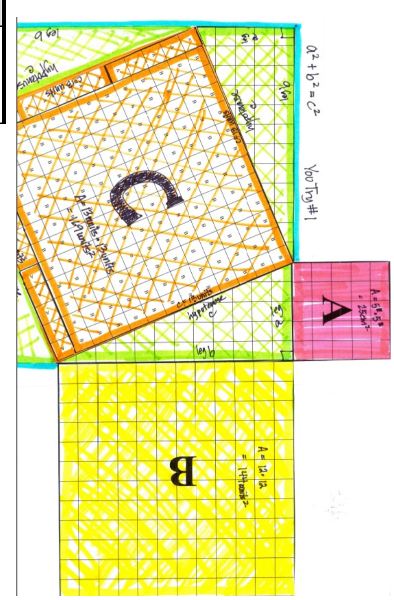


Pythagorean Theorem	Converse	Proof
$a^2 + b^2 = c^2$ $4^2 + 3^2 = c^2$ $(4 \bullet 4) + (3 \bullet 3) = c^2$ $16 + 9 = c^2$ $25 = c^2$ $\sqrt{25} = \sqrt{c^2}$ $\sqrt{5 \bullet 5} = \sqrt{c \bullet c}$ $5 = c$ ∴ The hypotenuse is 5 cm long	$a = 4,\ b = 3,\ c = 5$ $4^2 + 3^2 = 5^2$ $16 + 9 = 25$ $25cm^2 = 25cm^2$ ∴ It is a right angled triangle	$A_{Blue} = A_{Orange} + A_{\Delta}$ $289cm^2 = 169cm^2 + 120cm^2$ $289cm^2 = 289cm^2$ ∴ Pythagorean Theorem is proven true.

YT 1)

Square A	Square B	Square C	Green Triangles	Blue Square
$A = 5_{units} \bullet 5_{units}$ $= 25_{units}{}^2$	$A = 12_{units} \bullet 12_{units}$ $= 144_{units}{}^2$	$A = 13_{units} \bullet 13_{units}$ $= 169_{units}{}^2$	$A = 4\left(\frac{a \bullet b}{2}\right)$ $= 4\left(\frac{5 \bullet 12}{2}\right)$ $= 4(30)$ $= 120_{units}{}^2$	$A = 17_{units} \bullet 17_{units}$ $= 289_{units}{}^2$

Square A + Square B	Square C + Green Triangles
$25_{units}{}^2 + 144_{units}{}^2$ $= 169_{units}{}^2$	$169_{units}{}^2 + 120_{units}{}^2$ $= 289_{units}{}^2$



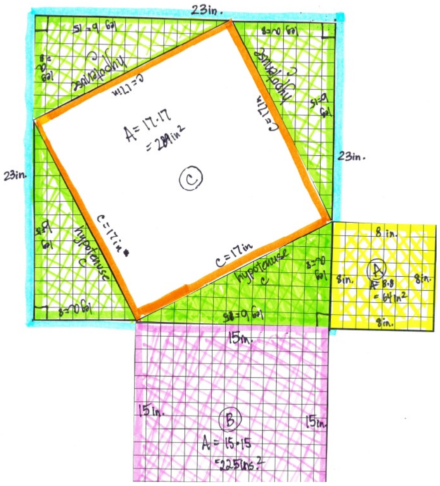
Pythagorean Theorem	Converse	Proof
$a^2 + b^2 = c^2$ $5^2 + 12^2 = c^2$ $(5 \bullet 5) + (12 \bullet 12) = c^2$ $25 + 144 = c^2$ $169 = c^2$ $\sqrt{169} = \sqrt{c^2}$ $\sqrt{13 \bullet 13} = \sqrt{c \bullet}$ $13 = c$ ∴ The hypotenuse is 13 cm long	$a = 5,\ b = 12,\ c = 13$ $5^2 + 12^2 = 13^2$ $25 + 144 = 169$ $169_{units}{}^2 = 169_{units}{}^2$ ∴ It is a right angled triangle.	$A_{Blue} = A_{Orange} + A_{\Delta}$ $289_{units}{}^2 = 169_{units}{}^2 + 120_{units}{}^2$ $289_{units}{}^2 = 289_{units}{}^2$ ∴ Pythagorean Theorem is proven true.

EX 2)

Square A	Square B	Square C	Green Triangles	Blue Square
$A = 8in \bullet 8in$ $= 64in^2$	$A = 15in \bullet 15in$ $= 225in^2$	$A = 17in \bullet 17in$ $= 289in^2$	$A = 4\left(\frac{8 \bullet 15}{2}\right)$ $= 4\left(\frac{120}{2}\right)$ $= 4(60)$ $= 240in^2$	$A = 23in \bullet 23in$ $= 529in^2$

Square A + Square B	Square C + Green Triangles
$64in^2 + 225in^2$ $= 289in^2$	$289in^2 + 240in^2$ $= 529in^2$

Pythagorean Theorem	Converse	Proof
$a^2 + b^2 = c^2$ $8^2 + 15^2 = c^2$ $(8 \bullet 8) + (15 \bullet 15) = c^2$ $64 + 225 = c^2$ $289 = c^2$ $\sqrt{289} = \sqrt{c^2}$ $\sqrt{17 \bullet 17} = \sqrt{c \bullet c}$ $17 = c$ ∴ The hypotenuse is 17 in. long	$a = 8,\ b = 15,\ c = 17$ $8^2 + 15^2 = 17^2$ $64 + 225 = 289$ $289in^2 = 289in^2$ ∴ It is a right angled triangle.	$A_{Blue} = A_{Orange} + A_{\Delta}$ $529in^2 = 289in^2 + 240in^2$ $529in^2 = 529in^2$ ∴ Pythagorean Theorem is proven true.

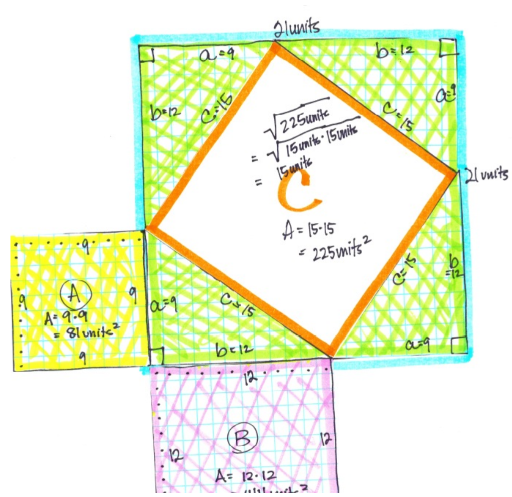


YT 2)

Square A	Square B	Square C	Green Triangles	Blue Square
$A = 9_{units} \bullet 9_{units}$ $= 81_{units}{}^2$	$A = 12_{units} \bullet 12_{units}$ $= 144_{units}{}^2$	$A = 15_{units} \bullet 15_{units}$ $= 225_{units}{}^2$	$A = 4\left(\frac{8 \bullet 15}{2}\right)$ $= 4\left(\frac{120}{2}\right)$ $= 4(60)$ $= 240in^2$	$A = 21_{units} \bullet 21_{units}$ $= 441_{units}{}^2$

Square A + Square B	Square C + Green Triangles
$= 81_{units}{}^2 + 144_{units}{}^2$ $= 225_{units}{}^2$	$= 225_{units}{}^2 + 216_{units}{}^2$ $= 441_{units}{}^2$

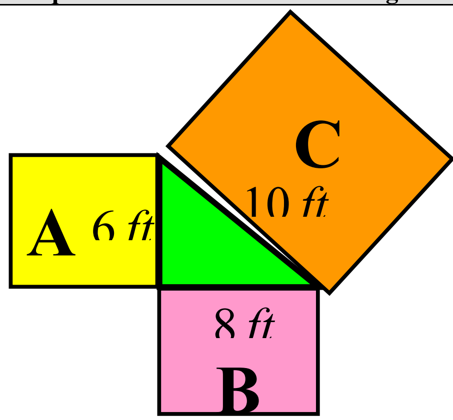
Pythagorean Theorem	Converse	Proof
$a^2 + b^2 = c^2$ $9^2 + 12^2 = c^2$ $(9 \bullet 9) + (12 \bullet 12) = c^2$ $81 + 144 = c^2$ $225 = c^2$ $\sqrt{225} = \sqrt{c^2}$ $\sqrt{15 \bullet 15} = \sqrt{c \bullet c}$ $15 = c$ ∴ The hypotenuse is 15 units long	$a = 8,\ b = 15,\ c = 17$ $9^2 + 12^2 = 15^2$ $81 + 144 = 225$ $225_{units}{}^2 = 225_{units}{}^2$ ∴ It is a right angled triangle.	$A_{Blue} = A_{Orange} + A_{\Delta}$ $441_{units}{}^2 = 225_{units}{}^2 + 216_{units}{}^2$ $441_{units}{}^2 = 441_{units}{}^2$ ∴ Pythagorean Theorem is proven true.



EX 3)

Square C	Square A	Square B	Side Length of Square B
$A = 10\,ft \bullet 10\,ft$ $= 100\,ft^2$	$A = 6\,ft \bullet 6\,ft$ $= 36\,ft^2$	$A = 8\,ft \bullet 8\,ft$ $= 64\,ft^2$	$b = 8\,ft$

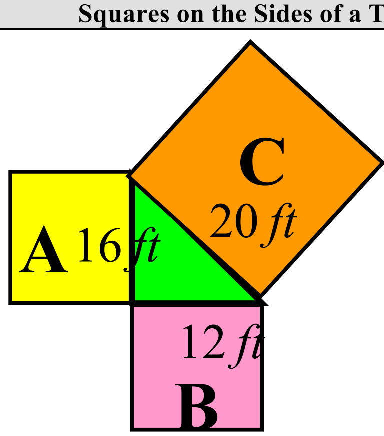
Square C − Square A
$= 100\,ft^2 - 36\,ft^2$ $= 64\,ft^2$

Method 1 Squares on the Sides of a Triangle	Method 2 Pythagorean Theorem
	$a^2 + b^2 = c^2$ $6^2 + b^2 = 10^2$ $(6 \bullet 6) + b^2 = (10 \bullet 10)$ $36 + b^2 = 100$ $36 - 36 + b^2 = 100 - 36$ $b^2 = 64$ $\sqrt{b^2} = \sqrt{64}$ $\sqrt{b \bullet b} = \sqrt{8 \bullet 8}$ $b = 8$ ∴ The leg is 8 ft long.

YT 3)

Square C	Square A	Square B	Side Length of Square B
$A = 20\,ft \bullet 20\,ft$ $= 400\,ft^2$	$A = 16\,ft \bullet 16\,ft$ $= 256\,ft^2$	$A = 12\,ft \bullet 12\,ft$ $= 144\,ft^2$	$b = 12\,ft$

Square C − Square A
$= 400\,ft^2 - 256\,ft^2$ $= 144\,ft^2$

Method 1 Squares on the Sides of a Triangle	Method 2 Pythagorean Theorem
	$a^2 + b^2 = c^2$ $16^2 + b^2 = 10^{22}$ $(16 \bullet 16) + b^2 = (20 \bullet 20)$ $256 + b^2 = 400$ $256 - 256 + b^2 = 400 - 256$ $b^2 = 144$ $\sqrt{b^2} = \sqrt{144}$ $\sqrt{b \bullet b} = \sqrt{12 \bullet 12}$ $b = 12$ ∴ The leg is 12 ft long.

YT 4a)

$$a = 9, b = 11, c = 14$$
$$a^2 + b^2 = c^2$$
$$9^2 + 11^2 = 14^2$$
$$(9 \bullet 9) + (11 \bullet 11) = (14 \bullet 14)$$
$$81 + 121 = 196$$
$$202 \neq 196$$
$\therefore$ It is not a Pythagorean Triple.

YT 4b)

$$a = 12, b = 35, c = 37$$
$$a^2 + b^2 = c^2$$
$$12^2 + 35^2 = 37^2$$
$$(12 \bullet 12) + (35 \bullet 35) = (37 \bullet 37)$$
$$144 + 1{,}225 = 1{,}369$$
$$1{,}369 = 1{,}369$$
$\therefore$ It is a Pythagorean Triple.

Warm-Up

CST/CAHSEE:

$$\sqrt{225}$$
$$= \sqrt{15 \bullet 15}$$
$$= 15$$
A

$$25^2 = 25 \bullet 25 = 625$$
$$35^2 = 35 \bullet 35 = 1{,}225$$
$$45^2 = 45 \bullet 45 = 2{,}025$$

Review:

B

Current:

B

Eliminate distracter C because the square root of 2 is an irrational number.
Eliminate distracter D because the missing side length can not be larger than 17. The perfect square of 17 is 289. Therefore the square root of 514 would be greater than 17.

Other:

B

You can eliminate A since the hypotenuse is the longest side of a right triangle.
3-4-5 is a Pythagorean Triple.