Grade Level/Course: Geometry
Lesson/Unit Plan Name: Law of Sines
Rationale/Lesson Abstract: Students will derive the Law of Sines and then use it to find the missing side lengths and angles of an oblique triangle. Students will then extend this principle to right triangles providing an alternative method to right triangle trigonometry.
Timeframe: 120 minutes
Common Core Standard(s): **G.SRT.10** Prove the Laws of Sines and Cosines and use them to solve problems. **G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.

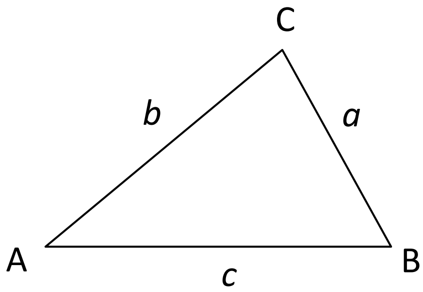
Instructional Resources/Materials:
Warm up, student note-taking guide, paper, pencil, and access to a calculator.

Answers to Warm Up:

1) Yes 2) No 3) Yes 4) No 5) Yes	EF, DE, DF By finding all the angles in the triangle we can order the angle measures from least to greatest. The Triangle Inequality Theorem says the sides opposite of those angles will be in that same order.
$\sin x° = \frac{11}{14}$ $x° = \sin^{-1}\left(\frac{11}{14}\right)$ $x° = 51.786...°$ $x° \approx 51.8°$	$\frac{10\sin 17°}{\sin 58°}$ $= 3.447...$ ≈ 3.4 This calculator work needs to be taught explicitly whether your students are using a scientific calculator, graphing calculator, or a non-scientific calculator and the table on page 10 of this lesson.

Activity/Lesson:
Pass out the note-taking guide and begin by defining the side lengths and angles. Then ask students if they notice anything about the placement of the letters (the side length a is opposite of angle A, the side length b is opposite of angle B, and the side length c is opposite of angle C). This observation is significant for using the Law of Sines.

Law of Sines:



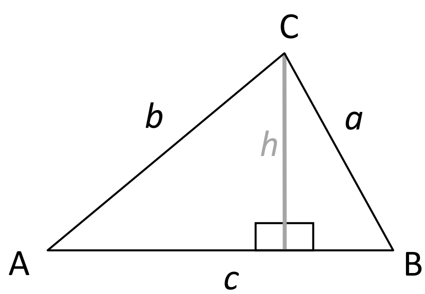
Side lengths: a, b, c (lower case)

Angles: $\angle A, \angle B, \angle C$ (upper case)

Think Pair Share – Is $\sin A = \frac{a}{b}$ ***? Why or why not?***

Share some thoughts from the class and make sure someone points out that this triangle is not a right triangle.

Draw in the height of triangle ABC from vertex C and label it "h". Point out that you have created two right triangles and then have students fill in the two sine ratios from the two right triangles.



$\sin A = \frac{h}{b}$ $\qquad$ $\sin B = \frac{h}{a}$

Solve each equation for h, substitute and then manipulate the equation to derive part of the Law of Sines:

$\sin A = \frac{h}{b}$ $\qquad$ $\sin B = \frac{h}{a}$

$\text{b}\bullet \sin A = b \bullet \frac{h}{b}$ $\qquad$ $a \bullet \sin B = a \bullet \frac{h}{a}$

$b \sin A = h$ $\qquad$ $a \sin B = h$

$$b \sin A = a \sin B \quad (\textit{substitution})$$

$$\frac{b \sin A}{ab} = \frac{a \sin B}{ab}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Point out that the other triangle on the page is the same triangle just rotated so that side a is on the bottom. Derive the rest of the Law of Sines (depending on the level of your students they might be able to do this on their own).

$$\sin B = \frac{h_2}{c} \qquad \sin C = \frac{h_2}{b}$$

$$c \bullet \sin B = c \bullet \frac{h_2}{c} \qquad b \bullet \sin C = b \bullet \frac{h_2}{b}$$

$$c \sin B = h_2 \qquad b \sin C = h_2$$

$$c \sin B = b \sin C \quad (substitution)$$

$$\frac{c \sin B}{bc} = \frac{b \sin C}{bc}$$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

Write out the Law of Sines on the next page of the note-taking guide:

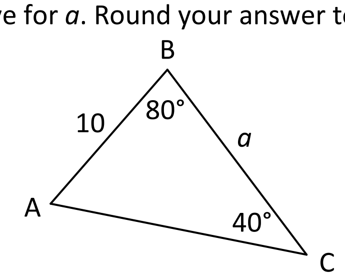
Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Continue to example 1. Call on different students randomly to answer the following questions below:

<u>Example 1</u>

Solve for *a*. Round your answer to the nearest tenth.



Are we solving for a side length or an angle measure? (side length)

Do we know the measure of the angle opposite of *a*? (no)

How can we find it? (Triangle Sum Theorem)

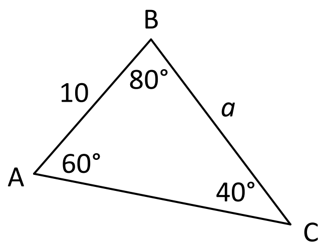
$$m\angle A + m\angle B + m\angle C = 180°$$

$$m\angle A + 80° + 40° = 180°$$

$$m\angle A + 120° = 180°$$

$$m\angle A = 60°$$

Write the result of $m\angle A$ ***in the diagram:***



$$\frac{\sin 60°}{a} = \frac{\sin 40°}{10}$$

$$\frac{a}{\sin 60°} = \frac{10}{\sin 40°}$$

Reciprocating the proportion before solving allows the students to be more adept with manipulating proportions.

$$\frac{a}{\sin 60°} \bullet \sin 60° = \frac{10}{\sin 40°} \bullet \sin 60°$$

Multiplying by $\sin 60°$ allows students to recognize that $\sin 60°$ is one number.

$$a = \frac{10 \sin 60°}{\sin 40°}$$

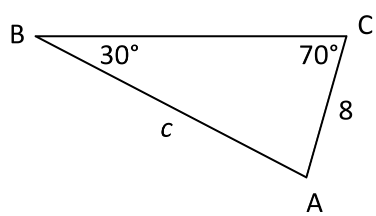
$$a = 13.472...$$

$$a \approx 13.5$$

Be aware that students might need help putting this in their calculators

Have students do the "You Try". After a few minutes, have students share their work with a partner.

Solve for *c*. Round your answer to the nearest tenth.



$$\frac{\sin 70°}{c} = \frac{\sin 30°}{8}$$

$$\frac{c}{\sin 70°} = \frac{8}{\sin 30°}$$

$$\frac{c}{\sin 70°} \bullet \sin 70° = \frac{8}{\sin 30°} \bullet \sin 70°$$

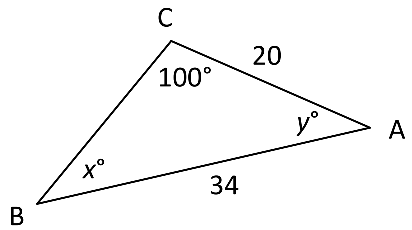
$$a = \frac{8 \sin 70°}{\sin 30°}$$

$$a = 15.035...$$

$$a \approx 15.0$$

Continue to example 2:

Solve for *x* and *y*. Round answers to the nearest tenth.



Choral Response:

Are we solving for side lengths or angle measures? (angle measures)

Do we know the side length opposite of one of the angles? (yes)

Which angle? (*x*)

Start writing the formula out, pause, and have students assist you.

$$\frac{\sin x°}{?}$$

$$\frac{\sin x°}{20}$$

$$\frac{\sin x°}{20} = \frac{?}{?}$$

$$\frac{\sin x°}{20} = \frac{\sin 100°}{?}$$

$$\frac{\sin x°}{20} = \frac{\sin 100°}{34}$$

Solve the proportion:

$$\frac{\sin x°}{20} = \frac{\sin 100°}{34}$$

$$\frac{\sin x°}{20} \bullet 20 = \frac{\sin 100°}{34} \bullet 20$$

$$\sin x° = \frac{20 \sin 100°}{34}$$

$$\sin x° = 0.57929...$$

$$x° = \sin^{-1}(0.57929...)$$

$$x° = 35.401...°$$

$$x° \approx 35.4°$$

After solving for x, find y using the triangle sum theorem:

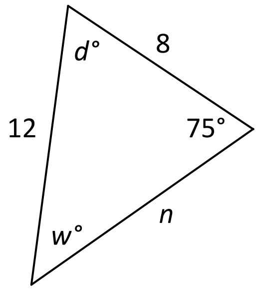
$$x° + y° + 100° = 180°$$

$$35.4° + y° + 100° \approx 180°$$

$$y° + 135.4° \approx 180°$$

$$y° \approx 44.6°$$

Continue to example 3:
Solve for *d, n,* and *w*. Round answers to the nearest tenth.



Start by writing the numerators of the entire Law of Sines. Then give students a minute to fill in the denominators with the appropriate side lengths:

$$\frac{\sin d°}{?} = \frac{\sin w°}{?} = \frac{\sin 75°}{?}$$

$$\frac{\sin d°}{n} = \frac{\sin w°}{8} = \frac{\sin 75°}{12}$$

Then point out that the first ratio has two variables, the second ratio has one variable and the last ratio has no variables. Using a proportion with the last two ratios creates an equation with only one variable, w, and allows us to solve for its value:

$$\frac{\sin w°}{8} = \frac{\sin 75°}{12}$$

$$8 \bullet \frac{\sin w°}{8} = 8 \bullet \frac{\sin 75°}{12}$$

$$\sin w° = \frac{8 \sin 75°}{12}$$

$$w° = \sin^{-1}\left(\frac{8 \sin 75°}{12}\right)$$

$$w° = 40.087...°$$

$$w° \approx 40.1°$$

Write the result of w in the diagram and then find d using the Triangle Sum Theorem:

$$d° + w° + 75° = 180°$$
$$d° + 40.1° + 75° \approx 180°$$
$$d° + 115.1° \approx 180°$$
$$d° \approx 64.9°$$

Rewrite the entire proportion with the estimated angle measures and then set up a proportion with two ratios to solve:

$$\frac{\sin 64.9°}{n} \approx \frac{\sin 38.5°}{8} \approx \frac{\sin 75°}{12}$$

$$\frac{\sin 64.9°}{n} \approx \frac{\sin 75°}{12}$$

$$\frac{\sin 64.9°}{\sin 75°} \approx \frac{n}{12}$$

Switching the means in the proportion before solving allows the students to be more adept with manipulating proportions.

$$12 \bullet \frac{\sin 64.9°}{\sin 75°} \approx 12 \bullet \frac{n}{12}$$

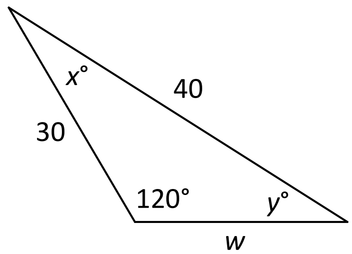
$$\frac{12 \sin 64.9°}{\sin 75°} \approx n$$

$$11.250... \approx n$$

$$11.3 \approx n$$

Have the students do the "You Try". Make sure to give students ample time to complete. Find a student with good syntax and clear work and have them explain to the class what they did.

Solve for *w, x,* and *y*. Round answers to the nearest tenth.



Solving for *y*:

$$\frac{\sin 120°}{40}=\frac{\sin y°}{30}$$

$$\frac{30\sin 120°}{40}=\sin y°$$

$$\sin^{-1}\left(\frac{30\sin 120°}{40}\right)=y°$$

$$40.5\approx y°$$

Solving for *x*:

$$x°+y°+120°=180°$$

$$x°+40.5°+120°\approx 180°$$

$$x°+160.5°\approx 180°$$

$$x°\approx 19.5°$$

Solving for *w*:

$$\frac{\sin 120°}{40}=\frac{\sin x°}{w}$$

$$\frac{\sin 120°}{40}\approx\frac{\sin 19.5°}{w}$$

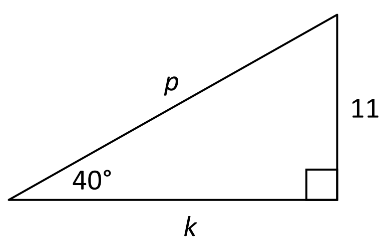
$$\frac{w}{40}\approx\frac{\sin 19.5°}{\sin 120°}$$

$$w\approx\frac{40\sin 19.5°}{\sin 120°}$$

$$w\approx 15.4$$

Continue to example 4:

Solve for *p* and *k*. Round answers to the nearest tenth.



What kind of triangle do we have? (Right Triangle)

Does the Law of Sines work on right triangles? (Let's find out)

Solving for *p:*

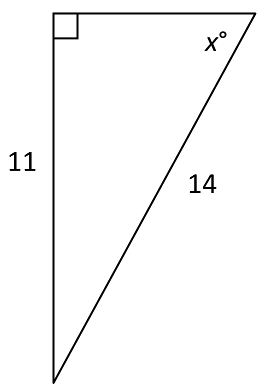
Using Right Triangle Trigonometry:	Using the Law of Sines:
$\sin 40°=\frac{11}{p}$	$\frac{\sin 40°}{11}=\frac{\sin 90°}{p}$
$\frac{\sin 40°}{1}=\frac{11}{p}$	$\frac{\sin 40°}{11}=\frac{1}{p}$ ← Take the opportunity to show that $\sin 90°=1$
$\frac{p}{1}=\frac{11}{\sin 40°}$	$\frac{11}{\sin 40°}=\frac{p}{1}$
$p=\frac{11}{\sin 40°}$	$\frac{11}{\sin 40°}=p$
$p\approx 17.1$	$17.1\approx p$

Solving for *k:*

Using Right Triangle Trigonometry:	Using the Law of Sines:
$\tan 40° = \frac{11}{k}$ $\frac{\tan 40°}{1} = \frac{11}{k}$ $\frac{k}{1} = \frac{11}{\tan 40°}$ $k = \frac{11}{\tan 40°}$ $k \approx 13.1$	(After finding the missing angle to be 50°) $\frac{\sin 40°}{11} = \frac{\sin 50°}{k}$ $\frac{k}{11} = \frac{\sin 50°}{\sin 40°}$ $11 \bullet \frac{k}{11} = 11 \bullet \frac{\sin 50°}{\sin 40°}$ $k = \frac{11 \sin 50°}{\sin 40°}$ $k \approx 13.1$

Possible Exit Ticket/Additional You Try:

Use the Law of Sines to solve for *x* in the right triangle. Round your answer to the nearest tenth.



Answer:

$$\frac{\sin x°}{11} = \frac{\sin 90°}{14}$$

$$\frac{\sin x°}{11} = \frac{1}{14}$$

$$\sin x° = \frac{11}{14} \quad \leftarrow \quad \text{(same with right triangle trigonometry)}$$

$$x° = \sin^{-1}\left(\frac{11}{14}\right)$$

$$x° = 51.8°$$

Warm-Up

CCSS: 7.RP.2

Determine whether each proportion is equivalent to:

$$\frac{a}{b}=\frac{3}{5}$$

1) $\frac{b}{a}=\frac{5}{3}$ Ⓐ Yes Ⓑ No

2) $\frac{a}{5}=\frac{3}{b}$ Ⓐ Yes Ⓑ No

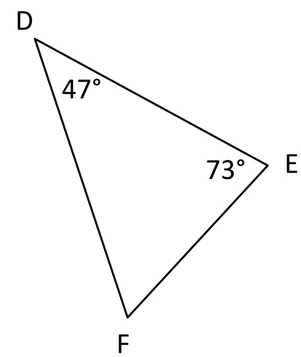
3) $\frac{5}{b}=\frac{3}{a}$ Ⓐ Yes Ⓑ No

4) $\frac{a}{3}=\frac{5}{b}$ Ⓐ Yes Ⓑ No

5) $\frac{a+a}{b+b}=\frac{3}{5}$ Ⓐ Yes Ⓑ No

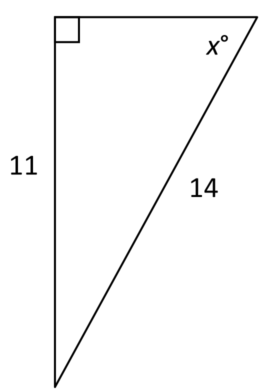
Review: G.GMD.6

List the side lengths in order from least to greatest. Justify your reasoning.



Current: G.SRT.8

Solve for *x*. Round to the nearest tenth.



Calculator Practice:

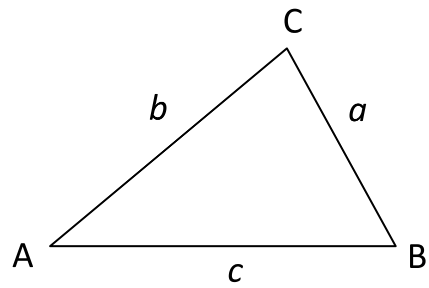
Use a calculator to evaluate $\frac{10\sin 17^\circ}{\sin 58^\circ}$ **and round to the nearest tenth.**

Table of Trigonometric Ratios

Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175
2°	.0349	.9994	.0349
3°	.0523	.9986	.0524
4°	.0698	.9976	.0699
5°	.0872	.9962	.0875
6°	.1045	.9945	.1051
7°	.1219	.9925	.1228
8°	.1392	.9903	.1405
9°	.1564	.9877	.1584
10°	.1736	.9848	.1763
11°	.1908	.9816	.1944
12°	.2079	.9781	.2126
13°	.2250	.9744	.2309
14°	.2419	.9703	.2493
15°	.2588	.9659	.2679
16°	.2756	.9613	.2867
17°	.2924	.9563	.3057
18°	.3090	.9511	.3249
19°	.3256	.9455	.3443
20°	.3420	.9397	.3640
21°	.3584	.9336	.3839
22°	.3746	.9272	.4040
23°	.3907	.9205	.4245
24°	.4067	.9135	.4452
25°	.4226	.9063	.4663
26°	.4384	.8988	.4877
27°	.4540	.8910	.5095
28°	.4695	.8829	.5317
29°	.4848	.8746	.5543
30°	.5000	.8660	.5774
31°	.5150	.8572	.6009
32°	.5299	.8480	.6249
33°	.5446	.8387	.6494
34°	.5592	.8290	.6745
35°	.5736	.8192	.7002
36°	.5878	.8090	.7265
37°	.6018	.7986	.7536
38°	.6157	.7880	.7813
39°	.6293	.7771	.8098
40°	.6428	.7660	.8391
41°	.6561	.7547	.8693
42°	.6691	.7431	.9004
43°	.6820	.7314	.9325
44°	.6947	.7193	.9657
45°	.7071	.7071	1.0000

Angle	Sine	Cosine	Tangent
46°	.7193	.6947	1.0355
47°	.7314	.6820	1.0724
48°	.7431	.6691	1.1106
49°	.7547	.6561	1.1504
50°	.7660	.6428	1.1918
51°	.7771	.6293	1.2349
52°	.7880	.6157	1.2799
53°	.7986	.6018	1.3270
54°	.8090	.5878	1.3764
55°	.8192	.5736	1.4281
56°	.8290	.5592	1.4826
57°	.8387	.5446	1.5399
58°	.8480	.5299	1.6003
59°	.8572	.5150	1.6643
60°	.8660	.5000	1.7321
61°	.8746	.4848	1.8040
62°	.8829	.4695	1.8807
63°	.8910	.4540	1.9626
64°	.8988	.4384	2.0503
65°	.9063	.4226	2.1445
66°	.9135	.4067	2.2460
67°	.9205	.3907	2.3559
68°	.9272	.3746	2.4751
69°	.9336	.3584	2.6051
70°	.9397	.3420	2.7475
71°	.9455	.3256	2.9042
72°	.9511	.3090	3.0777
73°	.9563	.2924	3.2709
74°	.9613	.2756	3.4874
75°	.9659	.2588	3.7321
76°	.9703	.2419	4.0108
77°	.9744	.2250	4.3315
78°	.9781	.2079	4.7046
79°	.9816	.1908	5.1446
80°	.9848	.1736	5.6713
81°	.9877	.1564	6.3138
82°	.9903	.1392	7.1154
83°	.9925	.1219	8.1443
84°	.9945	.1045	9.5144
85°	.9962	.0872	11.4301
86°	.9976	.0698	14.3007
87°	.9986	.0523	19.0811
88°	.9994	.0349	28.6363
89°	.9998	.0175	57.2900

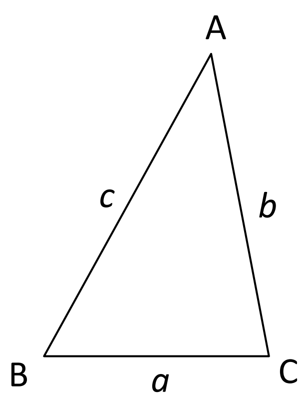
Law of Sines: Note-Taking Guide



Side lengths: ______________________________

Angles: ______________________________

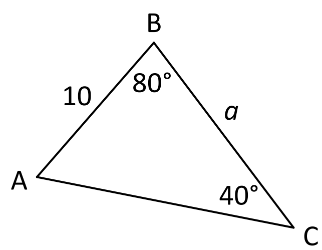
$\sin A = \frac{\quad}{\quad}$ $\sin B = \frac{\quad}{\quad}$



Law of Sines:

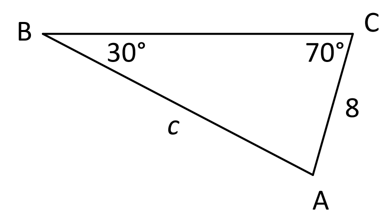
Example 1

Solve for *a*. Round your answer to the nearest tenth.



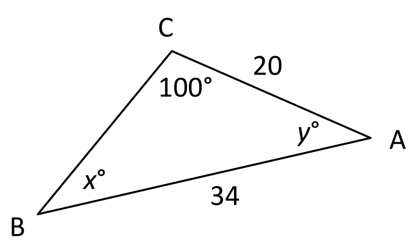
You Try

Solve for *c*. Round your answer to the nearest tenth.



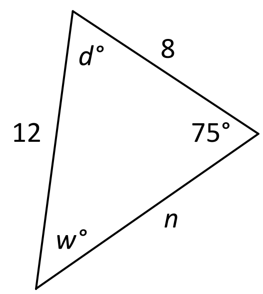
Example 2

Solve for *x* and *y*. Round answers to the nearest tenth.



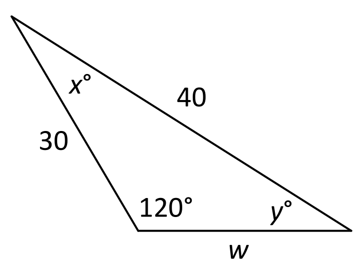
Example 3

Solve for *d, n,* and *w*. Round answers to the nearest tenth.



You Try

Solve for *w, x,* and *y*. Round answers to the nearest tenth.



Example 4

Solve for p and k. Round answers to the nearest tenth.