## Warm-Up Activity

- Copy the warm-up back to back, using the **same** copy on both sides. Students will do each proof two ways, once on the front and once on the back, if time permits.

- Photocopy, cut out and place in bags, the statements and reasons for the Flow Chart Proof. (The steps are provided below, in order, as they should appear in the proof.)

- Students should complete the warm-up in pairs. (There are **many** possible proofs!)

- The first problem is to be done using a flowchart proof. Students will piece together this proof by pairing Statements with appropriate Reasons. Then, students will connect each pair of Statements/Reasons in a logical order using arrows. They will tape the proof in place, on the front of the page, once they feel confident that it is complete.

- On the back of the page, if time permits, students will do the same proof another way, using a proof method of choice. (Paragraph, Two-Column or Flow Chart proof.)

- The second problem is to be done using a proof method of choice. Again, if time permits, students will do the same proof another way.

- Have four pairs present the two proofs, two pairs for each problem, each pair presenting a different proof.
    - The students should use two different ways of **organizing** the proofs.
        - Paragraph proof, Flow Chart proof and Two Column proof
    - The students should use two different **approaches** to proving the end result.
        - Congruent triangles (SSS, ASA, etc… Parallel lines, etc…)

- The following points are important and will be essential to the activity that follows:

    - In the first proof, concerning $\Delta$ABC, draw students' attention to the fact that $\angle$ BAM and $\angle$ CAM are congruent. Have students give another name to $\overline{AM}$; they may call it an angle bisector since it created two congruent angles within $\angle$BAC.

    - In the second proof, concerning square PQRS, draw students' attention to the fact that $\angle$PQS, $\angle$RSQ, $\angle$ PSQ and $\angle$ RQS are ALL congruent to one another. There are many ways to explain this, however, we may state that $\overline{QS}$ is not only the diagonal of square PQRS but is also an angle bisector of both right angles Q and S. Therefore, all four angles must measure $45^\circ$.

## **<u>Steps for Flow Chart Proof:</u>**

(In logical order)

| | |
|---|---|
| $\Delta$ABC is equilateral<br>$\overline{AM}$ is the $\perp$ bisector of $\overline{BC}$ | Given |
| $\overline{BM} \cong \overline{MC}$ | Definition of perpendicular bisector |
| $\overline{AB} \cong \overline{AC}$ | Definition of equilateral $\Delta$ |
| $\overline{AM} \cong \overline{AM}$ | Reflexive Property |
| $\Delta AMB \cong \Delta AMC$ | Side-Side-Side (SSS) |
| $\angle BAM \cong \angle CAM$ | Corresp. Parts of $\cong$ $\Delta$'s are $\cong$ (CPCTC) |

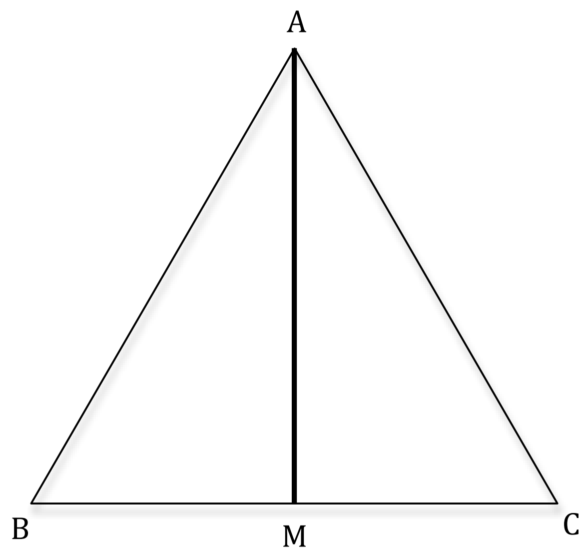
| | |
|---|---|
| **<u>Possible Solution</u>: Triangle Proof**<br>We are given that $\Delta$ABC is equilateral and $\overline{AM}$ is the perpendicular bisector of $\overline{BC}$. Therefore, $\angle$AMB and $\angle$AMC are right angles and are also congruent. $\overline{BM}$ is congruent to $\overline{CM}$ since M is the midpoint of $\overline{BC}$. $\overline{AM}$ is congruent to itself by the Reflexive Property. Therefore, $\Delta$AMB and $\Delta$AMC are congruent by Side-Angle-Side Congruence. Finally, $\Delta$BAM and $\Delta$CAM are congruent since Corresponding Parts of Congruent Triangles are Congruent. | **<u>Possible Solution</u>: Square Proof**<br>We are given that PQRS is a square. We may draw $\overline{SQ}$ since two points determine a segment. This creates two right triangles, $\Delta$PQS and $\Delta$RSQ. We know that $\angle$QPS and $\angle$SRQ are right angles since PQRS is a square. $\overline{PQ}$ is congruent to $\overline{RS}$ since all sides of a square are congruent by definition. Additionally, $\overline{SQ}$ is congruent to itself by the Reflexive Property. Therefore, the two triangles are congruent by Hypotenuse-Leg Congruence. Finally, $\angle$RQS and $\angle$PSQ and $\angle$PQS and $\angle$RSQ are congruent since Corresponding Parts of Congruent Triangles are Congruent. |

## Warm-Up

Complete the proof by creating a flow chart on one side. Prove a second way on the other.

**Given:** $\Delta ABC$ is equilateral.
$\overline{AM}$ is the $\perp$ bisector of $\overline{BC}$.

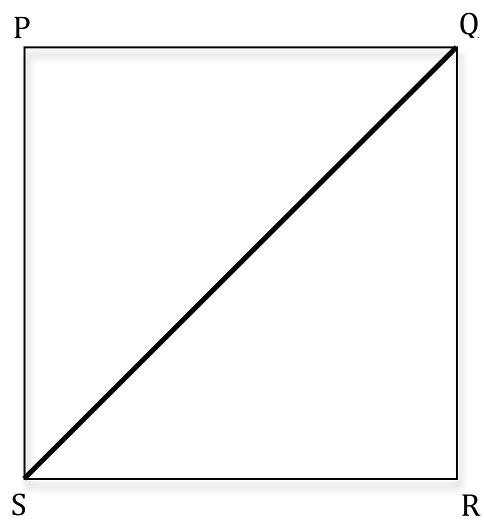
**Prove:** $\angle BAM \cong \angle CAM$




Complete the proof on one side using a method of your choice. Use another method on the back.

**Given:** Square $PQRS$

**Prove:** $\angle PQS \cong \angle RSQ$
$\angle PSQ \cong \angle RQS$



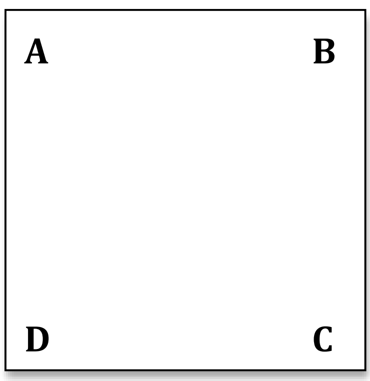
**Lesson Plan:**
(Written for a block schedule; feel free to spend one day on each type of triangle with a period schedule.)

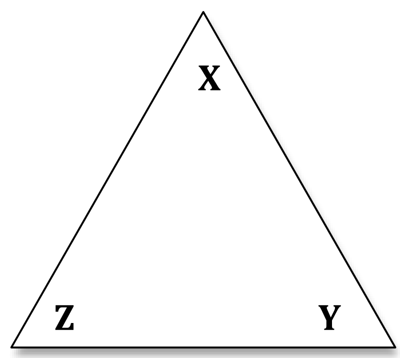
**Objective:** Students will use prior knowledge of the Pythagorean Theorem, triangles and basic Algebra in order to discover the side-angle relationships of special right triangles, specifically $30^\circ - 60^\circ - 90^\circ$ triangles and $45^\circ - 45^\circ - 90^\circ$ triangles.

**Materials**: Triangle/Square Backline Masters (one per pair), rulers (one per pair), Triangle/Square class posters, scissors, glue or tape, blank white paper (two per pair), pencils, practice problems, work samples.

**Investigation:**

**1**. Place students in appropriate pairs and pass out materials. Label the vertices of the triangle and the square. Make sure each pair is consistent and uses the same letters, in the same order on each figure. Each label placed on the figure should be placed **inside** the figure.
(See the diagram below.)




**2**. Have the pairs label everything they know about each figure and why. These labels should also be placed inside the figure. (Mark congruent parts, label angle measure, etc…)

- All sides of the triangle are congruent, all angles measure $60^\circ$.
- All sides of the square are congruent, all angles measure $90^\circ$.

**3**. Have two pairs come to the front and label what they found. Have them explain/justify each part. After this, tell students that since we do not know the length of each side, we will define the lengths using variables. The length of each side of the square will be "***m***" and the length of each side of the triangle will be "***2n***."

**4**. Next, have students carefully cut out each of the figures. Once they are cut out, tell students to fold the triangle in half, touching two of the vertices together, making a crease through the third vertex. (For example, fold triangle vertex Z so that it is on top of vertex Y, making a crease perpendicular to segment YZ, through vertex X.) Label the intersection of the fold and the base as point M. Help students understand why M is the Midpoint.

**5**. Have them fold along the diagonal of the square.
(Specify if they should fold vertex A onto vertex C or vertex B onto vertex D.)

**6**. Once the folds have been made, draw a dashed line over each fold using a ruler.

**7**. Now, have students label what they know. Ask them to state **why** they know.
(For this portion, ask students to refer to the warm-up to provide justification.)

- The fold in the triangle creates two congruent, $30^\circ - 60^\circ - 90^\circ$ triangles. Students may say the following: it creates a perpendicular bisector from the vertex angle to the base of the triangle; it bisects the vertex angle, creating two congruent, $30^\circ$ angles, etc… Also, the "short side" of the triangle now has length "n."
- The fold in the square creates two congruent, $45^\circ - 45^\circ - 90^\circ$ triangles. Students may say the following: it bisects the vertices, creating two congruent $45^\circ$ angles, etc…

**8**. Once these have been stated, have students cut along the dotted lines and label new information. Each student in the pair should receive half of the equilateral triangle and half of the square. Label the corresponding items on the poster in the front of the room.

**9**. Have students glue each triangle to each half of the paper, making sure to glue each off to the side to leave room for work. Label the top of each half as "$30^\circ - 60^\circ - 90^\circ$ Triangle" and "$45^\circ - 45^\circ - 90^\circ$ Triangle."

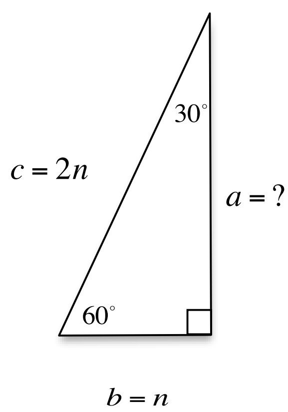
**10**. Ask students to state what they believe the measures of the "missing" sides to be. (The missing sides are the "long" side of the $30^\circ - 60^\circ - 90^\circ$ triangle and the hypotenuse of the $45^\circ - 45^\circ - 90^\circ$ triangle.) Once students have given various ideas, ask them if there is a way to find the lengths of the sides **exactly**. Hopefully, they will give the Pythagorean theorem as a place to start!

**11**. Using the Pythagorean theorem, find the lengths of the missing sides algebraically. Depending on the level of skill and independence of your students, you may choose to do the following:

- Walk the students through finding one of the missing sides; have them find the other with their partner.
- Walk the students through finding both of the missing sides.
- Start each problem and have them find each missing side with their partner.

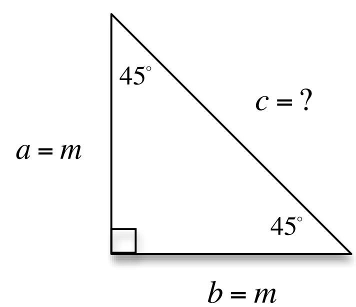
# <u>Completed Student Investigation</u>:

## $30^\circ - 60^\circ - 90^\circ$ **Triangle**



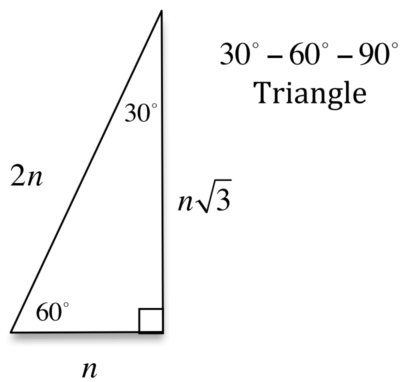

$$a^2 + b^2 = c^2$$
$$a^2 + (n)^2 = (2n)^2$$
$$a^2 + n^2 = (2)^2 \bullet (n)^2$$
$$a^2 + n^2 = 4n^2$$
$$a^2 + n^2 - n^2 = 4n^2 - n^2$$
$$a^2 = 3n^2$$
$$\sqrt{a^2} = \sqrt{3n^2}$$
$$a = \sqrt{3 \bullet n^2}$$
$$a = \sqrt{3} \bullet \sqrt{n^2}$$
$$a = n\sqrt{3}$$

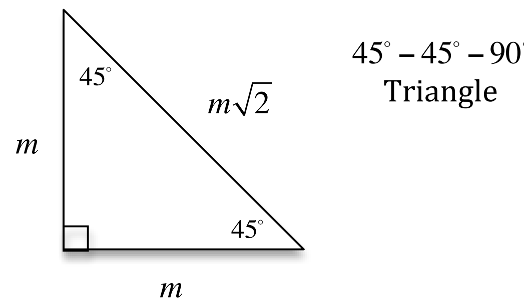
## $45^\circ - 45^\circ - 90^\circ$ **Triangle**




$$a^2 + b^2 = c^2$$
$$(m)^2 + (m)^2 = c^2$$
$$m^2 + m^2 = c^2$$
$$2m^2 = c^2$$
$$\sqrt{2m^2} = \sqrt{c^2}$$
$$\sqrt{2 \bullet m^2} = c$$
$$\sqrt{2} \bullet \sqrt{m^2} = c$$
$$m\sqrt{2} = c$$
$$c = m\sqrt{2}$$

**12**. Once you have found the lengths of the missing sides, label them. (See below.)



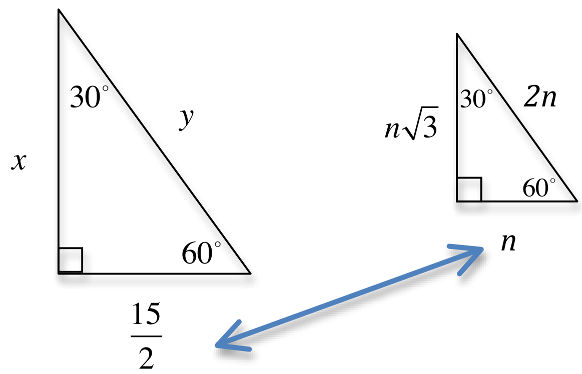

**13**. Explain to the students that each side length is a general representation of the lengths of the sides of the special right triangles. Since all special right triangles are also **similar** triangles, we may use these formulas to help us find the unknown lengths of sides of **all** special right triangles.

**14**. Move into the example problems to illustrate how to apply what we have just learned. Show students how to turn their figures so that they match up with corresponding sides of the sample problems. Help them to understand that once they have found corresponding sides, they must set the side length equal to the formula for that specific side in order to solve for "m" or "n." Help them understand that once they have found that value, they may use it to find the lengths of all of the other sides. Also, help them to draw a reference directly on the paper if they are having a difficult time finding the corresponding sides.

*If this is being used for non-block classes, use the problems on the next pages as quick examples and let the problems labeled "You Try" serve as an Exit Ticket.

## Example 1

Find the lengths of the missing sides.
(Find corresponding sides, set the lengths equal)




$$n = \frac{15}{2}$$

$$2(n) = (2)\frac{15}{2}$$

$$2n = 15$$

$$\therefore y = 15$$

$$n = \frac{15}{2}$$

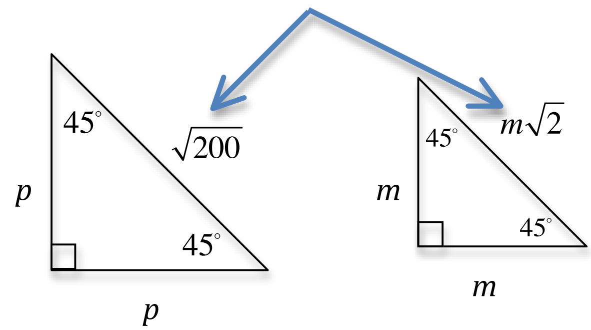
$$x = n\sqrt{3}$$

$$x = (\frac{15}{2})\sqrt{3}$$

$$\therefore x = \frac{15\sqrt{3}}{2}$$

## Example 2

Find the lengths of the missing sides.
(Find corresponding sides, set the lengths equal)




$$m\sqrt{2} = \sqrt{200}$$

$$m\sqrt{2} = \sqrt{100 \bullet 2}$$

$$m\sqrt{2} = \sqrt{100} \bullet \sqrt{2}$$

$$m\sqrt{2} = 10\sqrt{2}$$

$$\therefore m = 10$$

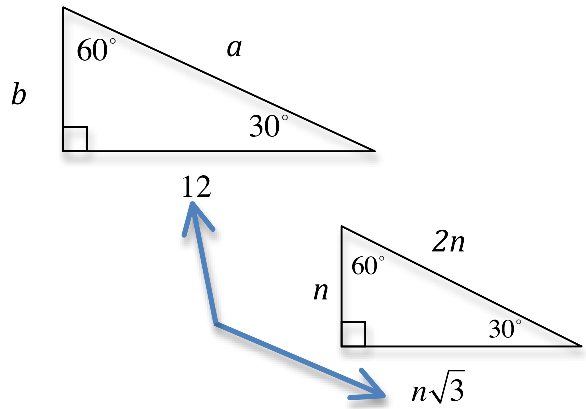
$$m = 10$$

$$p = m$$

$$\therefore p = 10$$

## You Try!

Find the lengths of the missing sides.




$n\sqrt{3} = 12$

$\frac{n\sqrt{3}}{\sqrt{3}} = \frac{12}{\sqrt{3}}$

$n = \frac{12}{\sqrt{3}}$

$n = \frac{12}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}}$

$n = \frac{12\sqrt{3}}{\sqrt{3 \bullet 3}}$

$n = \frac{12\sqrt{3}}{\sqrt{9}}$

$n = \frac{12\sqrt{3}}{3}$

$n = \frac{3 \bullet 4 \bullet \sqrt{3}}{3}$

$n = 4\sqrt{3}$

$n = 4\sqrt{3}$

$b = n$

$\therefore b = 4\sqrt{3}$

$a = 2n$

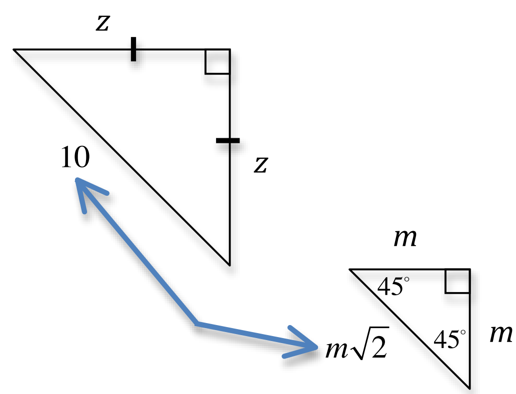
$a = 2(4\sqrt{3})$

$a = 2 \bullet 4 \bullet \sqrt{3}$

$\therefore a = 8\sqrt{3}$

## You Try!

Find the lengths of the missing sides.




$m\sqrt{2} = 10$

$\frac{m\sqrt{2}}{\sqrt{2}} = \frac{10}{\sqrt{2}}$

$m = \frac{10}{\sqrt{2}}$

$m = \frac{10}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}}$

$m = \frac{10\sqrt{2}}{\sqrt{2 \bullet 2}}$

$m = \frac{10\sqrt{2}}{\sqrt{4}}$

$m = \frac{10\sqrt{2}}{2}$

$m = \frac{2 \bullet 5 \bullet \sqrt{2}}{2}$

$m = 5\sqrt{2}$

$m = 5\sqrt{2}$

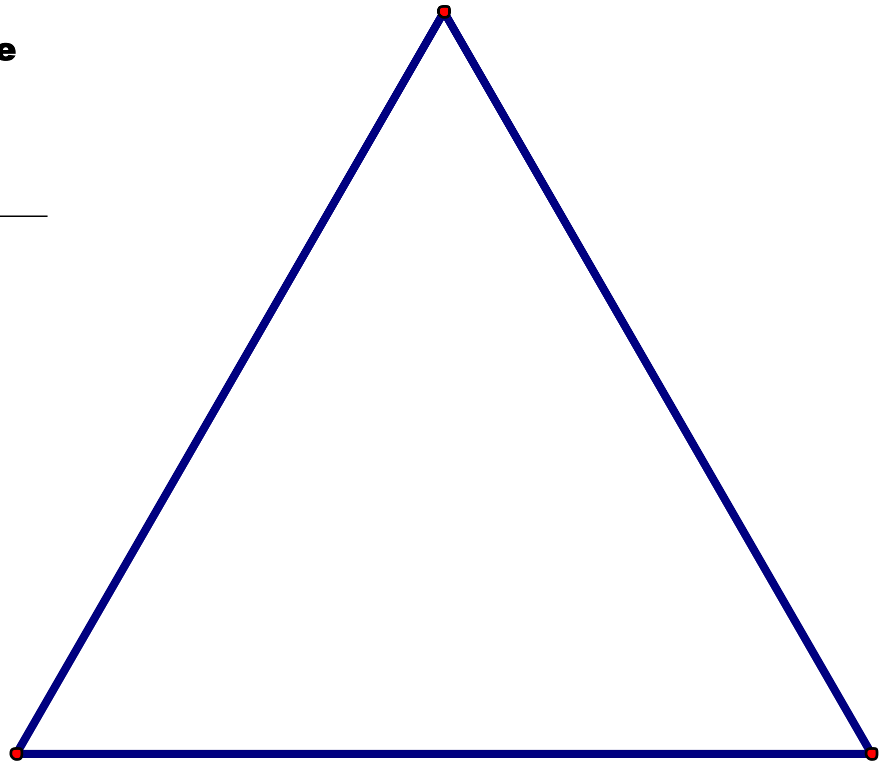
$z = m$

$\therefore z = 5\sqrt{2}$

### Equilateral Triangle

All sides are: ______

All angles have measure: ______



### Square

All sides are: ______

All angles measure: ______