Investigating Similar Triangles and Understanding Proportionality: Lesson Plan

Purpose of the lesson: This lesson is designed to help students to discover the properties of similar triangles. They will be asked to determine the general conditions required to verify or prove that two triangles are similar and specifically understand the concept of proportionality. This lesson is intended to be used as a way to introduce these concepts with the idea that formal postulates for proving triangle similarity will be supplied later.

Warm-Up: This warm-up contains review problems regarding triangle congruence and parallel lines and transversals.

Materials: Student handouts with black line masters of triangles, tracing paper, rulers, warm-ups, investigations, student pairings, and calculators (optional). Have an extra worksheet on hand that addresses similarity so that students who finish early may practice more while others finish.

Special Note: The black line masters may be used in different ways depending on the needs of your students and the resources you have for making copies. You may choose to photocopy a single triangle onto a sheet of colored paper and staple it to the back of the student handout to display in class when finished with the activity. You may also choose to simply photocopy it to the back of page five of the student handout. That way, you could create student handouts so that each student in a pair has a different triangle to work with. Also, you may choose to photocopy the black line masters onto card stock and re-use them in each class. Do as you see fit for your classroom.

Directions: After the warm-up, refer to the lesson opener shown below. Be sure to awaken students' prior knowledge the best you can by referring to previous topics such as congruent triangles. After this discussion, place students in pairs and assign one partner to collect materials. Remind students how to measure using centimeters. Be sure to point out that the edge of the ruler may not indicate zero centimeters; they need to begin measuring the side length at zero centimeters which may be slightly inward from the edge of the ruler. Circulate and help students to verify side and angle measures. **(This would be a great day to ask your coach to come in!)** As students discover properties of similarity, help them to generalize their thoughts and write sentences explaining their findings.

Once all students have finished, debrief questions 10-13 as a class. This would be a good time to use some of your favorite debriefing strategies such as Think-Pair-Share, Think-Aloud, etc.

After you have agreed upon the conditions required for triangle similarity, debrief the three practice problems, numbers 14-16. It is a good idea to invite pairs of students to write their solutions on transparencies or on the whiteboard and have pairs share their findings with the class. Also, have two or three pairs display their work for question #16. They will likely come up with many different answers and not all of them may be correct. If this happens, pose their questions to the class and see if students can see how to adjust the triangles to satisfy the conditions for similarity.

Closing: Ask students to write a summary of what they learned during the investigations. Use fill-in-the-blank statements such as "Today I learned that triangles can be congruent and ______________." (Similar) "In order for two triangles to be considered similar, all three ________________ _____________(Corresponding angles) must be congruent and all three pairs of ______________ ______________ (Corresponding sides) must be _________________________. (Proportional) Feel free to vary these statements or write different statements. You may also consider having students' complete statements regarding two triangles. For example, give students three congruence statements about the angles of two triangles. Have them determine and fill in the missing corresponding angles. Do the same with the proportions of the sides. Have fun!

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NOTE: Depending on your book and your department, it may be best to discuss how you would like students to write and compare ratios of corresponding sides. In this investigation, we compare the larger triangle to the smaller triangle but your book or colleagues may do it differently.

Lesson Opener (Teacher Copy):

Objective: This lesson is designed to help students to discover the properties of similar triangles and to specifically understand the concept of proportionality. They will be asked to determine the general conditions required to verify or prove that two triangles are similar. This lesson is intended to be used as a way to introduce these concepts with the idea that formal postulates for proving triangle similarity will be supplied later.

- **Ask the class:** List all Triangle Congruence Postulates that you know. Draw a picture of congruent triangles with the corresponding parts indicated for each postulate. (Quickly review each postulate.)

SSS	ASA	SAS	AAS	HL (Right triangles)

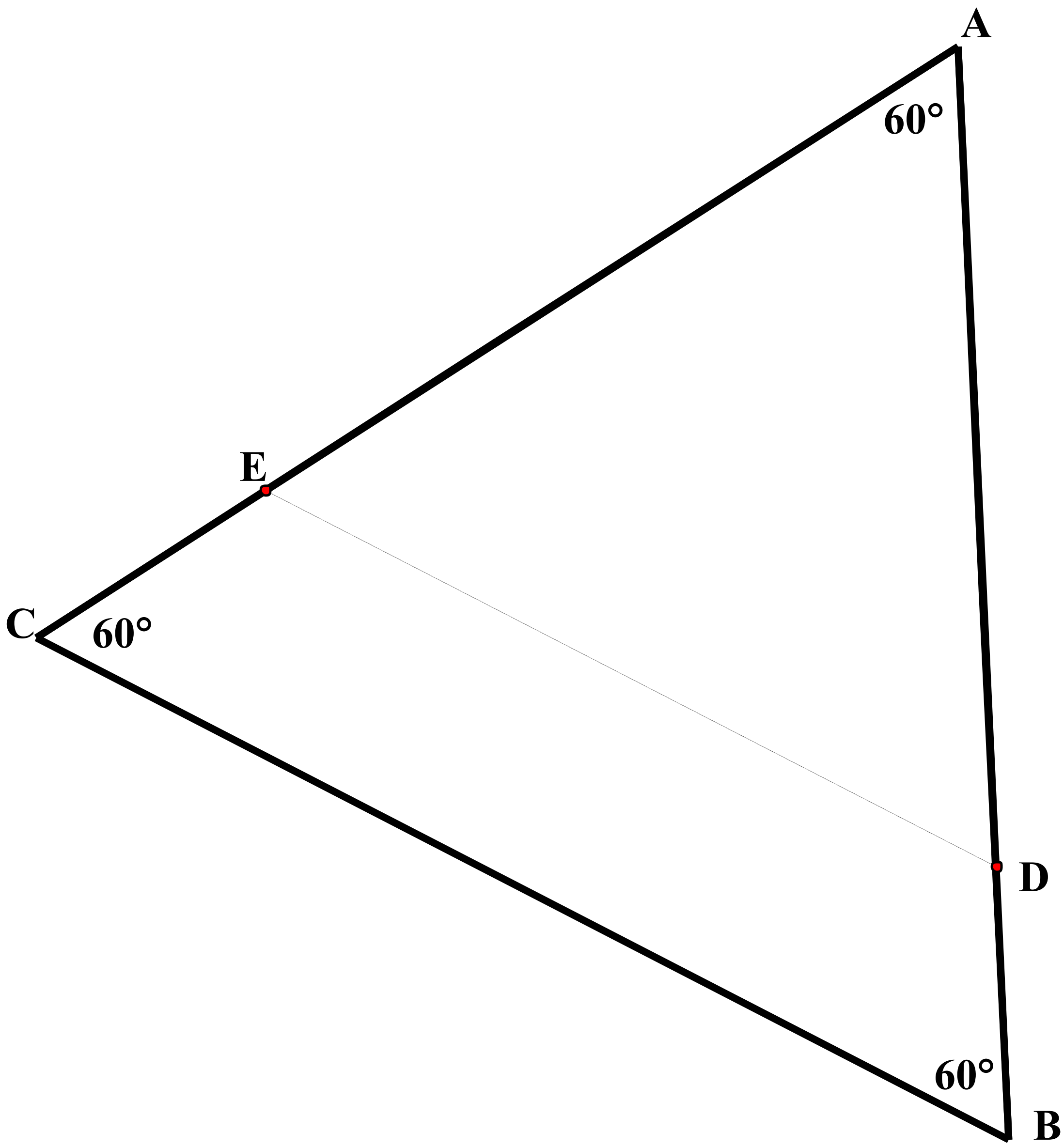
- **Ask the class:** Do you believe that having all three angles of a triangle congruent is another way to prove triangle congruence? Is AAA a triangle congruence postulate? Why or why not?
    - Remind them that a general definition of congruence states that figures must be the same size and the same shape in order to be considered congruent. Refer to the symbol for congruence ($\cong$). Compare this to the symbol for equality and discuss that this refers to the equality of values whereas congruence refers to figures having the same size and shape. Allow the students to discuss. Call on a few students to share their thoughts. Encourage them to draw pictures to support their claims.

- Explain to the students that the measures of the angles of a triangle do not dictate the lengths of the sides. Show several examples of pairs of triangles that have the same shape (where corresponding angles are congruent) but that are not the same size. Try to reach a consensus that the triangles have the same shape but not the same size.

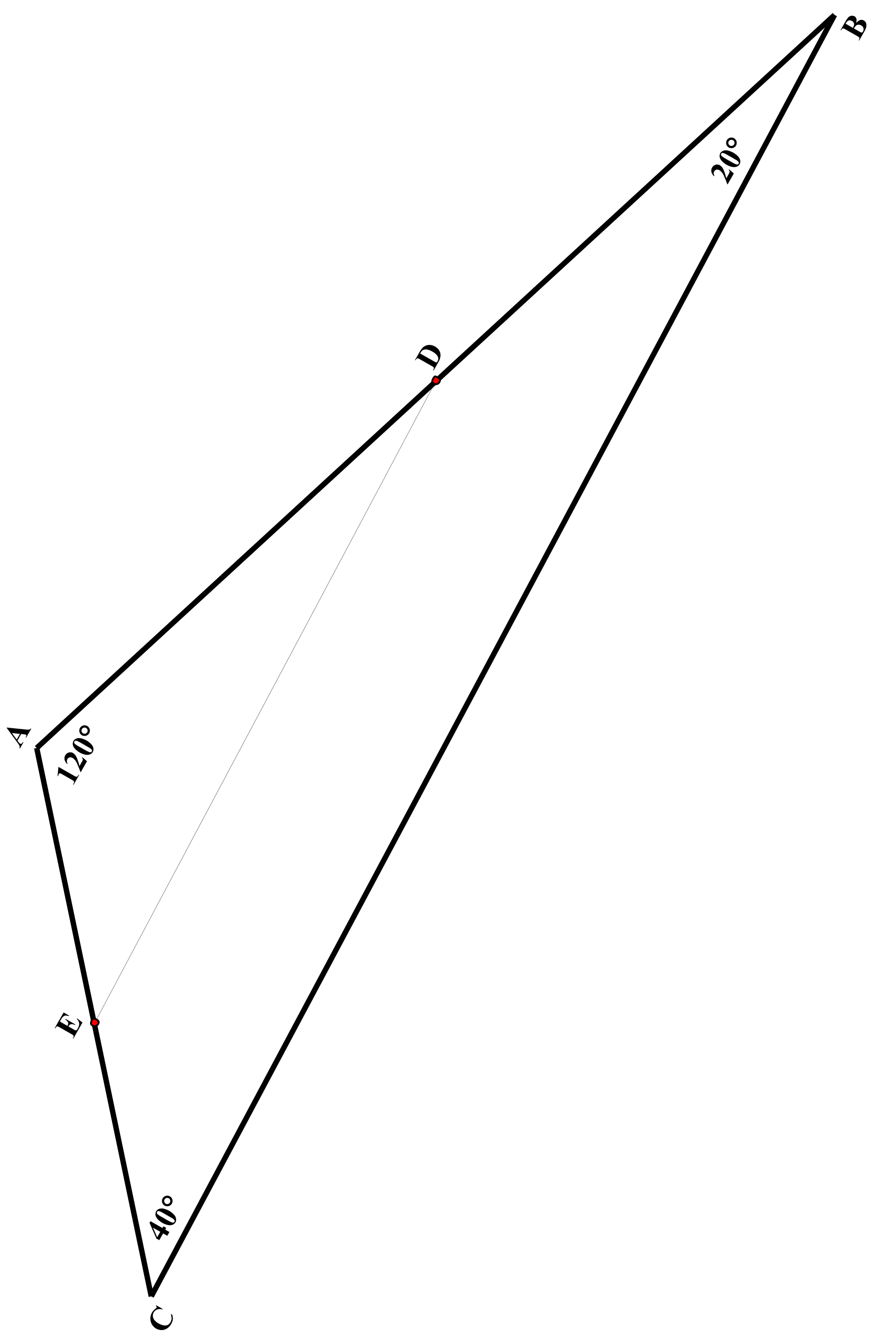
- **Ask the class:** Can you think a word that you could use to describe these triangles that look very much alike but that we have all agreed, are not congruent? (Hopefully, they will come up with the word **similar!)**
    - Introduce the symbol for similarity (~) and compare it to the previous symbols for congruence and equality. Explain that this symbol, when written alone, can be used to denote that two figures have the same shape but not necessarily the same size.

- **Ask the class:** Do you think that, in the same way congruent triangles have interesting properties; similar triangles will have interesting properties? Today, we will find out what those properties are!

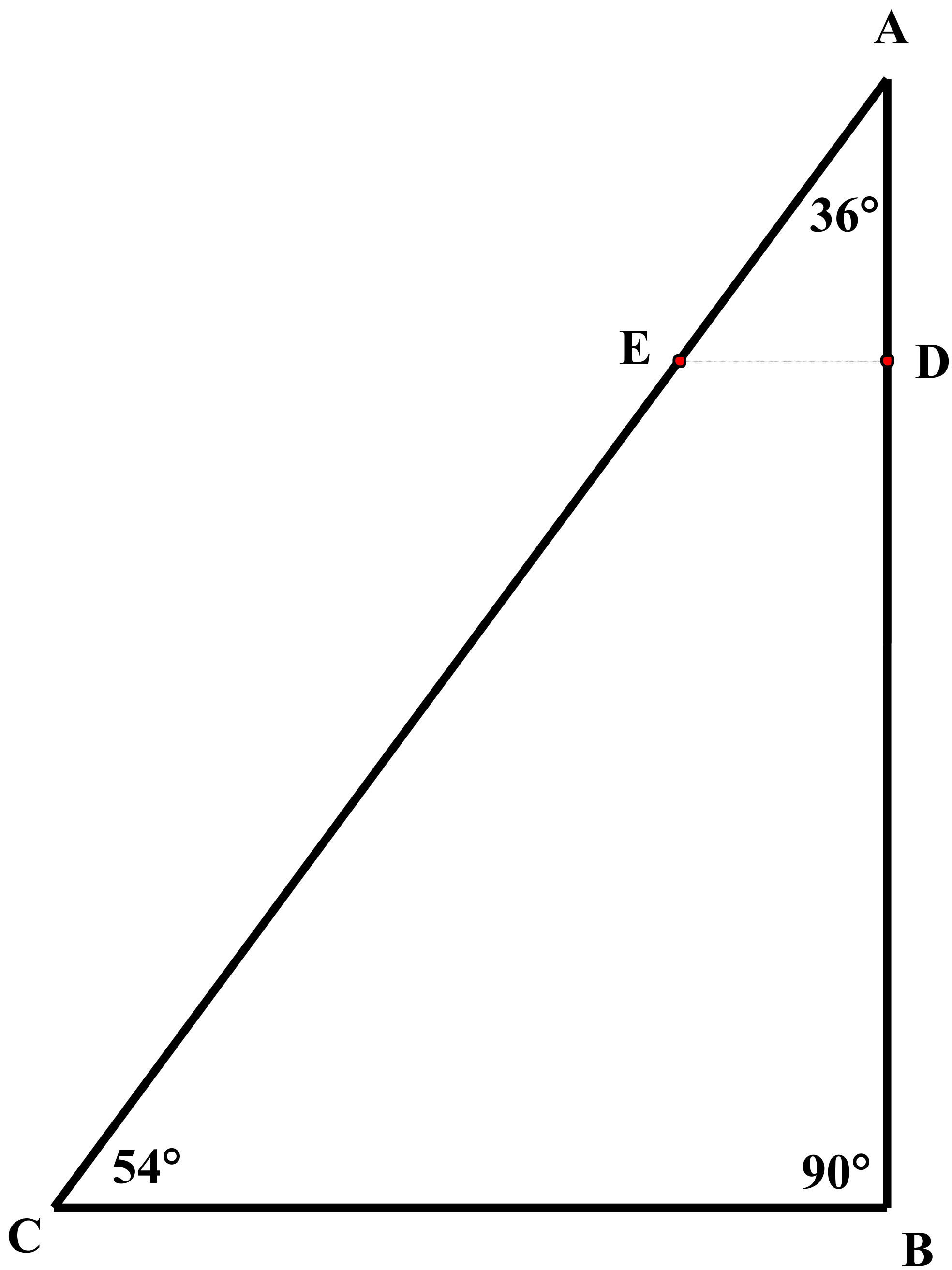
Triangle #1



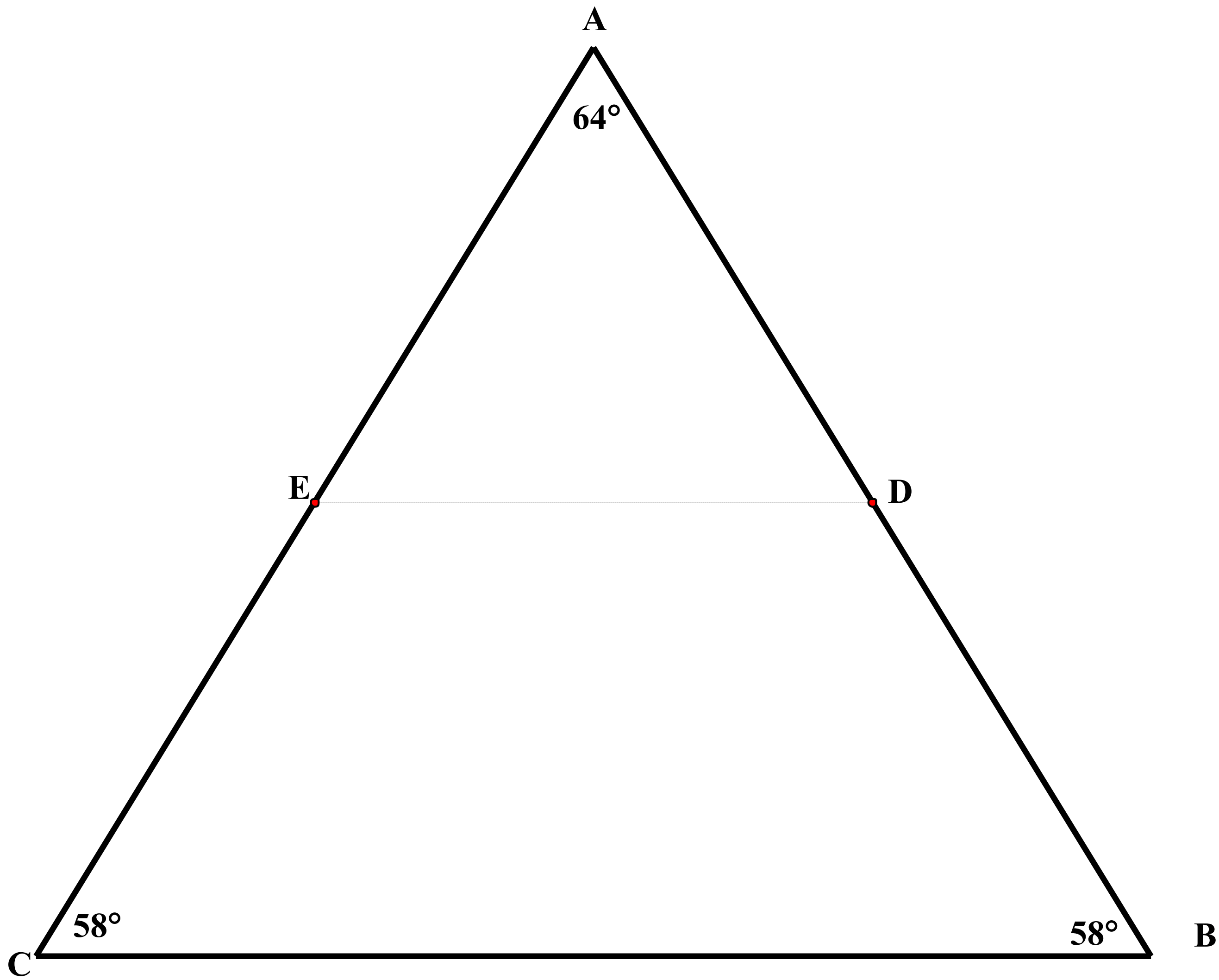
Triangle #2



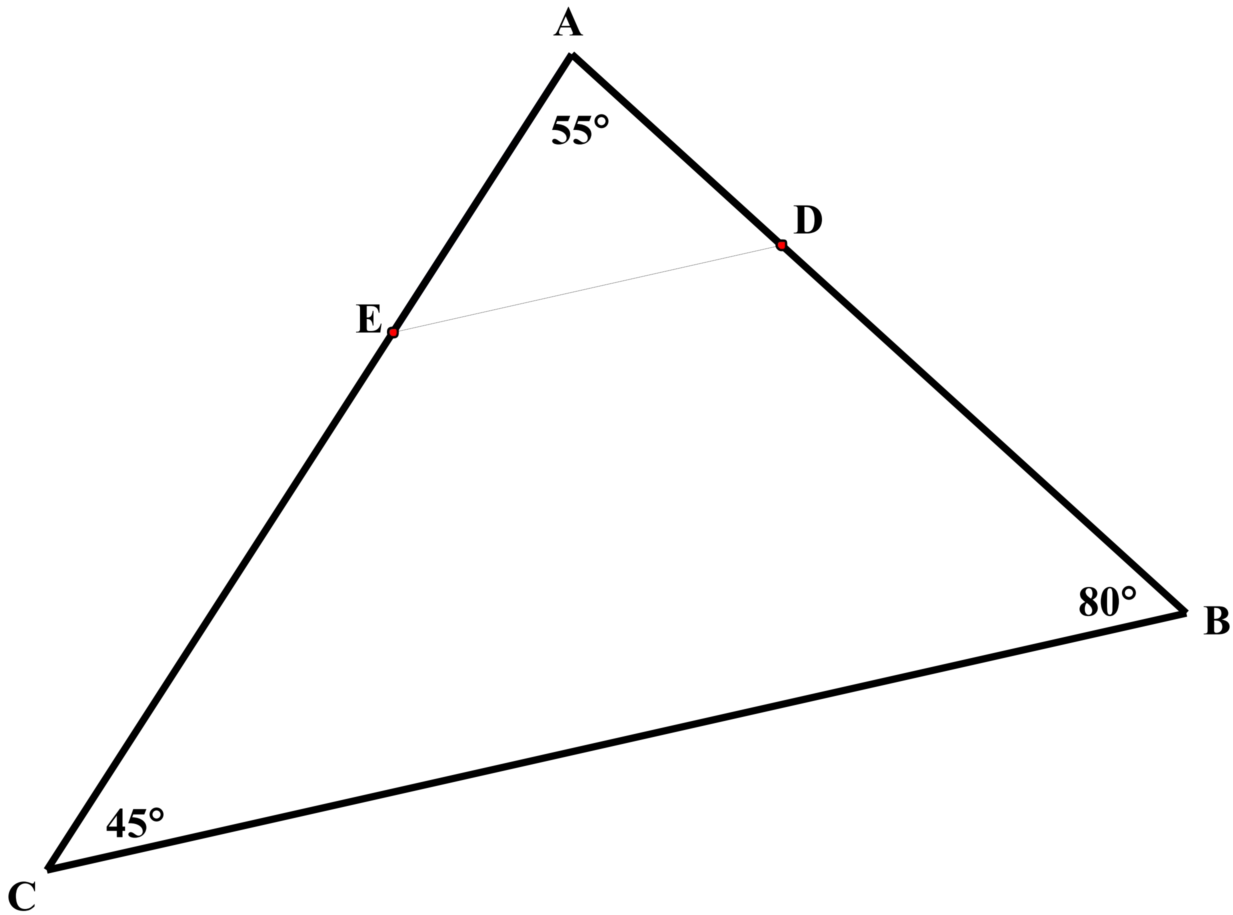
Triangle #3



Triangle #4



Triangle #5



Investigating Similar Triangles and Understanding Proportionality: Lesson Plan

Objective: This lesson is designed to help you discover the properties of similar triangles and to specifically understand the concept of proportionality. You will be determining the general conditions required to verify or prove that two triangles are similar.

1. List all **Triangle Congruence Postulates** that you know. There are five!
   Draw a picture of congruent triangles with the corresponding parts indicated for each postulate.

				(Right triangles)

2. Do you believe that having all three angles of a triangle congruent is another way to prove triangle congruence? Is AAA a triangle congruence postulate? Why or why not?

   Symbol for Congruence: ________________ Symbol for Equality: ________________

3. Draw several examples of pairs of triangles that have the same shape (corresponding angles are congruent) but that are not the same size.

4. Can you think a word that you could use to describe these triangles that look very much alike but that we have all agreed, are not congruent? ____

   Symbol for _________________: ________________

Directions:

Identify the two triangles in your picture, ΔABC (the larger triangle) and ΔADE (the smaller triangle).
You will be asked to identify and record certain measurements from each triangle in the chart below.

1. Using your ruler, measure the lengths of the sides of your larger triangle, ΔABC **in centimeters**. You will be measuring sides $\overline{AB}, \overline{BC}$ and $\overline{AC}$. Round to the nearest tenth of a centimeter. **Record the measurements below.**

2. Using your ruler, measure the lengths of the sides of the smaller triangle $\overline{AD}, \overline{DE}$ and $\overline{AE}$ **in centimeters.** Round to the nearest tenth of a centimeter. **Record the measurements below.**

3. Record the angle measures of your larger triangle, ΔABC. You will be recording $\angle A, \angle B$ and $\angle C$. Verify that the **sum** of the angles is 180°.

4. **Using your ruler**, connect the two points D and E to create $\overline{DE}$, a segment **parallel** to segment $\overline{BC}$. Try to be careful and precise!

5. Using what you know about parallel lines and transversals, find the measures of $\angle ADE$ and $\angle AED$. **Record the information below**. What reasons could you give for why these angles have these measures?

____

____

6. What is the measure of $\angle DAE$? Why?

____

Steps 1-6: Record measurements here.

Measurements for ΔABC		**Measurements for ΔADE**	
$m\overline{AB}$	$m\angle A$	$m\overline{AD}$	$\angle DAE$
$m\overline{BC}$	$m\angle B$	$m\overline{DE}$	$m\angle ADE$
$m\overline{AC}$	$m\angle C$	$m\overline{AE}$	$m\angle DEA$

7. Using your ruler, your pencil and a piece of tracing paper, trace the smaller triangle, ΔADE. Glue or tape it to your paper next to ΔABC. You now have two similar triangles.

8. In the table below, **identify and list the corresponding sides and the corresponding angles** of your two triangles. Also, **label** each of the side lengths and angle measures **on the two pictures**. (Label ΔABC on the paper and label ΔADE on the tracing paper.)

Corresponding Sides	Corresponding Angles

9. **Create ratios using the corresponding sides of the two triangles**. Refer to your chart above for help. Write the ratios as shown in the table below. Once you have set up the ratios, find the quotient. (Use your calculator to find the answer to the division problem!)

Ratio #1 $\dfrac{\overline{AB}}{\overline{AD}}$	**Ratio #2** $\dfrac{\overline{BC}}{\overline{DE}}$	**Ratio #3** $\dfrac{\overline{AC}}{\overline{AE}}$

10. What do you notice about the **ratios** of the **corresponding sides**?

*We say that the sides are **proportional** because the ratios of the corresponding sides are ________________.

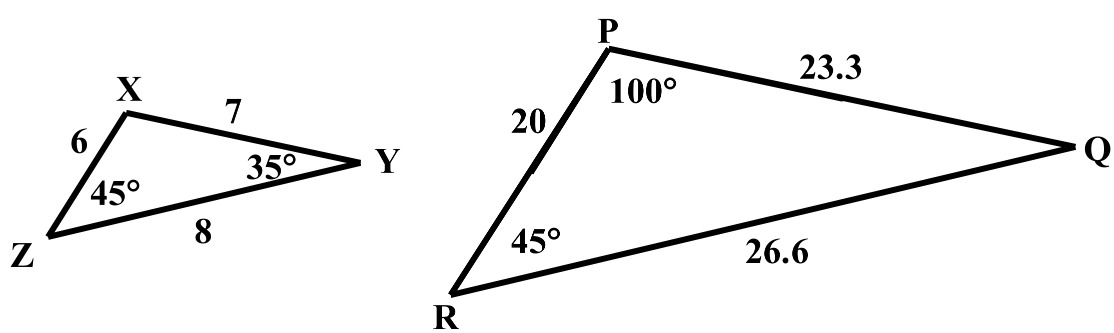
11. What did you notice about the **measures of corresponding angles?**

12. What have you discovered about **similar triangles?**

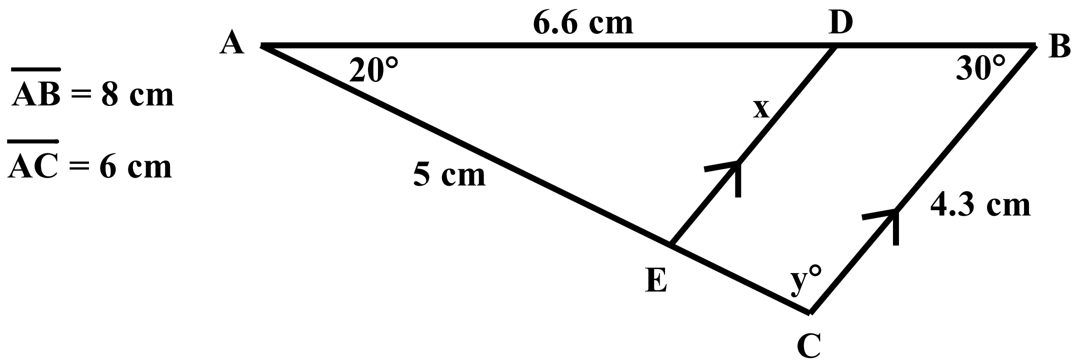
13. In your opinion, what **conditions** must be met in order for triangles to be considered similar? Do you think that these same conditions could apply to any closed figure? (Hexagon? Pentagon?)

Applying what you have learned:

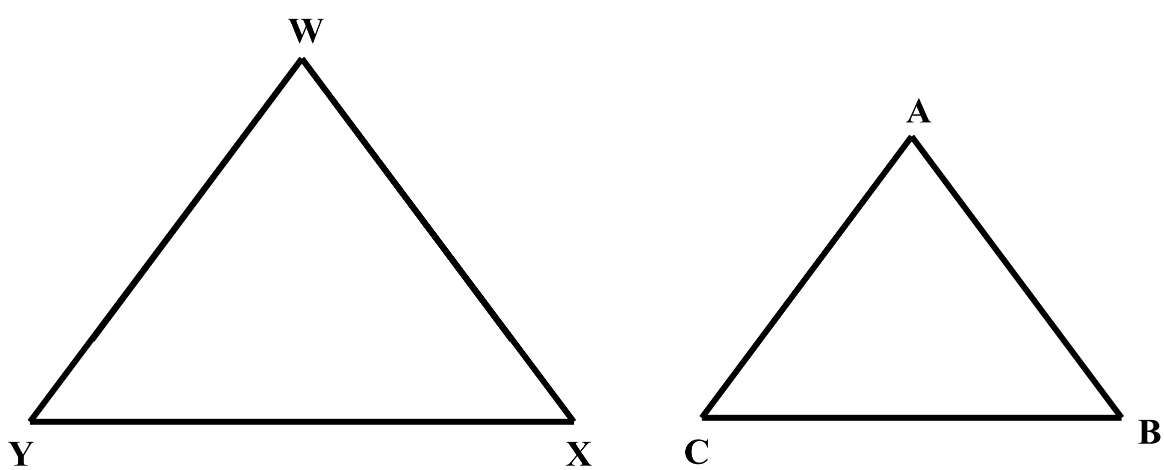
14. The two triangles below are similar. Explain why.
(*Hint:* Check all measures of corresponding angles and compare ratios of corresponding sides.)



15. For what values of *x* and *y* are the two triangles similar?
(*Hint:* The sides must be proportional; you will have to write a proportion.)



16. Here are two triangles that appear to be similar. Assign angle measures and side lengths that will **make** your two triangles similar. Have your partner verify that you created similar triangles.

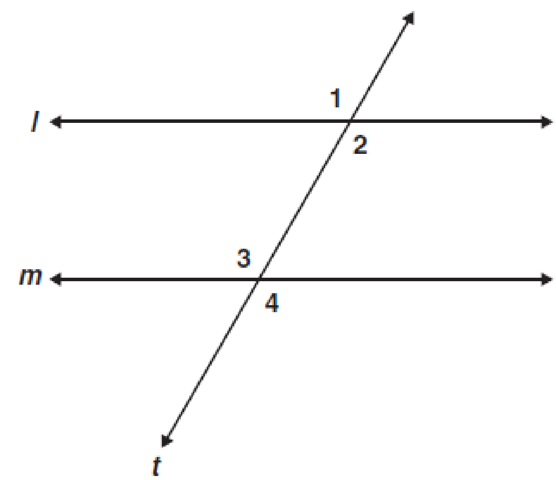


Warm-Up

CST/CAHSEE

In the diagram below, $\angle 1 \cong \angle 4$.

Which of the following conclusions does *not* have to be true?



A. $\angle 3$ and $\angle 4$ are supplementary angles.

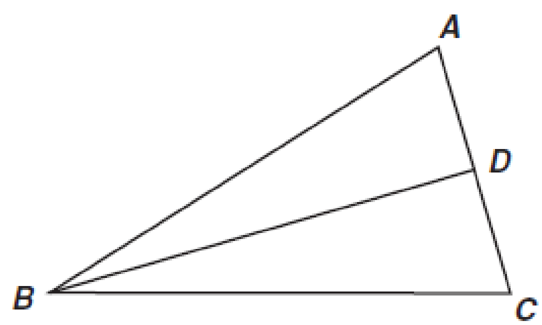
B. Line *l* is parallel to line *m*.

C. $\angle 1 \cong \angle 3$

D. $\angle 2 \cong \angle 3$

What do we call the pairs of angles in answer choices C and D?

Review



Use the proof to answer the question below.

Given: $\overline{AB} \cong \overline{BC}$; D is the midpoint of $\overline{AC}$.
Prove: $\Delta ABD \cong \Delta CBD$

Statement	**Reason**	
1. $\overline{AB} \cong \overline{BC}$; D is the midpoint of $\overline{AC}$.	1. Given	**A.** *AAS*
2. $\overline{AD} \cong \overline{CD}$	2. Definition of Midpoint	**B.** *ASA*
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive Property	**C.** *SAS*
4. $\Delta ABD \cong \Delta CBD$	4. **?**	**D**. *SSS*

Today's Standards:
CA State Standard Geometry 4.0: Students prove basic theorems involving congruence and similarity.
CA State Standard Geometry 5.0: Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.

Warm-Up

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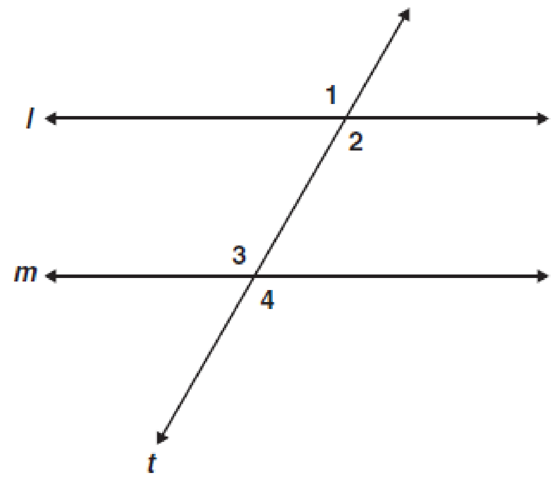

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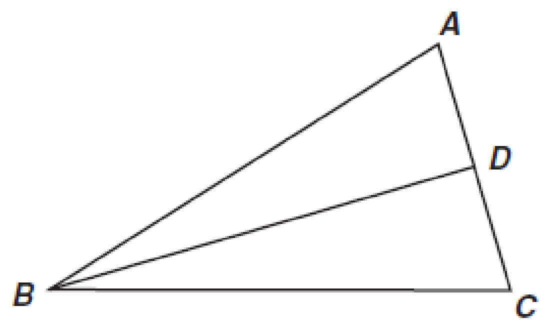

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