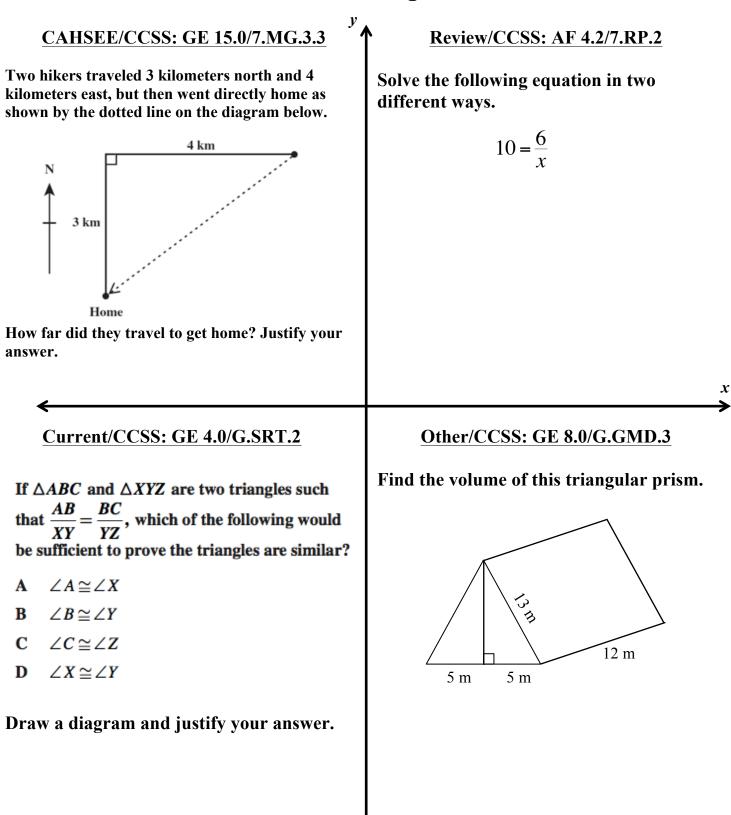
# Warm-Up



## Introduction to Trigonometric Functions – Day 1 Geometry

**Students will be able to** define the basic trigonometric ratios and identify the reference angle, opposite side, adjacent side, and hypotenuse of given right triangles.

**CA State Standard Geometry 18.0:** Students know the definitions of the trigonometric functions defined by the angles of a right triangle.

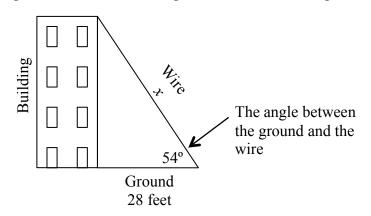
**CA Common Core Standard G.SRT.6**: Understand that, by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

### Materials needed:

- Print out of the 30°-60°-90° similar triangles. (Enough for 1 per student or 1 per group)
- Calculators (optional)

### I. Discovery

- a. Give pairs or groups of students the page with the two similar 30°-60°-90° triangles.
- b. Instruct students to record and calculate the ratio of the left leg to the hypotenuse of the smaller triangle  $\left(\frac{left \ leg}{hypotenuse}\right)$ . Have them do the same for the larger triangle. Continue in this way until all ratios have been computed for both triangles.
  - i. Ask students "What do you notice about the left leg/hypotenuse ratio for both triangles? What about the other ratios?"
- c. After the students have come to this conclusion on their own, say, "These fixed numbers are called trigonometric ratios. The value of the trigonometric ratio depends only on the acute angles in a triangle and not the length of the sides. As long as the angles are the same (as in similar triangles), the trigonometric ratio will be the same."
- d. Introduce 'missing side' trig problem to explain why we have to use trigonometry. Present it as a real world problem. "I am trying to hang a wire from the top of this building to the ground. I need to know the exact length of the wire I need to buy, but I can't measure something that huge. What I can do is measure the distance from the building to the spot on the ground I want to place the wire, and then measure the angle the wire would make to the ground. I can then use trig to determine what length of wire to purchase."



#### П. Definition

- a. Introduce the three most common trig functions and their abbreviations: sine (sin), cosine (cos), and tangent (tan).
- b. "Each of these ratios depends on one of the triangle's acute angles. This acute angle is called the reference angle. In your discovery, we used the orientation of the triangles to determine which sides to use for which ratio, but in future problems, we will use the reference angle."
- c. Introduce the definitions of trig ratio, explaining that the words "Opposite" and "Adjacent" depend on the reference angle.
  - i. Sine  $(\sin) = \frac{Opposite}{Hypotenuse}$

ii. Cosine (cos) =  $\frac{Adjacent}{...}$ Hypotenuse

- iii. Tangent  $(tan) = \frac{Opposite}{Adjacent}$
- iv. Explain the use of SOH CAH TOA as a way to memorize these ratios.

#### III. Labeling Parts of a Right Triangle

- a. Complete the two "I do" problems from the worksheet. Share your identification and labeling through a Think Aloud.
  - i. "First. I identify the reference angle. If more than one acute angle is given, follow directions on which to use."
  - ii. "Second, I identify the hypotenuse and label it with an H. The hypotenuse, as we learned from the Pythagorean theorem, is the side opposite the right angle."
  - iii. "Third, I identify the opposite side and label it with an O. The opposite side is the leg opposite (or across from) our reference angle."
  - iv. "Lastly, I identify the adjacent side and label it with an A. The adjacent side is the leg between the reference angle and the right angle."
- b. Have students complete the "We do" problem using a Think-Pair.
  - i. Give students time to identify the reference angle, hypotenuse, opposite and adjacent sides.
  - ii. Afterwards, instruct students to compare their work with their partner's.
    - 1. "Turn to your partner and check if your answers are the same. Tell your partner how you knew to label each part of the triangle in that way."
- c. Give students time to complete the four "You try" problems independently by using the reference angle as instructed. When time is up, display the answers. Allow students to ask questions if needed.

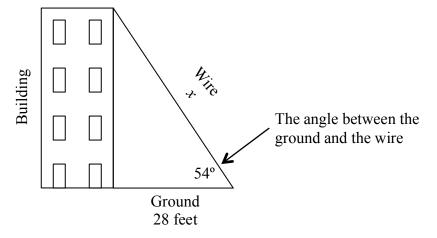
#### IV. **Creating Trigonometric Ratios**

- a. Complete the "I do" problem in the second part of the notes.
  - i. Set up the three common trig ratios using 56° as a reference angle and SOH-CAH-TOA as a means to memorize the definitions.

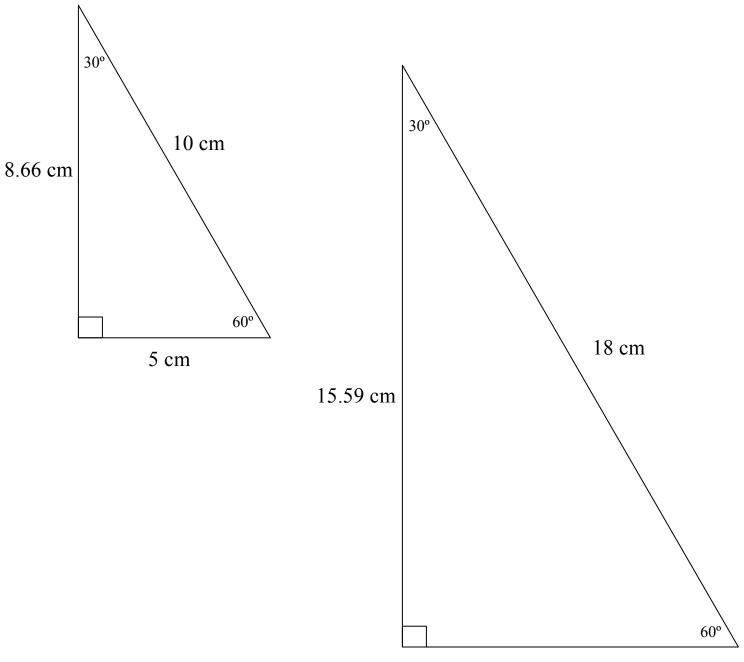
- ii. Explain why 34° is listed as part of this problem (using the triangle sum theorem, 34° must be the other acute angle in the given triangle). Find the ratios using 34° as your reference angle.
- b. Give students time to complete the two "We do" problems in partners. Then, have partners come to the document camera to display and explain their answers.
- c. Give students time to complete the three "You try" problems independently. Then, have individual students come to the document camera to display and explain their answers.

### V. Closure

a. Return to the original "missing side" trig problem and ask the following questions:



- i. Which ratio(s) could you create with the given information? Why only this/these ratio(s)?
- ii. Find the measure of the unknown angle (the angle between the wire and the building). Using that angle as your reference, what new ratio could you create with this information? Could you follow this process for any right triangle?





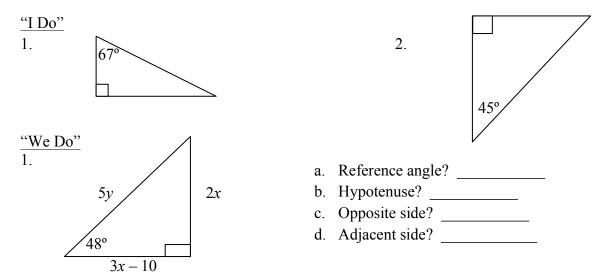
### **Introduction to Trigonometric Functions – Day 1**

### I. Discovery

	Smaller Triangle	Larger Triangle
left leg		
hypotenuse		
bottom leg		
hypotenuse		
bottom leg		
left leg		

### II. Definition

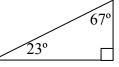
### III. Labeling Parts of a Right Triangle

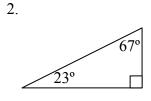


<u>"You Try"</u>

Label each side of the triangle with respect to the angle named as a reference angle.

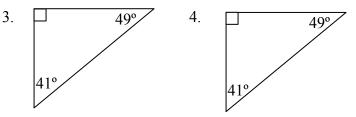
1.



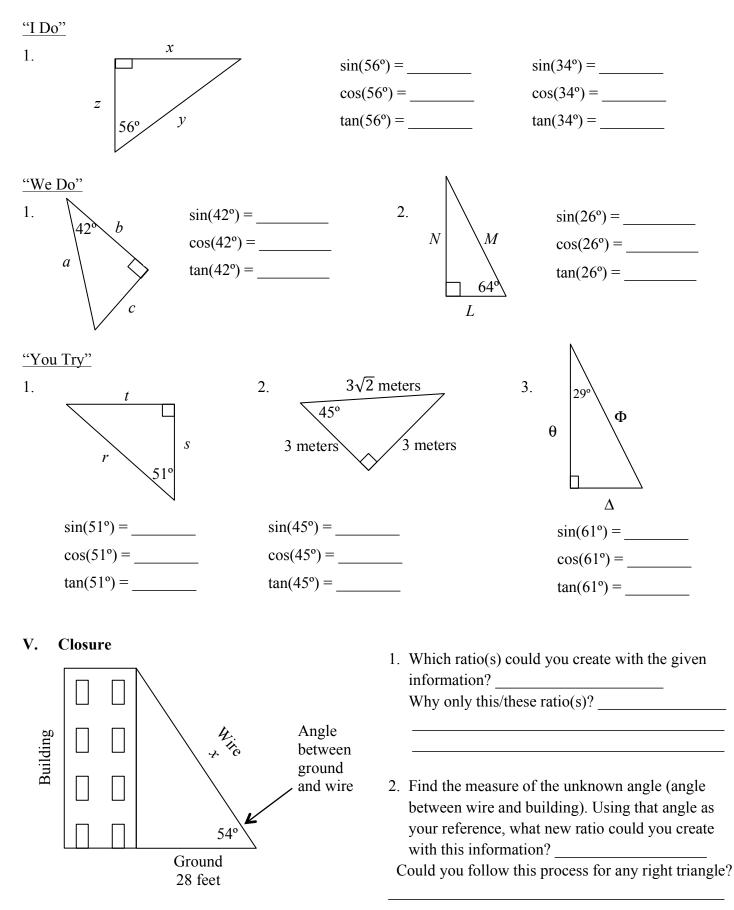


Use 23° as a reference angle

Use 67° as a reference angle



Use 41° as a reference angle Use 49° as a reference angle

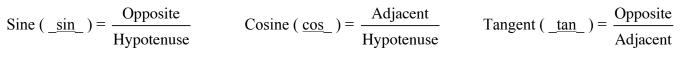


### Introduction to Trigonometric Functions - Day 1

### I. Discovery

	Smaller Triangle	Larger Triangle
left leg hypotenuse	$\frac{8.66 \text{ cm}}{10 \text{ cm}} = 0.866$	$\frac{15.59 \text{ cm}}{18 \text{ cm}} \approx 0.8661$
bottom leg hypotenuse	$\frac{5 \text{ cm}}{10 \text{ cm}} = 0.5$	$\frac{9 \text{ cm}}{18 \text{ cm}} = 0.5$
bottom leg left leg	$\frac{5 \text{ cm}}{8.66 \text{ cm}} \approx 0.5774$	$\frac{9 \text{ cm}}{15.59 \text{ cm}} \approx 0.5773$

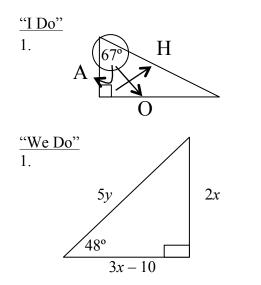
### **II.** Definition



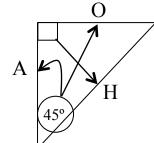
CAH

## SOH

### III. Labeling Parts of a Right Triangle



2.

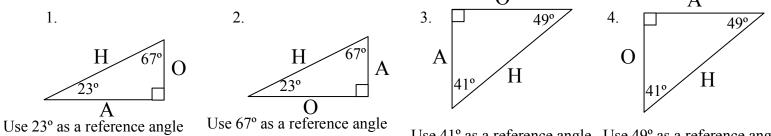


TOA

- e. Reference angle? <u>48°</u>
- f. Hypotenuse? <u>5y</u>
- g. Opposite side? 2x
- h. Adjacent side? 3x 10

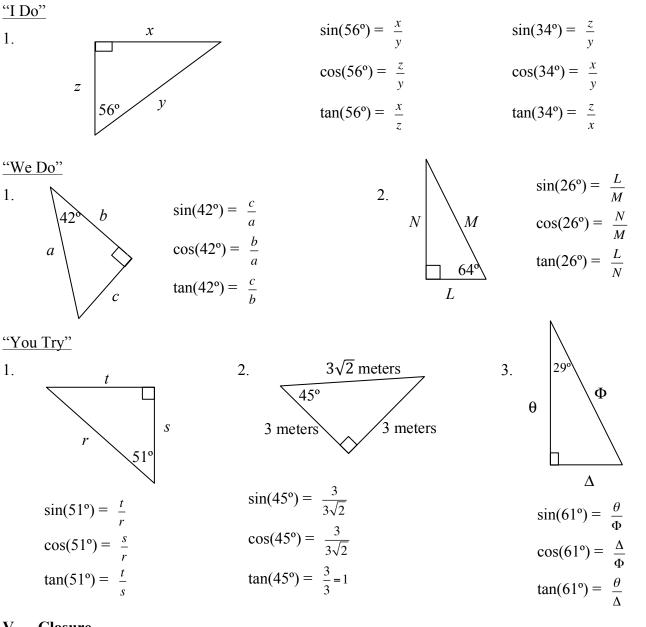
### <u>"You Try"</u>

Label each side of the triangle with respect to the angle named as a reference angle.

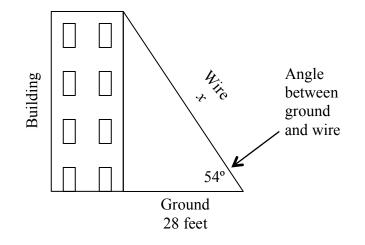


Use 41° as a reference angle Use 49° as a reference angle

### IV. Creating Trigonometric Ratios



V. Closure



1. Which ratio(s) could you create with the given information? <u>Cosine</u>

Why only this/these ratio(s)? <u>Because we only</u> <u>have info for the adjacent side (Ground 28 feet) and</u> <u>hypotenuse (Wire x). Cosine = adjacent/hypotenuse.</u>

2. Find the measure of the unknown angle (angle between wire and building). Using that angle as your reference, what new ratio could you create with this information? <u>Sine</u>

Could you follow this process for any right triangle? Yes. You know the acute angles are complementary.

## Introduction to Trigonometric Functions – Day 2 High School Geometry

**Students will be able to** solve problems involving missing sides of a right triangle using trigonometric ratios.

**CA State Standard Geometry 19.0:** Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side.

**CA Common Core Standard G.SRT.8:** Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems.

### Materials needed:

- Calculators
- Table of Trigonometric Ratios
- Copied and cut-out "Pass the Problem Cards" enough for one per pair of students

### I. Why do we use trig?

- a. Review with students the Pythagorean theorem and how it can only be used with right triangles where there are two given sides and one missing side. Label the blank triangle with the information necessary in order to use the Pythagorean theorem. Refer to the first two example problems (the 30°–60°–90° and 45°–45°–90° triangles) and ask students:
  - i. "Would you be able to use the Pythagorean theorem to solve for the missing side in these triangles? Why or why not?" << No. Because we do not have two given sides of the right triangle. >>
- b. Introduce the 30°-60°-90° triangle missing side problem. Tell students: "I recognize this as a special right triangle. Since the bottom angle is 60°, the top angle must be 30° and this is a 30-60-90 triangle. Even though we can not use the Pythagorean theorem for this problem, we now have two ways of solving for the missing side." Solve for the missing side using both methods.
  - i. METHOD 1: Using properties of special right triangles: Reflect the 30°–60°–90° triangle to create an equilateral triangle.
  - ii. METHOD 2: Setting up and solving a trigonometric ratio.
- c. Introduce the 45°–45°–90° triangle missing side problem in a similar way and solve for the missing side using both methods.
  - i. METHOD 1: Using properties of special right triangles: Prove that the 45°–45°–90° triangle is an isosceles right triangle. The acute angles are congruent; therefore the sides across from those angles are congruent.
  - ii. METHOD 2: Setting up and solving a trigonometric ratio.
- d. Introduce the last missing side problem in this section.
  - i. "Can we solve this triangle as well using both of the methods we have been using? Why or why not?" << No. Because this is not a special right triangle, so we cannot solve it using special right triangle properties. >>
  - ii. "So, how can we solve for the missing side of this triangle?" << Using trigonometric ratios! >>

iii. "We will use trigonometric ratios to solve for missing sides of right triangles like this one, where we can not use the Pythagorean theorem, or properties of special right triangles."

### II. Finding missing sides

- a. "I Do" Problems
  - i. Do a Think-Aloud for each of these two problems. For the first problem, say:
    - 1. "First I check if I can use any other method to find this missing side. Can I use Pythagorean theorem? No. Can I use properties of special right triangles? No. So the only other way I know how to find a missing side in a right triangle is using trigonometric ratios."
    - 2. "Now, I need to identify my reference angle so that I can create the appropriate ratio. After I have found my reference angle, I use it to identify and label the hypotenuse, opposite, and adjacent sides of the triangle."
    - 3. "Usually, only one side is given and the other side is our missing side. In this case, the side that is given is the hypotenuse and our missing side is the adjacent side. Because the two sides involved are the hypotenuse and the adjacent, I know I must use cosine because cosine is the adjacent over the hypotenuse (CAH)."
    - 4. "Now, I set up the ratio where the cosine of  $47^{\circ}$  equals *x* over 12. I know that the cosine of  $47^{\circ}$  is a constant, so I will look at my Trigonometric Ratio chart and replace  $\cos(47^{\circ})$  with it's decimal value."
    - 5. "From here, we solve the problem either by clearing the fraction from the equation or cross-multiplying."
    - 6. At this point, use whichever method you prefer to finish this problem.
  - ii. Complete the second "I Do" problem by doing a similar Think-Aloud.
- b. "We Do" Problems
  - i. Give each pair of students the set of four problem cards and inform them that they only have one set of problems for the both of them.
  - ii. Instruct students to choose one partner to be Student A and one to be Student B. Record these names on their student note sheet.
  - iii. Direct students on how to complete these problems by passing the paper back and forth after each new step of the problem.
    - 1. "Student A begins with the first problem and must correctly label the reference angle, hypotenuse, opposite and adjacent sides. Then, A passes the problem to Student B. Student B checks the work of Student A and then continues the next step of the problem, which is to set up the correct trigonometric ratio. B then passes the problem back to A who checks the previous work and solves the ratio. This passing continues after each step of the problem until the problem is solved. When we are ready to begin with problem two, Student B will do the first step and then continue the passing."
    - 2. Give students a set time limit to complete the first problem. When that time limit has passed, have one pair come up to the board to show and justify their answer.
  - iv. Continue with this process until all four problems have been completed and reviewed in front of the class.

- c. "You Do" Problems
  - i. Give students a set time limit to complete these problems independently. When all students are finished or the time limit is up, review the answers to these problems and answer any lingering questions.

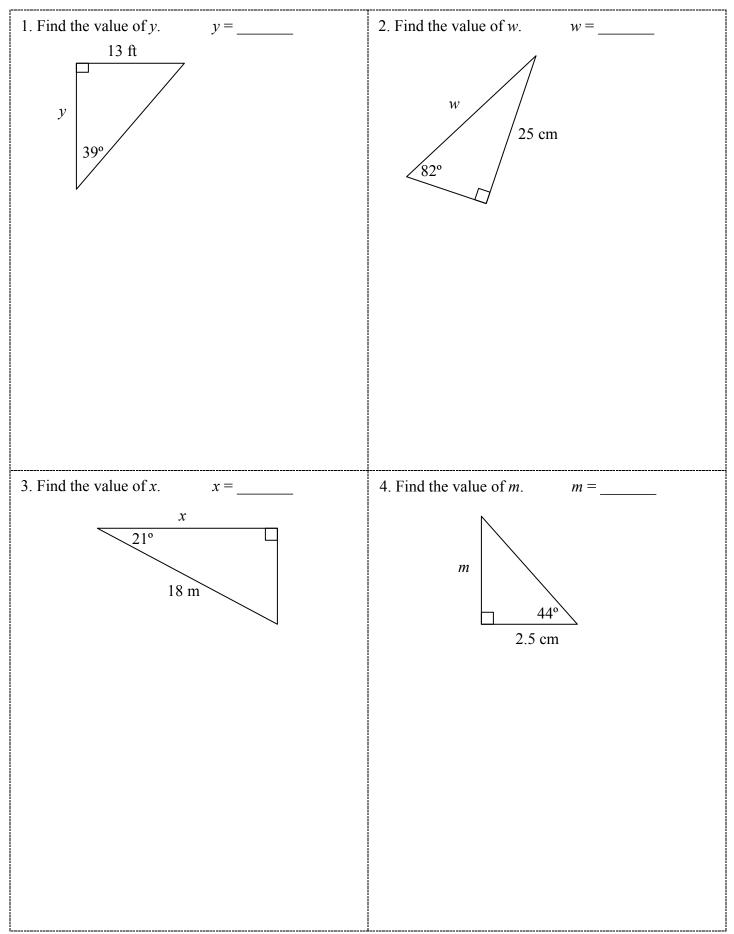
### III. Closure

a. Have students complete the closure problem by finding the value of *y* and answering the constructive response questions related to this question.

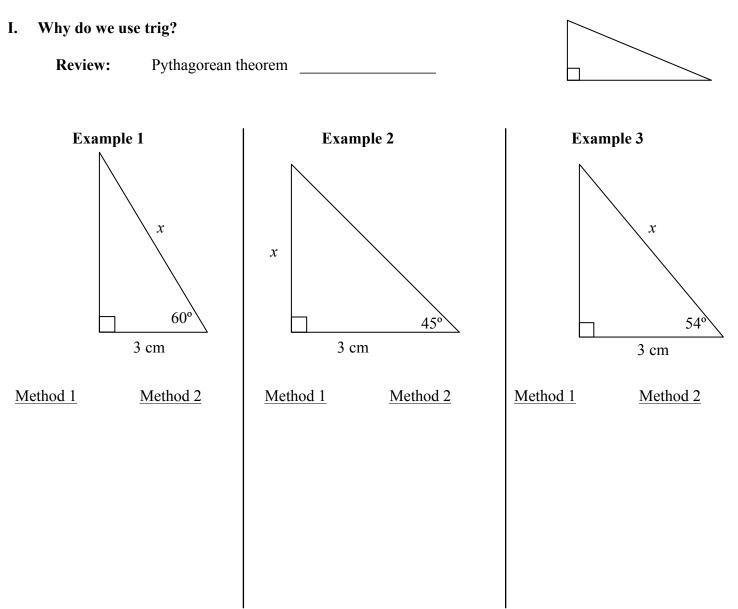
### **Table of Trigonometric Ratios**

Degrees	Sine	Cosine	Tangent	Degrees	Sine	Cosine	Tangent
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524	48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699	49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875	 50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051	 51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228	 52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405	 53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584	54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763	55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944	56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126	57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309	58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493	59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679	60	0.8660	0.5000	1.7321
16	0.2756	0.9613	0.2867	61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057	62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249	 63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443	64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640	 65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839	 66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040	 67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245	 68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452	 69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663	70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877	 71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095	72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317	73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543	74	0.9613	0.2756	3.4874
30	0.5000	0.8660	0.5774	75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009	76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249	 77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494	78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745	79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002	80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265	81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536	82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813	83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098	84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391	85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693	86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004	87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325	88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657	89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1.0000	90	1.0000	0.0000	

### Pass the Problem Cards



### Introduction to Trigonometric Functions – Day 2

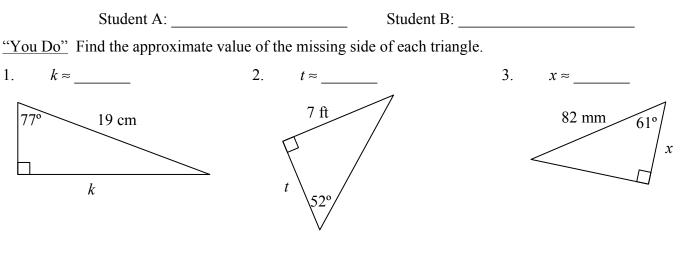


### II. Finding missing sides

"I Do" Find the approximate value of the missing side of each triangle.

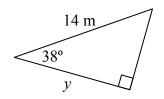


<u>"We Do"</u> Find the approximate value of the missing side of each triangle.



### III. Closure

a) What is the approximate value of y in the triangle below?  $y \approx$  \_\_\_\_\_



**b)** List two other methods we have used to find missing sides of right triangles. For each alternate method, draw a triangle whose missing side could be found by that method. Why will none of these other methods work with the above problem?

Method 1 Description of method:

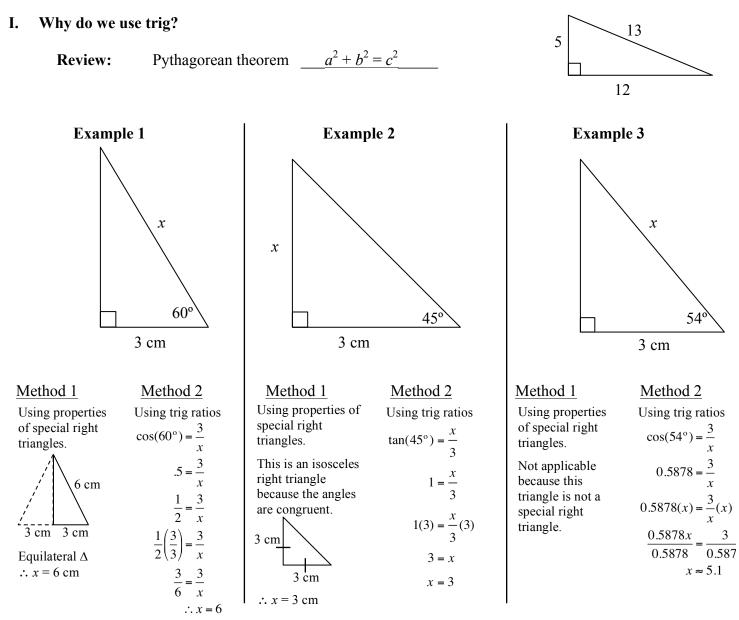
Example triangle:

Why won't this method work?

Method 2 Description of method:

Example triangle:

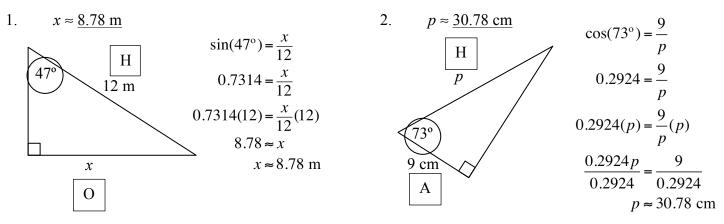
Why won't this method work?



Introduction to Trigonometric Functions – Day 2

### II. Finding missing sides

"I Do" Find the approximate value of the missing side of each triangle.



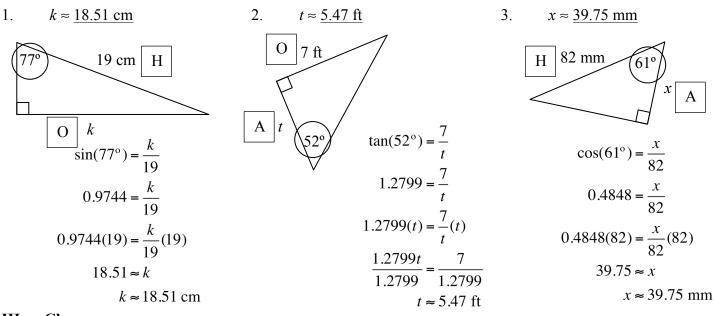
### Page 17 of 18

"We Do" Find the approximate value of the missing side of each triangle.

### Student A: \_\_\_\_\_ Stu

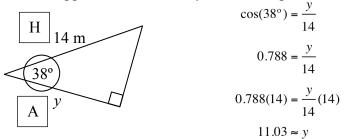
\_\_\_\_ Student B: \_\_\_\_

"You Do" Find the approximate value of the missing side of each triangle.

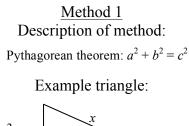


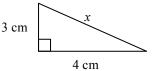
### III. Closure

a) What is the approximate value of y in the triangle below?  $y \approx 11.03 \text{ m}$ 



**b)** List two other methods we have used to find missing sides of right triangles. For each alternate method, draw a triangle whose missing side could be found by that method. Why will none of these other methods work with the above problem?

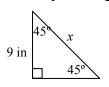




### Why won't this method work?

This method won't work with the above problem because in order to use the Pythagorean theorem we must have at least 2 sides of the triangle given to us. In the above problem, we only have one side given.

### <u>Method 2</u> Description of method: Special right triangle properties. 30-60-90 or 45-45-90 triangles. Example triangle:



Why won't this method work?

This method won't work with the above problem because it is not a special right triangle. In order to apply special right triangle properties, the triangle needs to be a 30-60-90 or 45-45-90 triangle. The above problem is neither of these.