Grade Level/Course: Geometry
Lesson/Unit Plan Name: Introduction to Conditional Probability
Rationale/Lesson Abstract: This lesson is designed to introduce conditional probability by building on students' prior knowledge of probability. The lesson focuses on showing the entire sample space to find conditional probabilities. The lesson incorporates examples with Venn diagrams and two-way tables. It wraps up with a definition of independent events.
Timeframe: 60 minutes
Common Core Standard(s): **S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. **S.CP.3** Understand the conditional probability of A given B as $\frac{P(\text{A and B})}{P(\text{B})}$ and <u>interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.</u>

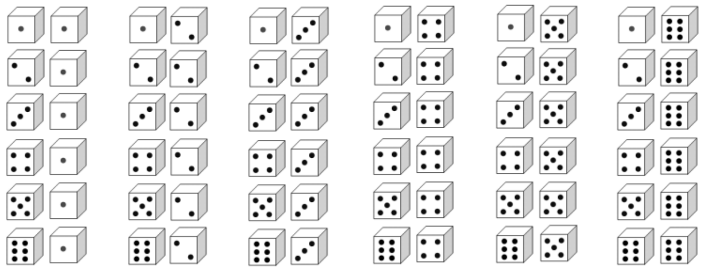
Instructional Resources/Materials:
Warm up (pg. 8), rolling two dice outcomes (pg. 11), student note-taking guide (pg. 9 & 10), and pencil.

Answers to Warm Up:

a) $P(\text{roll a 5}) = \frac{1}{6} \approx 17\%$ b) $P(\text{roll an even}) = \frac{3}{6} = \frac{1}{2} = 50\%$ c) $P(\text{roll at least a 3}) = \frac{4}{6} = \frac{2}{3} \approx 67\%$	*Obviously yes. The driver is spreading his focus to more than just driving and not concentrating on just the road. This makes the probability of getting into an accident more likely.* *(point out that there are two different probabilities here – the probability of getting into an accident & the probability of getting into an accident if you are texting)*
Probabilities will vary. *(point out that the weather today might be an indicator of the weather this weekend . For example, if it is gloomy/rainy it might be more likely to rain this weekend than if there hasn't been a cloud in weeks)*	The probabilities should be the complement of those in quadrant III. *(point out that complementary probabilities are different than complementary angles. This means that the two probabilities should add to 100% or 1)*

Activity/Lesson:

Show the possible outcomes of rolling two dice (pg. 11) :



"What is the probability of rolling doubles?"

Call on a student and have them explain their reasoning.

$$P(doubles) = \frac{6}{36}$$

← outcomes that are doubles (6)

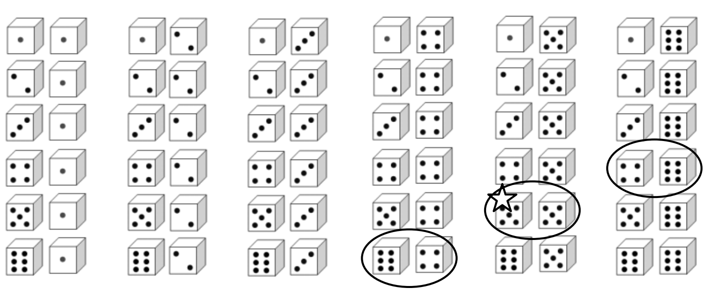
← total possible outcomes (36)

$$= \frac{1}{6}$$

$$\approx 16.7\%$$

"Taja rolled a ten (the sum of her dice). Knowing that she rolled a ten, what is the probability that she rolled doubles? Would the probability change? Why or why not?

Think Pair Share – Give students enough time to think about the question by themselves before engaging in conversation with their partner. After partners discuss with one another, call on students to share with the class to generate discussion.



Since Taja rolled a ten, then her possible outcomes are restricted to 4-6, 5-5, or 6-4. Since one of those outcomes is doubles (5-5), then:

"Probability of doubles **given** a roll of ten"

$$P\left(doubles \mid roll\ of\ ten\right) = \frac{1}{3}$$

$$= 33\%$$

"If Taja had rolled a nine, would the probability of her getting doubles still be $\frac{1}{3}$?

Call on a student to explain that the probability would be 0. (If Taja rolled a nine, then her possible outcomes are restricted to 3-6, 4-5, 5-4, or 6-3, none of which are doubles)

*****Pass out student note taking guide**

Define conditional probability:

Conditional Probability – The probability that an event will occur, given that another event has already occurred.

Example 1: Two six-sided dice are rolled. Find the conditional probabilities below

a) $P(5 \text{ on } 2^{nd} \text{ die} \mid 5 \text{ on } 1^{st} \text{ die})$

1-1	2-1	3-1	4-1	5-1	6-1
1-2	2-2	3-2	4-2	5-2	6-2
1-3	2-3	3-3	4-3	5-3	6-3
1-4	2-4	3-4	4-4	5-4	6-4
1-5	2-5	3-5	4-5	5-5	6-5
1-6	2-6	3-6	4-6	5-6	6-6

"The event that already occurred was a 5 on the 1st die. Let's circle all outcomes with a 5 on the first die. Now of those outcomes let's put a star next to the outcomes that have a 5 on the 2nd die."

$$P(5 \text{ on } 2^{nd} \text{ di} \mid 5 \text{ on } 1^{st} \text{ di}) = \frac{1}{6}$$
$$\approx 16.7\%$$

b) $P(\text{doubles} \mid \text{sum of dice is at least 8})$

1-1	2-1	3-1	4-1	5-1	6-1
1-2	2-2	3-2	4-2	5-2	6-2
1-3	2-3	3-3	4-3	5-3	6-3
1-4	2-4	3-4	4-4	5-4	6-4
1-5	2-5	3-5	4-5	5-5	6-5
1-6	2-6	3-6	4-6	5-6	6-6

Ask these questions to randomly selected students:

"What is the event that already occurred?" (sum of the dice is at least 8)
"How many outcomes have a sum of at least 8?" (15 – show by circling)
"Of those 15 outcomes, how many are doubles?" (3)

$$P(\text{doubles} \mid \text{sum of dice is at least 8}) = \frac{3}{15}$$
$$= \frac{1}{5}$$
$$= 20\%$$

c) $P(\text{sum of dice is at least 8} \mid \text{doubles})$

1-1	2-1	3-1	4-1	5-1	6-1
1-2	2-2	3-2	4-2	5-2	6-2
1-3	2-3	3-3	4-3	5-3	6-3
1-4	2-4	3-4	4-4	5-4	6-4
1-5	2-5	3-5	4-5	5-5	6-5
1-6	2-6	3-6	4-6	5-6	6-6

Ask the students if they think the probability will be the same as b). Have them do it as a "you try". Have a student debrief their work with the doc cam if time permits.

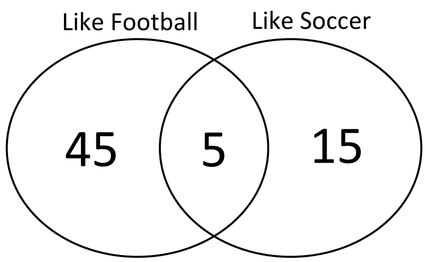
$$P(\text{sum of dice is at least 8} \mid \text{doubles}) = \frac{3}{6}$$
$$= \frac{1}{2}$$
$$= 50\%$$

Example 2: 65 students were surveyed about watching sports on tv. 50 students said they like watching football and 20 students said they like watching soccer. Of those students there are 5 who like both. If a random student is selected from the survey, find the probabilities below.

Ask this to the class:

"If there were 50 students who like football and 20 students who like soccer, then how come it says that only 65 students were surveyed?" (5 students were counted twice since they like football and soccer)

Use the information to set up a Venn Diagram with your class like this:



*Note: It might be easier to explain the setup of the Venn Diagram by explaining that the 5 in the middle represents the students who like watching both football and soccer.

Point out to the students that if you add up the numbers in your Venn Diagram it equals the total number of students surveyed: $45+5+15=65$.

For a-d do not simplify the probabilities. Leave them unsimplified as this will emphasize that the conditional probability restricts the sample space (if this is something you stress in your class, then do it after finding all the probabilities for a-d).

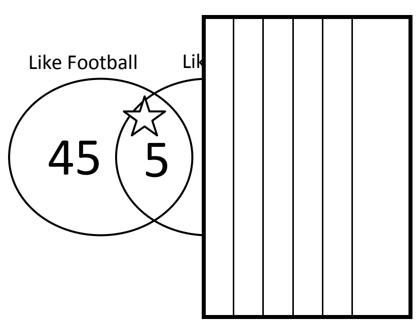
a) $P(\text{like football})=\frac{50}{65}$ ← Students who like football / ← total students surveyed

b) $P(\text{only like football})=\frac{45}{65}$ ← Students who only like football / ← total students surveyed

c) $P(\text{like both})=\frac{5}{65}$ ← Students who like both / ← total students surveyed

d) $P(\text{like soccer} \mid \text{like football})=\frac{5}{50}$

"Since it is given that the student likes football, then we are limiting the sample space to the 50 students who like football." (Cover up the right side of the Venn Diagram with a notecard)



You Try: Use the same Venn Diagram to find the following probabilities.

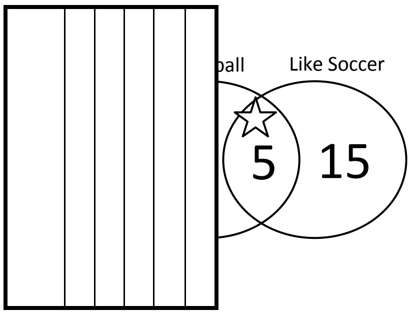
e) $P(\text{only like soccer})$

f) $P(\text{like football} \mid \text{like soccer})$

Answers:

e) $P(\text{only like soccer}) = \frac{15}{65}$ ← Students who only like soccer / ← total students surveyed

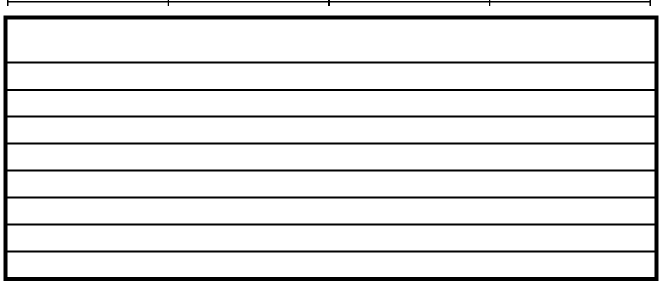
f) $P(\text{like football} \mid \text{like soccer}) = \frac{5}{20}$ ⇦ Of the 20 students who like soccer, 5 of them also like football



Example 3: The two-way table below shows the survey results of 150 high school students who were asked if they ate breakfast in the morning before school and their mood was assessed.

a) $P(\text{ate breakfast} \mid \text{good mood}) =$

	Ate Breakfast	Skipped Breakfast	Total
Good Mood	78	22	100



"Since it is given that the student is in a good mood, our sample space is restricted to these 100 students." (Cover up the bottom of the two-way frequency table with a note card or piece of paper)

$$P(\text{ate breakfast} \mid \text{good mood}) = \frac{78}{100} = 78\%$$

"Of the 100 students in a good mood, 78 of them ate breakfast."

b) $P(\text{good mood} \mid \text{ate breakfast}) = \dfrac{78}{90}$

$\approx 87\%$

Choral response:

"What is given? (The student ate breakfast)
How many students ate breakfast? (90 – show by circling)
Of those 90 students who ate breakfast, how many are in a good mood? (78 – show by putting a star)"

	Ate Breakfast	Skipped Breakfast	Total
Good Mood	☆ 78	22	100
Bad Mood	12	38	50
Total	(90)	60	150

Have students work on *c)* as a "you try". Have students share with a partner after a couple of minutes.

c) $P(\text{bad mood} \mid \text{skipped breakfast}) = \dfrac{38}{60}$

$\approx 63\%$

Example 4: A six-sided die is rolled and two coins are flipped. Find the following probabilities.

a) $P(\text{6 is rolled})$

1-HH	**2-HH**	**3-HH**	**4-HH**	**5-HH**	☆**6-HH**
1-HT	**2-HT**	**3-HT**	**4-HT**	**5-HT**	☆**6-HT**
1-TH	**2-TH**	**3-TH**	**4-TH**	**5-TH**	☆**6-TH**
1-TT	**2-TT**	**3-TT**	**4-TT**	**5-TT**	☆**6-TT**

"The sample space is provided with all 24 outcomes. A 6 is rolled on the die for 4 of the outcomes."

$P(\text{6 is rolled}) = \dfrac{4}{24}$

$= \dfrac{1}{6}$

b) $P(\text{6 is rolled} \mid \text{heads on both coins})$

(**1-HH**)	(**2-HH**)	(**3-HH**)	(**4-HH**)	(**5-HH**)	(☆**6-HH**)
1-HT	**2-HT**	**3-HT**	**4-HT**	**5-HT**	**6-HT**
1-TH	**2-TH**	**3-TH**	**4-TH**	**5-TH**	**6-TH**
1-TT	**2-TT**	**3-TT**	**4-TT**	**5-TT**	**6-TT**

"Restricting the sample space to the six outcomes with HH, only one of those outcomes has a 6 rolled."

$P(\text{6 is rolled} \mid \text{heads on both coins}) = \dfrac{1}{6}$

"Notice how both of the probabilities are the same! This is because getting heads on both coin flips has no effect on the probability of rolling a 6. Therefore these events are independent of one another."

Given two events A and B:

If $P(\mathrm{A} \mid \mathrm{B}) = P(\mathrm{A})$, then events A and B are independent events.

"Since events A and B are independent events and one event doesn't have an effect on the other, then it should follow that event A wouldn't have an effect on B as well or $P(\mathrm{B} \mid \mathrm{A}) = P(\mathrm{B})$. Let's see if this is true with the last example."

Have students write down this you try under the given box and work it out.
You Try:

c) $P(\text{heads on both coins})$

d) $P(\text{heads on both coins} \mid 6 \text{ is rolled})$

Answers:

c) $P(\text{heads on both coins})$

1-HH	2-HH	3-HH	4-HH	5-HH	6-HH
1-HT	2-HT	3-HT	4-HT	5-HT	6-HT
1-TH	2-TH	3-TH	4-TH	5-TH	6-TH
1-TT	2-TT	3-TT	4-TT	5-TT	6-TT

$$P(\text{heads on both coins}) = \frac{6}{24}$$
$$= 25\%$$

d) $P(\text{heads on both coins} \mid 6 \text{ is rolled})$

1-HH	2-HH	3-HH	4-HH	5-HH	6-HH
1-HT	2-HT	3-HT	4-HT	5-HT	6-HT
1-TH	2-TH	3-TH	4-TH	5-TH	6-TH
1-TT	2-TT	3-TT	4-TT	5-TT	6-TT

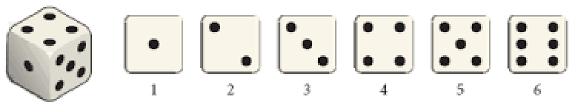
$$P(\text{heads on both coins} \mid 6 \text{ is rolled}) = \frac{1}{4}$$
$$= 25\%$$

"Rolling a 6 has no effect on the likelihood of getting heads on both coin flips because these two events are independent of one another."

Warm-Up

Review: 7.SP.7a

You roll a die. Find the probabilities below.



a) $P(\text{roll a } 5)$

b) $P(\text{roll an even})$

c) $P(\text{roll at least a } 3)$

Geometry: S.CP.5

Do you think the probability of getting into an accident increases if the driver is texting? Why or why not?

Review: 7.SP.5

Estimate the following probabilities.

a) $P(\text{it will rain this weekend})$

b) $P(\text{you get called on today})$

c) $P(\text{you go to college})$

Geometry: S.CP.1

Using your estimates from quadrant III, find the following probabilities.

a) $P(\text{it will not rain this weekend})$

b) $P(\text{you don't get called on today})$

c) $P(\text{you don't go to college})$

Definition of Conditional Probability:

Example 1: Two six-sided dice are rolled. Find the conditional probabilities below:

a) $P(5 \text{ on } 2^{nd} \text{ die} \mid 5 \text{ on } 1^{st} \text{ die})$ **b)** $P(\text{doubles} \mid \text{sum of dice at least } 8)$ **c)** $P(\text{sum of dice is at least } 8 \mid \text{doubles})$

1-1	2-1	3-1	4-1	5-1	6-1
1-2	2-2	3-2	4-2	5-2	6-2
1-3	2-3	3-3	4-3	5-3	6-3
1-4	2-4	3-4	4-4	5-4	6-4
1-5	2-5	3-5	4-5	5-5	6-5
1-6	2-6	3-6	4-6	5-6	6-6

1-1	2-1	3-1	4-1	5-1	6-1
1-2	2-2	3-2	4-2	5-2	6-2
1-3	2-3	3-3	4-3	5-3	6-3
1-4	2-4	3-4	4-4	5-4	6-4
1-5	2-5	3-5	4-5	5-5	6-5
1-6	2-6	3-6	4-6	5-6	6-6

1-1	2-1	3-1	4-1	5-1	6-1
1-2	2-2	3-2	4-2	5-2	6-2
1-3	2-3	3-3	4-3	5-3	6-3
1-4	2-4	3-4	4-4	5-4	6-4
1-5	2-5	3-5	4-5	5-5	6-5
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Example 2: 65 students were surveyed about watching sports on tv. 50 students said they like watching football and 20 students said they like watching soccer. Of those students there were 5 who like both. If a random student is selected from the survey, find the probabilities below.

a) $P(\textit{like football})$ **b)** $P(\textit{only like football})$ **c)** $P(\textit{like both})$ **d)** $P(\textit{like soccer} \mid \textit{like football})$

You Try: Use the same Venn Diagram to find the following probabilities.

e)

f)

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	Ate Breakfast	Skipped Breakfast	Total
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b) $P(\text{good mood} \mid \text{ate breakfast}) =$

c) $P(\text{bad mood} \mid \text{skipped breakfast}) =$

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1-HT	2-HT	3-HT	4-HT	5-HT	6-HT
1-TH	2-TH	3-TH	4-TH	5-TH	6-TH
1-TT	2-TT	3-TT	4-TT	5-TT	6-TT

b) $P(6 \text{ is rolled} \mid \text{heads on both coins})$

1-HH	2-HH	3-HH	4-HH	5-HH	6-HH
1-HT	2-HT	3-HT	4-HT	5-HT	6-HT
1-TH	2-TH	3-TH	4-TH	5-TH	6-TH
1-TT	2-TT	3-TT	4-TT	5-TT	6-TT

Given two events A and B:

You Try:

c)

d)

Rolling Two Dice