Grade Level/Course:
Geometry

Lesson/Unit Plan Name:
Finding the Equation of a Circle

Rationale/Lesson Abstract:
Students will find the equation of a circle using the Pythagorean Theorem. Through exploration, they will understand why they must subtract the *x*- & *y*- coordinates of the center of the circle when the center is not at the origin.

Timeframe:
2 - 55min. periods

Common Core Standard(s):

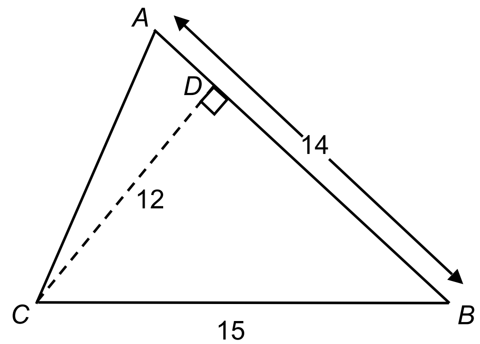
G.GPE.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem.

A.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Warm Up

Grade 8: 8.G.7

In this figure $\overline{AB}$ and $\overline{CD}$ are perpendicular. Select *all* of the following statements that are true. Justify your answers.



A) The length of $\overline{AD}$ is 5 units.

B) The perimeter of $\triangle CDB$ is 37 units.

C) The length of $\overline{AC}$ is 11 units.

D) The perimeter of $\triangle ABC$ is 42 units.

Grade 8: 8.G.6

Select *all* of the statements that are true about the described triangles. Justify your answers.

A) A triangle with side lengths 8 cm., 15 cm., and 17 cm., is a right triangle.

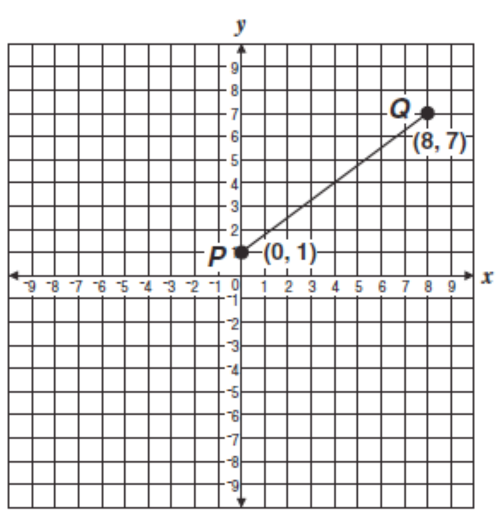
B) A triangle with side lengths 10 in., 20 in., and 24 in., is a right triangle.

C) A triangle with side lengths 9 ft., 12 ft., and 18 ft., is a right triangle.

D) A triangle with side lengths 12 m., 16 m., and 20 m., is a right triangle.

Grade 8: 8.G.8

What is the length of line segment $\overline{PQ}$ shown below?



Geometry: G.GPE.7

A triangle has vertices at the following coordinates: (10, 20), (7, 16), and (13, 16).

A) Find the perimeter of the triangle.

B) Find the area of the triangle.

C) Classify the triangle by the lengths of its sides.

Grade 8: 8.G.7

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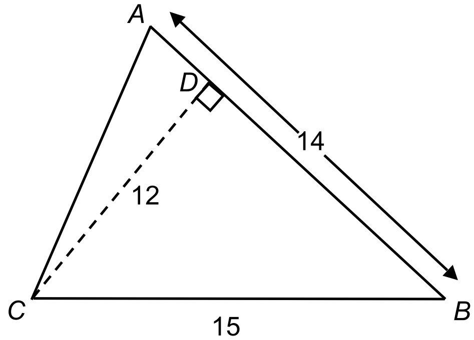

● The length of $\overline{AD}$ is 5 units.

B) The perimeter of $\triangle CDB$ is 37 units.

C) The length of $\overline{AC}$ is 11 units.

● The perimeter of $\triangle ABC$ is 42 units.

Grade 8: 8.G.6

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A) A triangle with side lengths 9 cm., 12 cm., and 18 cm., is a right triangle.

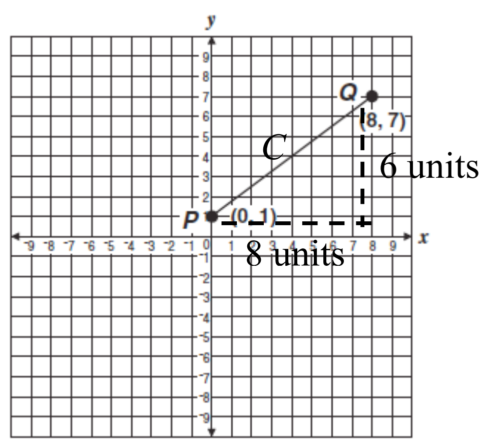
B) A triangle with side lengths 10 in., 20 in., and 24 in., is a right triangle.

● A triangle with side lengths 8 ft., 15 ft., and 17 ft., is a right triangle.

● A triangle with side lengths 12 m., 16 m., and 20 m., is a right triangle.

Grade 8: 8.G.8

What is the length of line segment $\overline{PQ}$ shown below?



$$c^2 = 8^2 + 6^2$$

$$c^2 = 64 + 36$$

$$c^2 = 100$$

$$c = \pm 10$$

$$\therefore \overline{PQ} = 10 \ units$$

Geometry: G.GPE.7

A triangle has vertices at the following coordinates: (10, 20), (7, 16), and (13, 16).

A) Find the perimeter of the triangle.

The perimeter is 16 units

B) Find the area of the triangle.

The area is 12 units^2

C) Classify the triangle by the lengths of its sides.

The triangle is an isosceles triangle.

Instructional Resources/Materials:

Ruler, ¼ inch Graph Paper, Paper, Pencil, Compass, Calculator, Scissors

Activity/Lesson:

In pairs: Give students two minutes to come up with characteristics of a circle. Now give them another minute to formulate a definition. Ask for answers.

Give students the definition of a circle: "A circle is defined as the set of all points in a plane at a given distance from a given point. How close is your definition to this one?"

Allow students time to answer. Draw a circle on the board with a point in the center as a model.

Ask students, "What do we call that given point?"

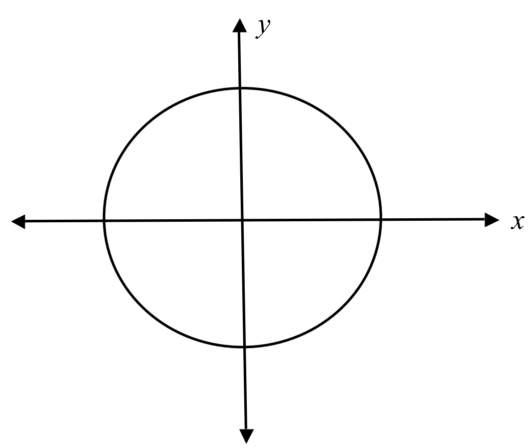
Choral Response: The Center

Ask students, "What term do we use to define the given distance from the center?"

Choral Response: The Radius

"Today we are going to use what we know about the Pythagorean Theorem and the radius of a circle to find the equation of a circle. Since a circle is defined by its radius and its center, we need to devise a way to finding the radius. This will give us the equation of the circle."

Using their rulers, have students draw a large 4-quadrant coordinate plane in the center of their graph paper, labeling the *x*- and *y*- axes and the origin. Next instruct them to use their compass to draw a circle on their graph with a radius between 2 and 3 inches, and the center of the circle at the origin. Let the students know that their measurements will be in units, not inches. Check their drawings to make sure they are correct.



Activity/Lesson continued:

Creating a right triangle.

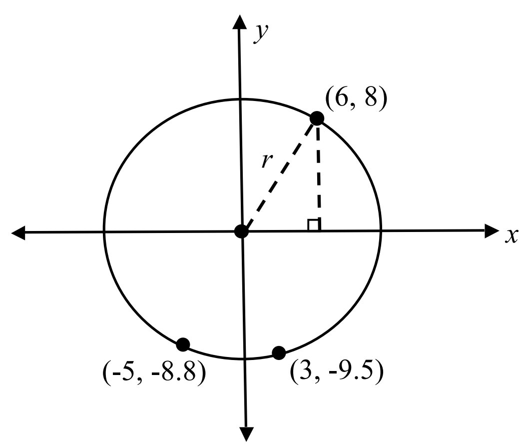
"Place a point on the edge of the circle that is not on the *x*- or *y*- axis, and one at the origin. Connect them with line segment to represent a radius. Label this distance *r*."

Ask students, "Since we are using the Pythagorean Theorem today, we need a right triangle. And since we define a circle by its center and radius, we need to use these also. How can we make a right triangle using the center of the circle and the radius? Is there more than one way? Share with your partner."

Give students a moment to discuss with each other.

"Create a right triangle with the radius and the center by drawing a line segment to connect the point on the circle with the *x*-axis or *y*-axis.

"Use the gridlines to write the coordinates of the point on the circle. If your point does not fall on the gridlines, estimate the coordinates to the nearest tenth."



Ask students, "What does the radius of the circle represent on the right triangle?"

Student Response: The radius represents the hypotenuse.

Ask students, "How can we find the lengths of the legs of the right triangle?"

Student Response: The coordinates of the point on the circle define the lengths of the legs.

Ask students, "How do we find the hypotenuse of a right triangle if we know the lengths of the legs?"

Student Response: With the Pythagorean Theorem; $a^2 + b^2 = c^2$

Ask Students, "In our case, what is c^2?"

Student Response: The radius squared.

Give the students a few minutes to find the length of the radius using the Pythagorean Theorem. After they find it, have them pick two other points on their circle in different quadrants and repeat the process and compare their radii.

Ask students, "Does it matter where we place the point on the circle, will the length of the radius always be the same?"

Student Response: Yes

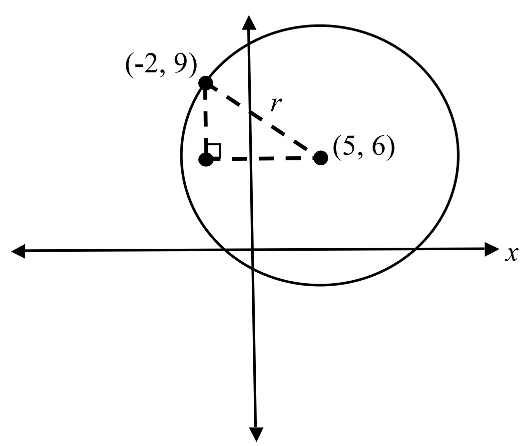
"Since this is the case, place the point (x, y) on your circle and write a general equation to find the radius. Compare your answer with your partner."

Have students share out their equations. $x^2 + y^2 = r^2$ or $r = \sqrt{x^2 + y^2}$

Part II

"Now turn your graph paper over and make another large 4-quadrant coordinate plane on your graph paper. This time pick a point about an inch away from the origin that is not on either axis to use as the center of your circle. Draw a circle and label your axes. Also, label the point at the center of your circle."

"Place a point on your circle where the gridlines meet. Write the coordinate of the point on the circle. Draw a line segment from this point to the center to create the radius. Label this distance r."



"Again we need to create a right triangle. How can we make a right triangle using the center and the radius? Can we use the x- and y- axes as we did before? Why not? Discuss this with your partners."

Student Response: If we use the axes, we will not get a right triangle.

"OK. Pick a point you can use to create a right triangle and draw the legs."

"Earlier we saw that the coordinates of the point on the circle corresponded with the lengths of the legs of the right triangle. Use the gridlines to find the lengths of the legs (in units). What are the lengths of the legs now?"

Student Response: Answers will vary.

"How do these leg lengths compare with the coordinates of the point on the circle? Why aren't they the same? How much do they differ by? Discuss with your partner."

Student Response: The leg lengths are not the same because the center of the circle is not at the origin. They differ by the coordinates of the center of the circle.

"Therefore what must we do to find the actual lengths of the legs of the triangle in order to use the Pythagorean Theorem?"

Student Response: The coordinates of the center have to be subtracted from the coordinates of the point on the circle.

"Find the lengths of the legs. Then find the length of the radius." Give students about 2 minutes for work.

"Suppose that you have a circle with the center at the point (h, k) and a point on the circle (x, y). Independently, write expressions to find the lengths of the legs. Once you have your answer, check it with your partner."

Student Answers: Leg1 $(x - h)$, Leg2 $(y - k)$

"Now using those expressions, write an equation to find the radius with your partner. Share out your answers."

Student Response: $(x - h)^2 + (y - k)^2 = r^2$ or $r = \sqrt{(x-h)^2 + (y-k)^2}$

You Try:

1) Find the equation of a circle with the center at $(-5, 0)$ and a radius of 7.

2) Find the radius and center of the circle $(x + 4)^2 + y^2 = 121$

3) The center of a circle is $(5, 6)$. One point on the circle is $(-2, 3)$. Find the equation of the circle.

Student Activity: Sorting Equations (20 minutes)

Organize the class into small groups of two or three students

Give each group of students a copy of Card Set: Equations, and a copy of Sorting Equations.

Have students cut out their equations and place the correct equation in the spaces given on the Sorting Equations worksheet. Some spaces will require the students to come up with their own equation.

Give the students the opportunity to present and justify their answers.

Sorting Equations

1. Take turns to place an equation card in one of the categories in the table.

2. When you place a card, explain how you came to your decision.

3. If you don't agree or understand, ask your partner to explain their reasoning.

4. Write additional information, or include a drawing as part of your explanation.

5. Some of your cards will be placed in the final column. You will need to figure out the coordinates for the center of the circle for all equations placed in this column.

6. Make up your own equations for any empty spaces.

You all need to agree on and explain the placement of every card.

Sorting Equations

	Center at (2, 1)	Center at (2, –1)	Center at (0, –1)	Center (__, __)
Radius of $\sqrt{5}$				
Radius of $\sqrt{10}$				
Radius of 5				
Radius of 10				

Card Set: Equations

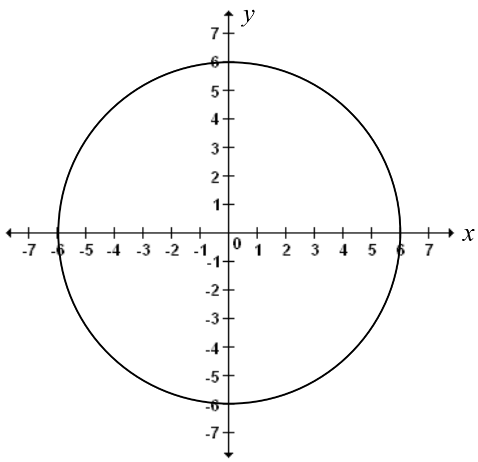
1. $(x-2)^2+(y-1)^2=25$	2. $(x+2)^2+(y-1)^2-100=0$	3. $x^2+(y+1)^2=25$
4. $(y-1)^2+(x-2)^2=5$	5. $(x+2)^2+(y-1)^2=10$	6. $x^2+(y+1)^2=100$
7. $(x-2)^2+(y-1)^2+15=25$	8. $(x-2)^2+(1+y)^2=100$	9. $(y+1)^2+x^2=10$
10. $(x-2)^2+(y+1)^2=10$	11. $(x-2)^2+(y+1)^2+4=9$	12. $(y+1)^2+(x-2)^2=25$

Sorting Equations: Answer Key

	Center at (2, 1)	Center at (2, –1)	Center at (0, –1)	Center (–2, 1)
Radius of $\sqrt{5}$	4. $(y-1)^2 + (x-2)^2 = 5$	11. $(x-2)^2 + (y+1)^2 + 4 = 9$	$x^2 + (y+1)^2 = 5$	$(x+2)^2 + (y-1)^2 = 5$
Radius of $\sqrt{10}$	7. $(x-2)^2 + (y-1)^2 + 15 = 25$	10. $(x-2)^2 + (y+1)^2 = 10$	9. $(y+1)^2 + x^2 = 10$	5. $(x+2)^2 + (y-1)^2 = 10$
Radius of 5	1. $(x-2)^2 + (y-1)^2 = 25$	12. $(y+1)^2 + (x-2)^2 = 25$	3. $x^2 + (y+1)^2 = 25$	$(x+2)^2 + (y-1)^2 = 25$
Radius of 10	$(x-2)^2 + (y-1)^2 = 100$	8. $(x-2)^2 + (1+y)^2 = 100$	6. $x^2 + (y+1)^2 = 100$	2. $(x+2)^2 + (y-1)^2 - 100 = 0$

Exit Ticket

A circle with a center at (0, 0) has a radius of 6 units. A point on the circle has coordinates (2, *y*). Find the value of *y*.



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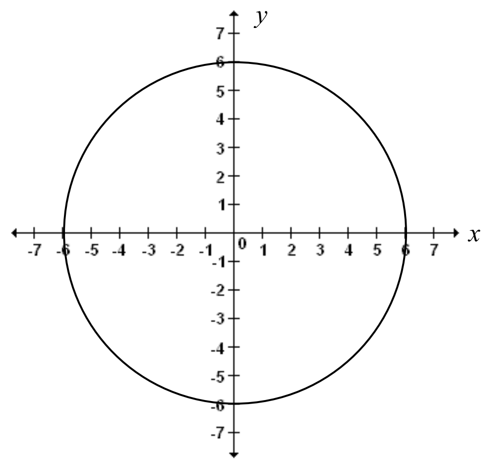

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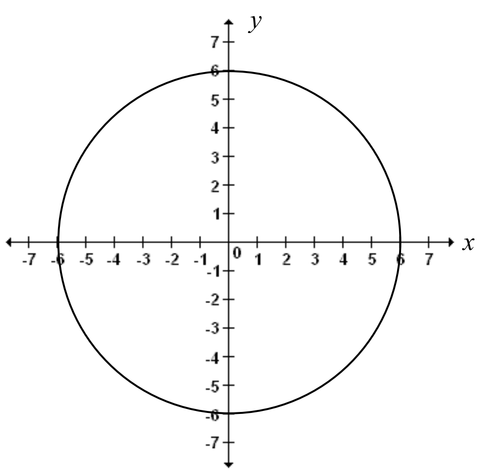

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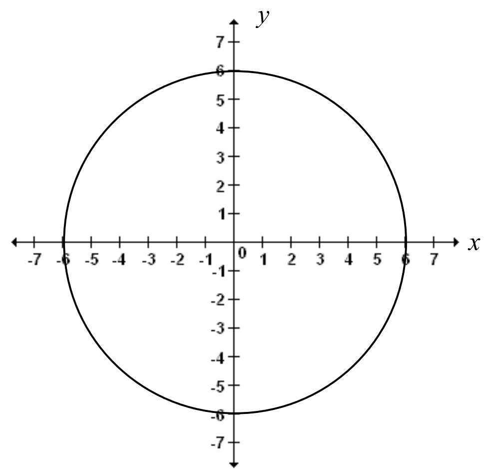

You Try Answers:

1) Find the equation of a circle with the center at (–5, 0) and a radius of 7.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x+5)^2 + (y-0)^2 = 7^2$$

$$(x+5)^2 + y^2 = 49$$

2) Find the radius and center of the circle $(x+4)^2 + y^2 = 121$

Center: (–4, 0) and radius = 11

3) The center of a circle is (5, 6). One point on the circle is (–2, 3). Find the equation of the circle.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(-2-5)^2 + (3-6)^2 = r^2$$

$$(-7)^2 + (-3)^2 = r^2$$

$$49 + 9 = r^2$$

$$58 = r^2$$

The equation of the circle is $(x-5)^2 + (y-6)^2 = 58$.

Exit Ticket Answer

A circle with a center at (0, 0) has a radius of 6 units.
A point on the circle has coordinates $(2, y)$. Find the value of y.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(2-0)^2 + (y-0)^2 = 6^2$$

$$4 + y^2 = 36$$

$$y^2 = 32$$

$$y = \sqrt{32} = 4\sqrt{2}$$

$$y \approx 5.66$$