Grade Level/Course: Geometry
Lesson/Unit Plan Name: Dilations Using Right Triangles
Rationale/Lesson Abstract: This lesson is designed to introduce dilations by using congruent right triangles from the center of dilation. It is designed to be more geometrical in design than algebraic computation of the coordinates. It begins with examples that dilate a point and then moves to other examples where the students are dilating a segment, and a triangle. It also contains several examples where the center of dilation is not the origin. Students will discover how a dilated line segment not going through the center will produce an image that is parallel to the preimage and the ratio of their lengths will be equal to the scale factor of the dilation. This lesson is to be used after teaching similar and congruent triangles with your students. It is intended to build off students' prior use of slope triangles* from Algebra 1. * A slope triangle is a right triangle drawn on a linear function to help find the slope. The hypotenuse lies on the graph of the line and the legs of the right triangle can be used to represent the vertical and horizontal change between two points. This vertical and horizontal change can be used to help calculate the slope of the line, $m=\frac{\Delta y}{\Delta x}$. Quadrant I of the warm up is designed for teachers to help assist students unfamiliar with the concept.
Timeframe: 2 days
Common Core Standard(s): **G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor: a.) A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b.) The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Instructional Resources/Materials:
Warm up (pg. 10), student note-taking guide (pg. 11-13), exit-ticket (pg. 14), highlighter and pencil.

Answers to Warm Up:

1) What is $\frac{1}{3}$ of 15?

15		
5	5	5

$\frac{1}{3} \bullet 15$

$= \frac{1}{3} \bullet \frac{15}{1}$

$= \frac{15}{3}$

$= 5$

2) What is $\frac{3}{4}$ of 8?

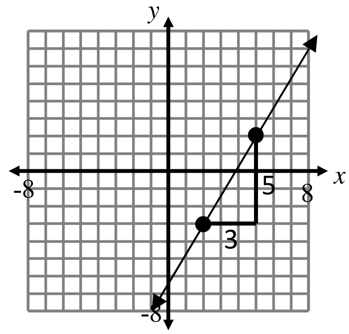
8			
2	2	2	2

$\frac{3}{4} \bullet 8$

$= \frac{3}{4} \bullet \frac{8}{1}$

$= \frac{24}{4}$

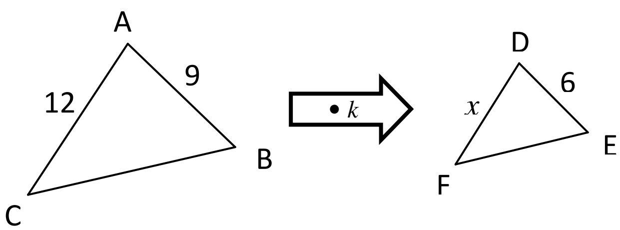
$= 6$



(draw the slope triangle between the two points)

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{5}{3}$$



$\overline{AB}$ *and* $\overline{DE}$ *are corresponding sides in the similarity statement so 9 times some scalar k will equal 6:*

$$9k = 6$$

$$\frac{9k}{9} = \frac{6}{9}$$

$$k = \frac{2}{3}$$

Now we can find x by multiplying 12 by $\frac{2}{3}$ *and we can circle the following statements that are true.*

1) $x = 8$

2) $DE = \frac{2}{3}(AB)$

3) $BC = \frac{2}{3}(EF)$

4) $m\angle D = \frac{2}{3}(m\angle A)$

Using Pythagorean Theorem:

$$6^2 + 8^2 = n^2$$

$$36 + 64 = n^2$$

$$100 = n^2$$

$$\sqrt{100} = \sqrt{n^2}$$

$$10 = n$$

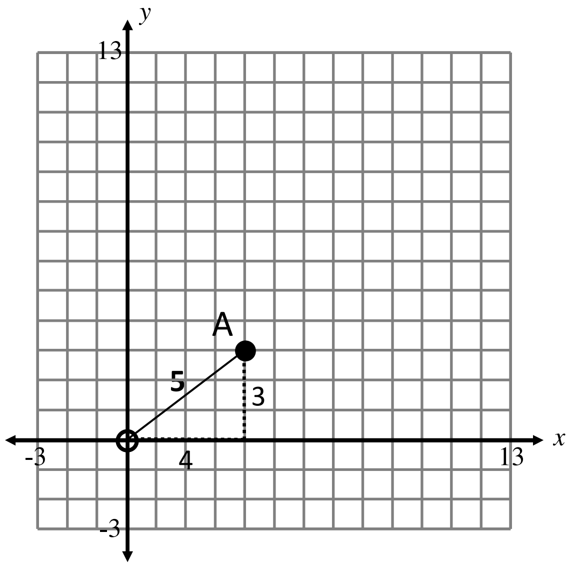
Activity/Lesson:

Start by giving the definition of Dilation to the students:

Dilation – A non-rigid transformation that enlarges or reduces a geometric figure by a scale factor relative to a point.

Example 1: Graph the image of point A after a dilation with scale factor of 3 centered at the origin.

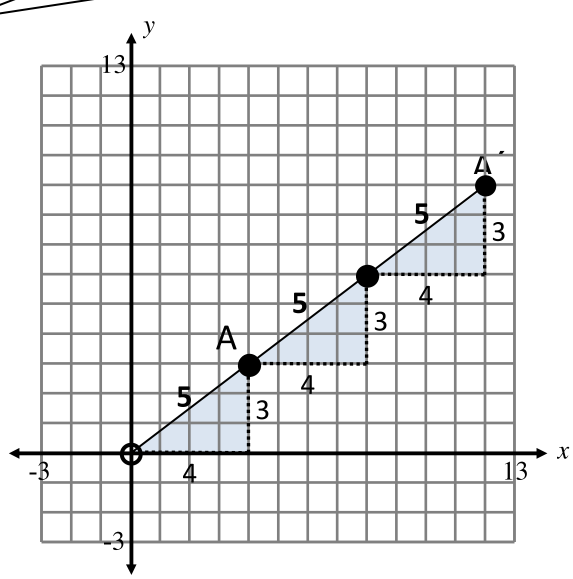
"In this example the center of dilation is the origin. Let's draw the slope triangle from the center to point A and find the lengths of the horizontal and vertical sides of our slope triangle."



"Using Pythagorean Theorem we can solve for the length of the hypotenuse."

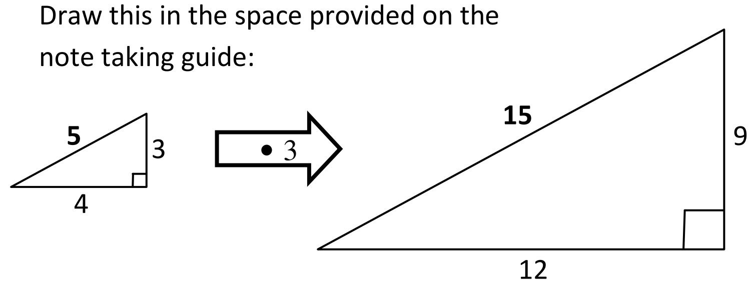
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$5 = c$$

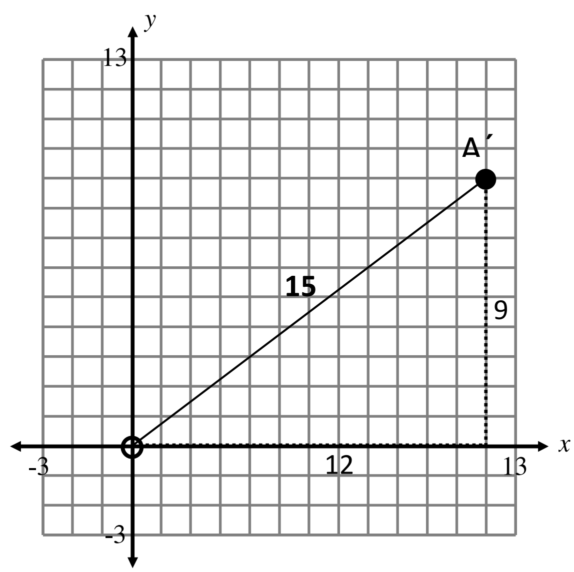
"The scale factor is 3 so we need to triple the distance between the center and the point. We can do that by creating 3 congruent slope triangles in the direction of point A."



"Another way we can do this is by tripling the sides of our slope triangle creating a similar triangle that is three times the size and then orienting the new triangle at the center."

Draw this in the space provided on the note taking guide:

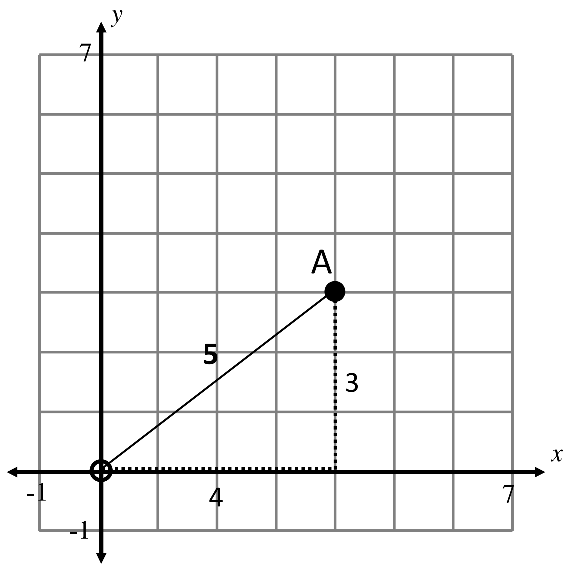




The coordinates of A´ are $(12,9)$.

Example 2: Graph the image of point A after a dilation with scale factor of $\frac{1}{2}$ centered at the origin.

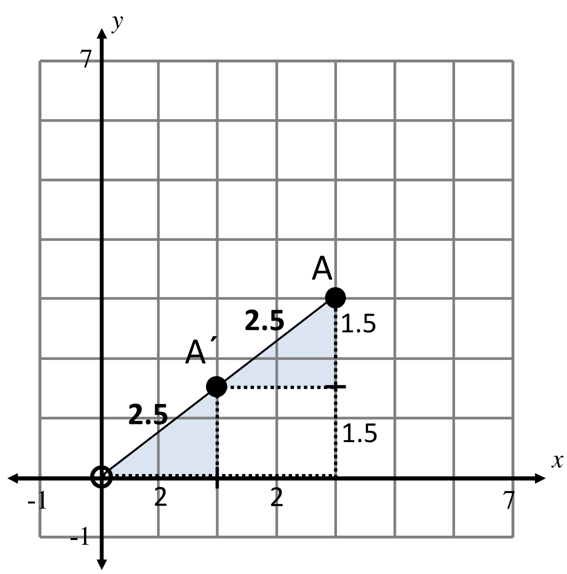
"In this example the center of dilation is the origin. Let's draw the slope triangle from the center to point A and find the lengths of the horizontal and vertical sides of our slope triangle."



<u>Think-Pair-Share:</u> What effect will a scale factor of $\frac{1}{2}$ have on point A and how do you think we are going to figure out where the image is after the dilation?

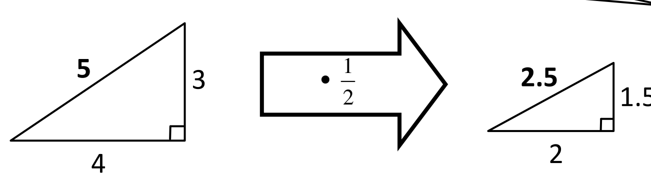
- Have some students share out and work through their reasoning to find the image of point A. Some possible solutions to finding the image are below.

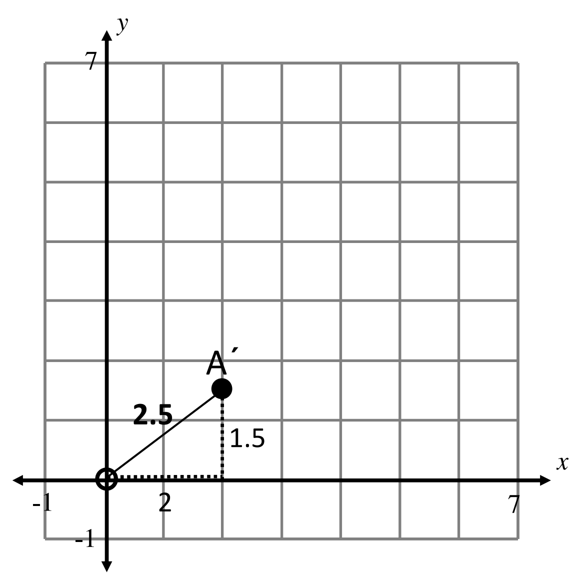
"The scale factor is $\frac{1}{2}$ so we need to cut the distance between the center and the point in half. We can do that by creating 2 smaller congruent slope triangles that end at point A."



The coordinates of A´ are $(2,\ 1.5)$.

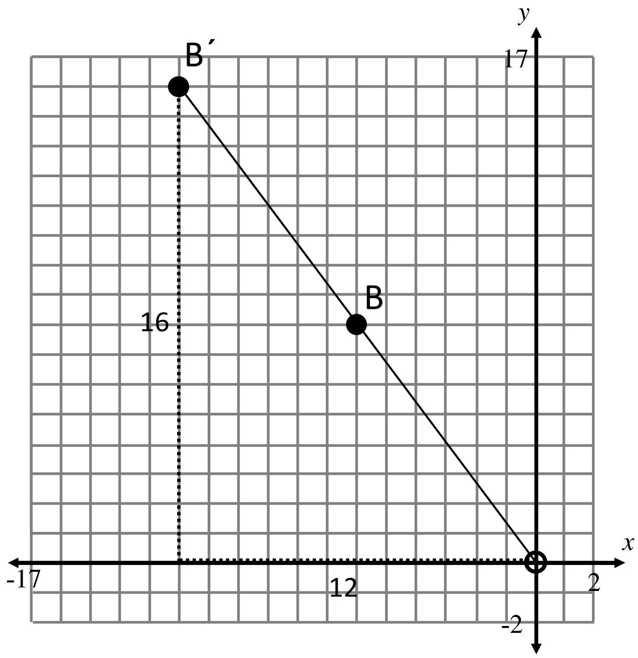
"Another way we can do this is by cutting the sides of our slope triangle in half creating a similar triangle that is half the size and then orienting the new triangle at the center."





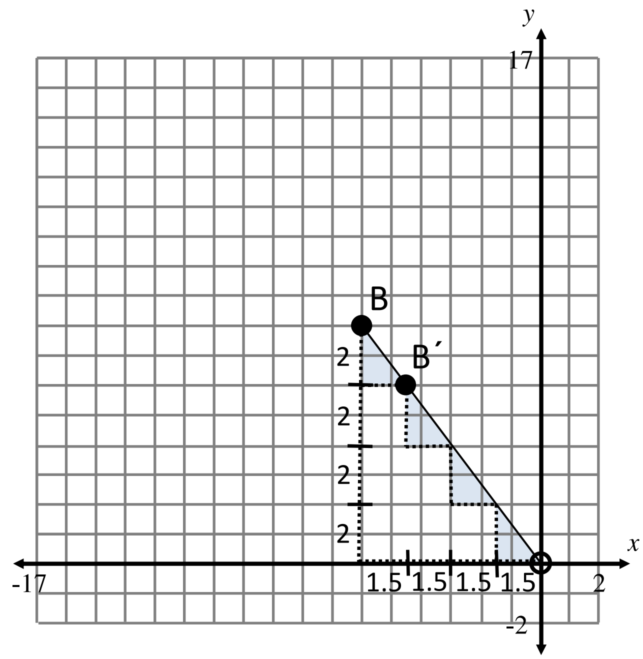
You Try: Graph the image of point B after each of the following dilations.

a) Dilation with scale factor of 2 centered at the origin.



The coordinates of B´ are $(-12,16)$**.**

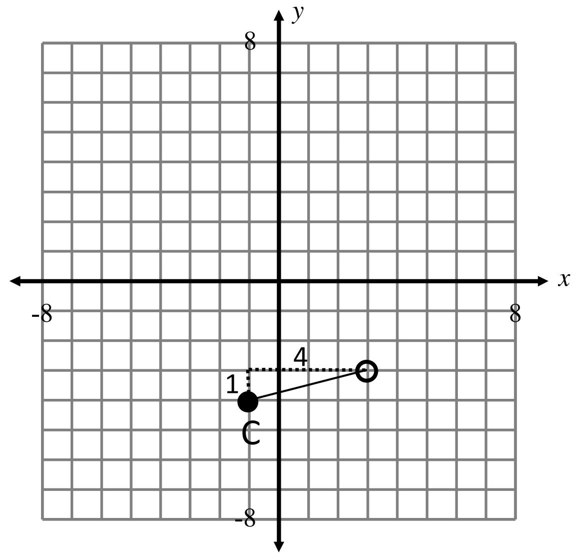
b) Dilation with scale factor of $\frac{3}{4}$ centered at the origin.



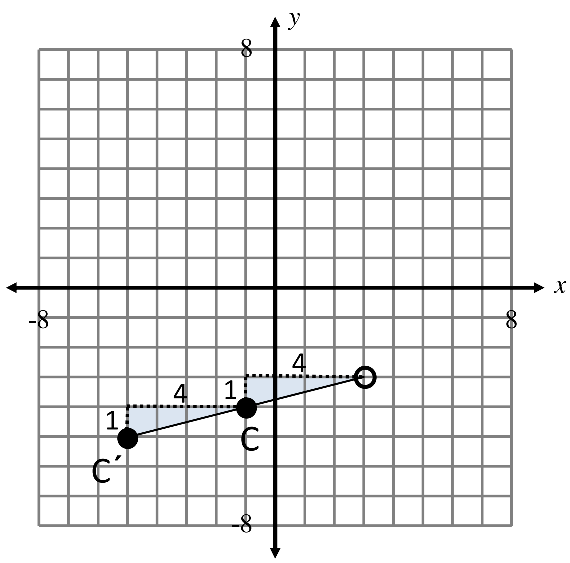
The coordinates of B´ are $(-4.5,6)$**.**

Example 3: Graph the image of point C after a dilation with scale factor of 2 centered at $(3,-3)$.

"In this example the center of dilation is no longer the origin. Let's plot the center and draw the slope triangle from the center to point C."

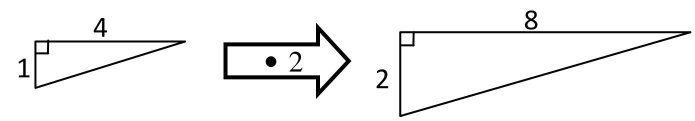


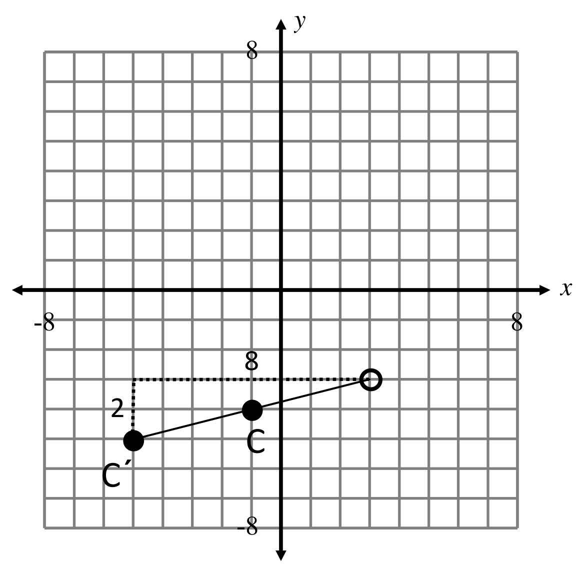
"The scale factor is 2 so we need to double the distance between the center and the point. We can do that by creating another congruent slope triangle continuing in the direction of point C or we can double the sides of our slope triangle creating a similar triangle that is twice the size."



The coordinates of C´ are $(-5,-5)$

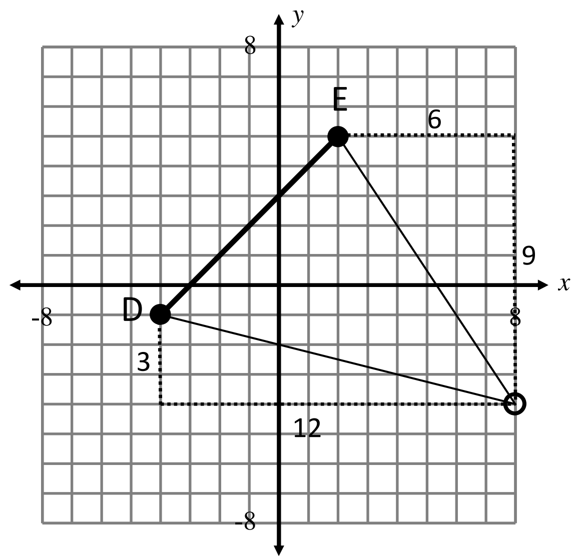
Note: Finding the length of the hypotenuse is irrelevant to finding the coordinates of the image. We know they are congruent triangles (by SAS) so we know the image is twice as far from the center.



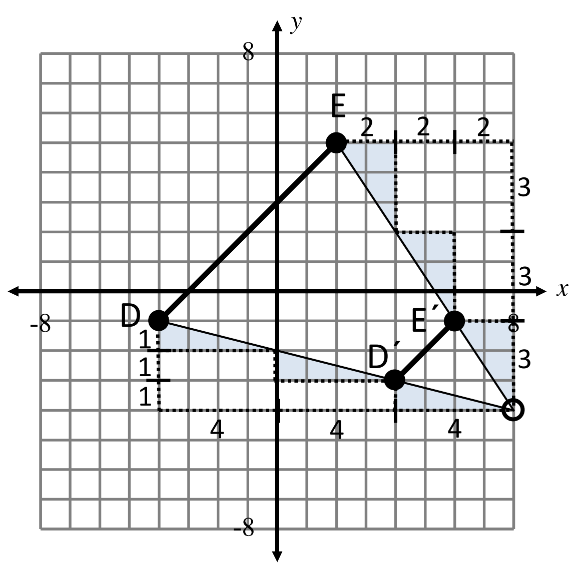


Example 4: Graph the image of segment $\overline{DE}$ after a dilation with scale factor of $\frac{1}{3}$ centered at $(8,-4)$.

"We need to dilate the entire segment. We can do this by focusing on dilating the two endpoints. Draw the slope triangles from the center to each of the two endpoints."

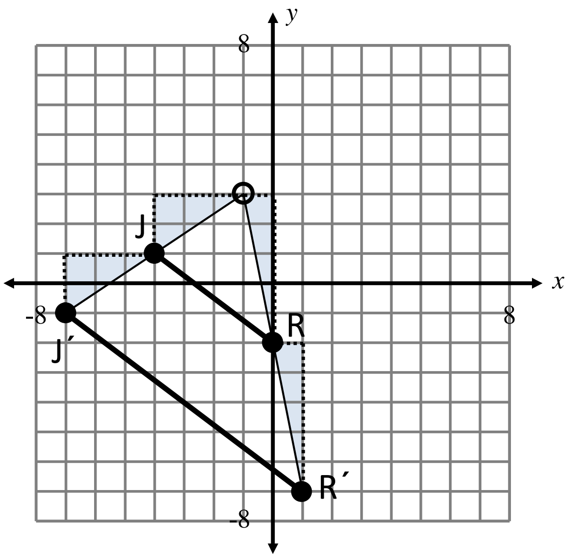


"The scale factor is $\frac{1}{3}$ so we need to cut the distance between the center and each of the endpoints in thirds. We can do that by creating 3 smaller congruent slope triangles that end at each of the endpoints."



Point out that this dilation is a reduction (the image is smaller than the preimage), because the scale factor is less than one.

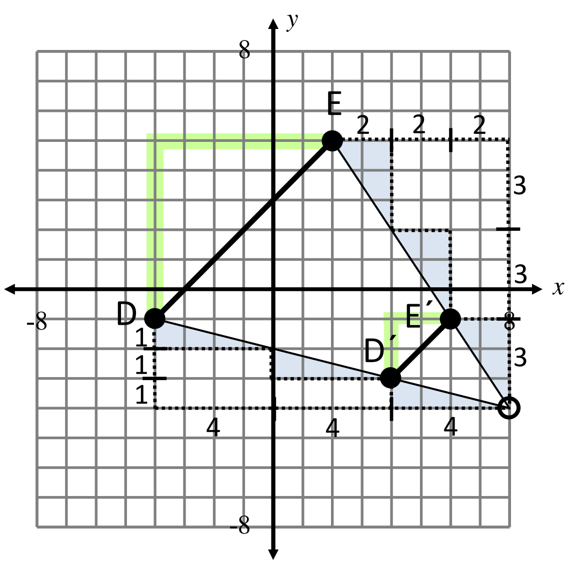
You Try: Graph the image of segment $\overline{JR}$ after a dilation with scale factor of 2 centered at $(-1, 3)$.



After the "You Try," point out that this dilation is an enlargement (the image is bigger than the preimage), because the scale factor is greater than one.

Example 4 revisited…

Let's take a closer look at the reduction we completed in Example 4. We used slope triangles to complete the dilation, but let's look at the slopes of the preimage and image (the two line segments).



*Draw the slope triangles for the two segments using a highlighter or some other distinct marker.

$m\overline{DE} = \frac{6}{6}$ $\qquad$ $m\overline{D'E'} = \frac{2}{2}$

$m\overline{DE} = 1$ $\qquad$ $m\overline{D'E'} = 1$

The slopes of the two line segments are equal. Therefore, the two line segments are parallel.

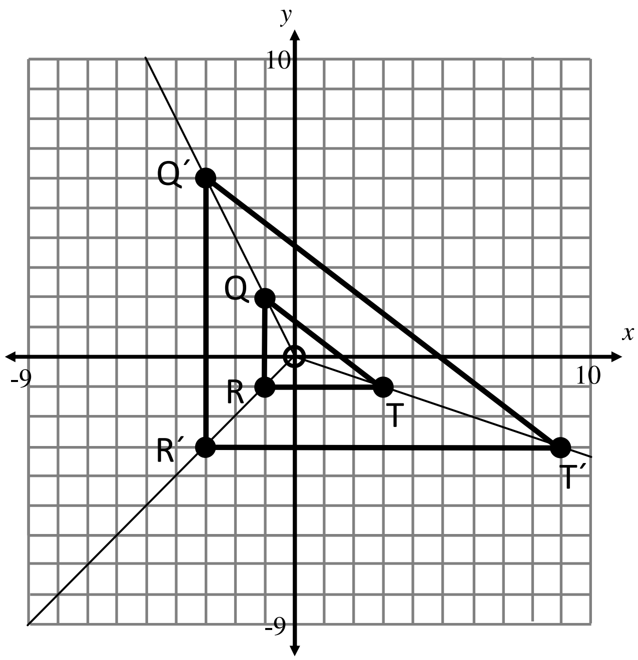
When you dilate a segment that doesn't go through the center of dilation, will it always produce a parallel segment? (YES)

Ask the students to find the slopes of both segments in the last "You Try" to verify:

$m\overline{JR} = -\frac{3}{4}$ $\qquad$ $m\overline{J'R'} = -\frac{6}{8}$

$m\overline{J'R'} = -\frac{3}{4}$

Example 5: Graph the image of ΔQRT after a dilation with scale factor of 3 centered at the origin.



a) **Find the side lengths of ΔQRT .**

$QR = 3$

$RT = 4$

$QT = 5$

b) **Graph the image of ΔQRT after a dilation with scale factor of 3 centered at the origin.**

c) **Find the side lengths of the image, $\Delta Q'R'T'$.**

$Q'R' = 9$

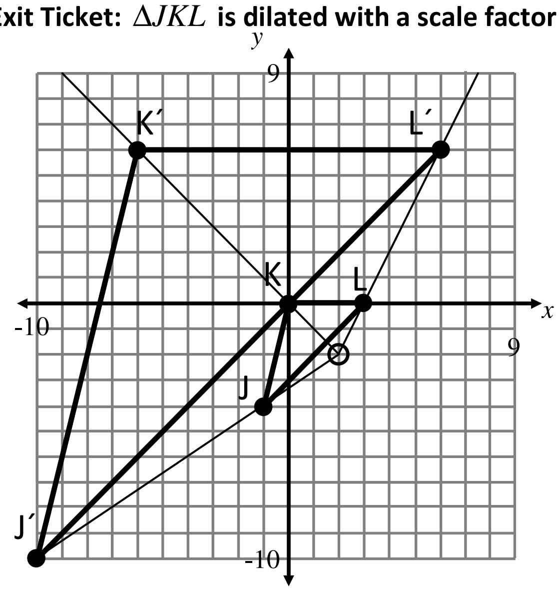
$R'T' = 12$

$Q'T' = 15$

<u>**Think-Pair-Share:**</u> What do you notice about the corresponding side lengths of the preimage and image? (The side lengths of the image are longer in the ratio given by the scale factor.)

Make sure to give enough time for students to think individually before sharing with a partner. Have some students share out.

Exit Ticket: ΔJKL is dilated with a scale factor of 4 centered at $(2,-2)$. Circle all true statements.



1. The dilation is classified as a reduction.
2. The coordinates of K' are $(-8,8)$.
3. The coordinates of J' are $(-10,-10)$.
4. $\overline{JK} \parallel \overline{J'K'}$
5. $K'L' = 13$
6. $J'L' = 4(JL)$

Warm-Up

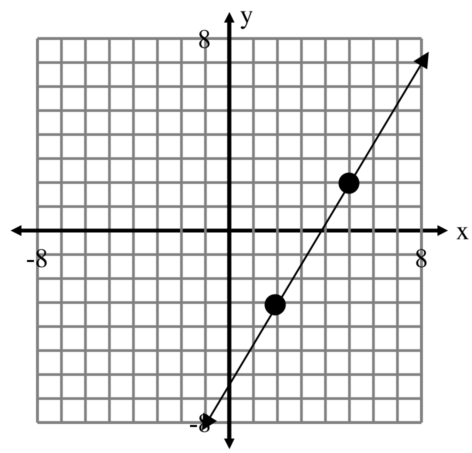
Review: 5.NF.4a

1) What is $\frac{1}{3}$ of 15?

2) What is $\frac{3}{4}$ of 8?

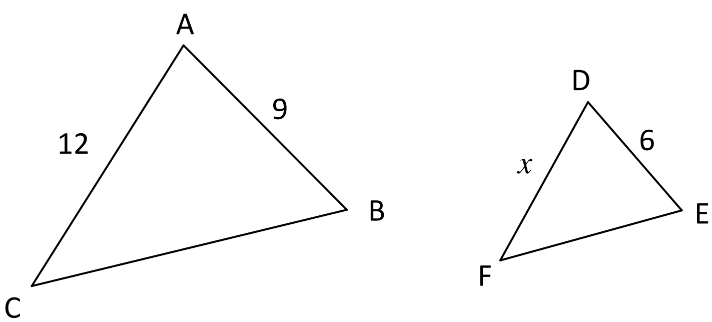
Review: 8.EE.5

Find the slope of the line below.



Current: G.SRT.2

Given that $\Delta ABC \sim \Delta DEF$, circle the following statements that are true.



1) $x = 8$

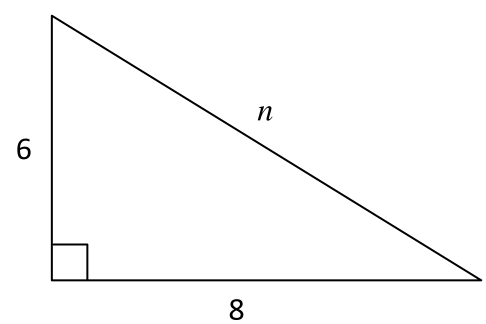
2) $DE = \frac{2}{3}(AB)$

3) $BC = \frac{2}{3}(EF)$

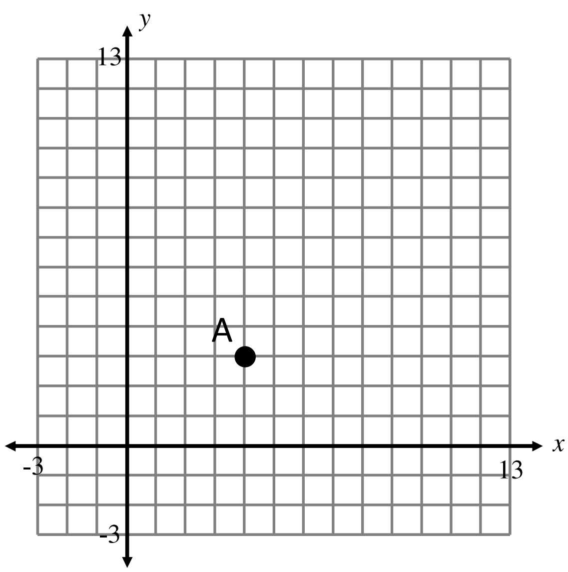
4) $m\angle D = \frac{2}{3}(m\angle A)$

Other: 8.G.7

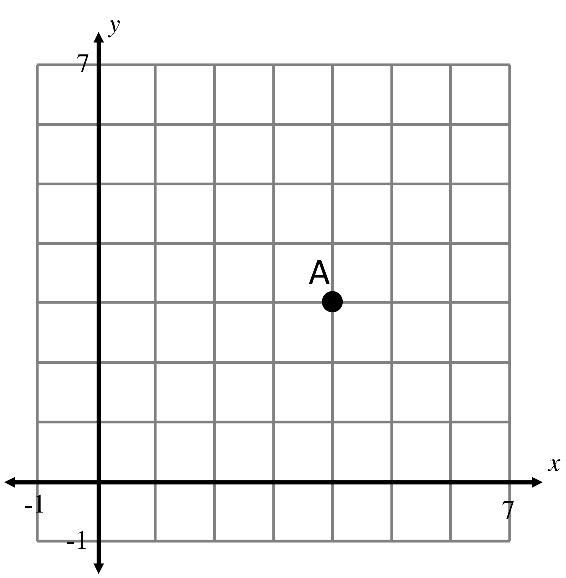
Solve for *n*.



Example 1: Graph the image of point A after a dilation with scale factor of 3 centered at the origin.



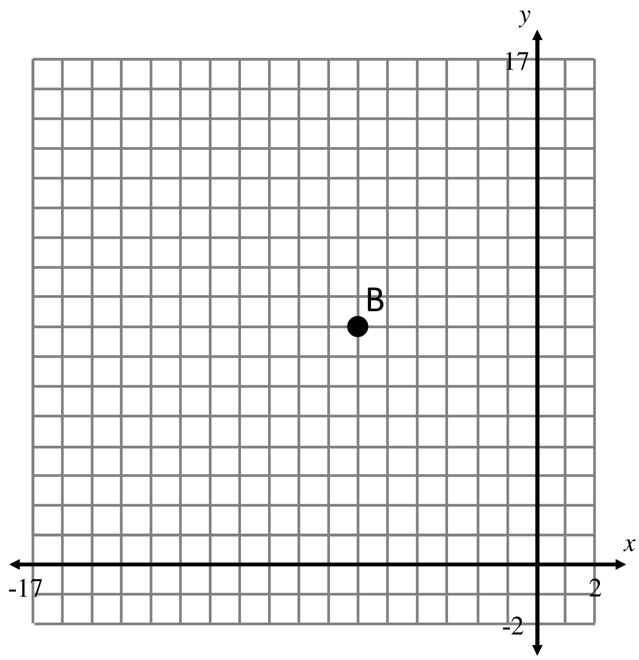
Example 2: Graph the image of point A after a dilation with scale factor of $\frac{1}{2}$ centered at the origin.



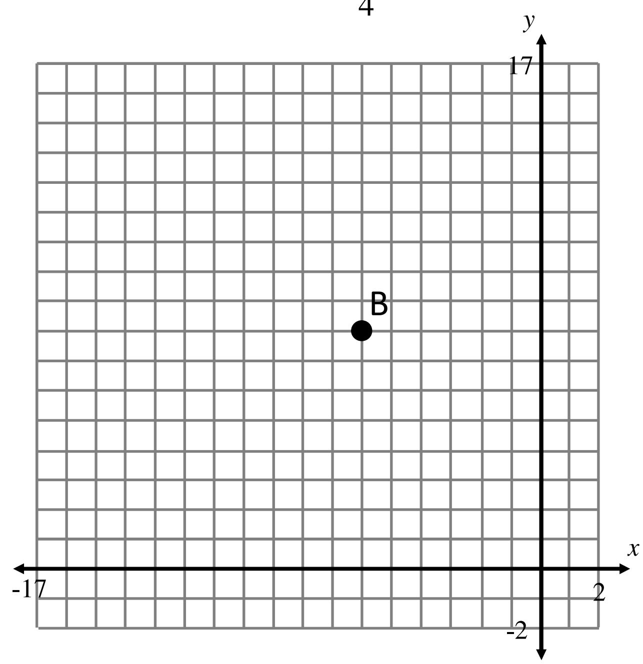
Dilations Using Right Triangles: Note-Taking Guide

You Try: Graph the image of point B after each of the following dilations.

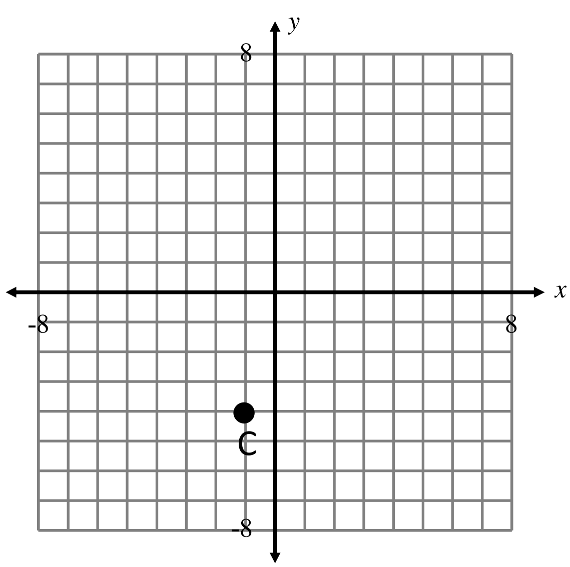
a) Dilation with scale factor of 2 centered at the origin.



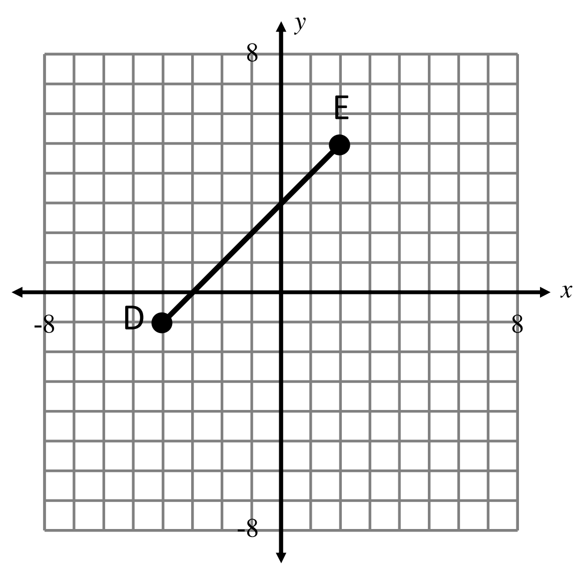
b) Dilation with scale factor of $\frac{3}{4}$ centered at the origin.



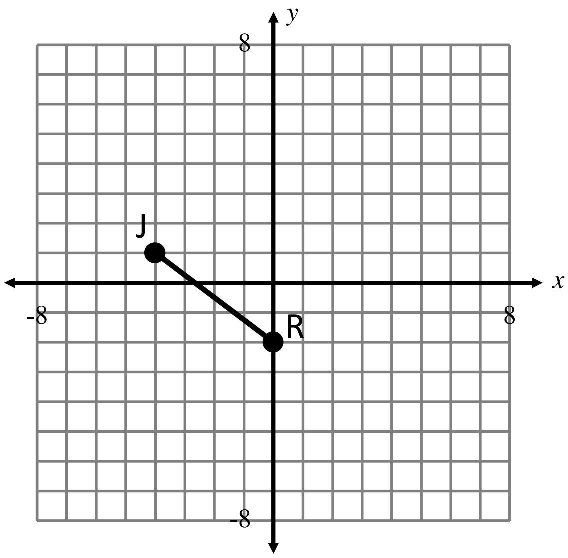
Example 3: Graph the image of point C after a dilation with scale factor of 2 centered at $(3,-3)$.



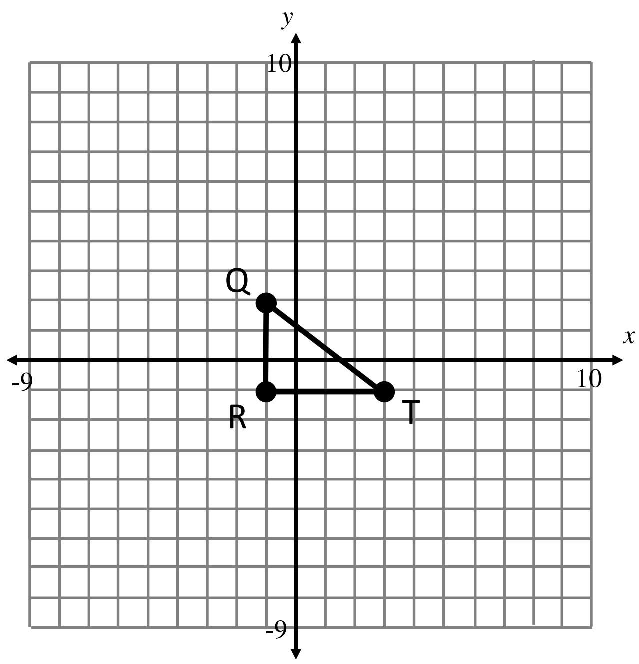
Example 4: Graph the image of segment $\overline{DE}$ after a dilation with scale factor of $\frac{1}{3}$ centered at $(8,-4)$.



You Try: Graph the image of segment $\overline{JR}$ after a dilation with scale factor of 2 centered at $(-1,3)$.



Example 5:



a) **Find the side lengths of ΔQRT.**

$QR =$

$RT =$

$QT =$

b) **Graph the image of ΔQRT after a dilation with scale factor of 3 centered at the origin.**

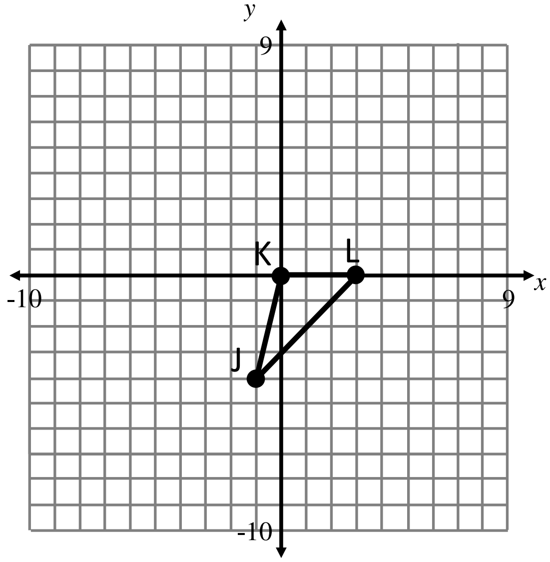
c) **Find the side lengths of the image, $\Delta Q'R'T'$.**

$Q'R' =$

$R'T' =$

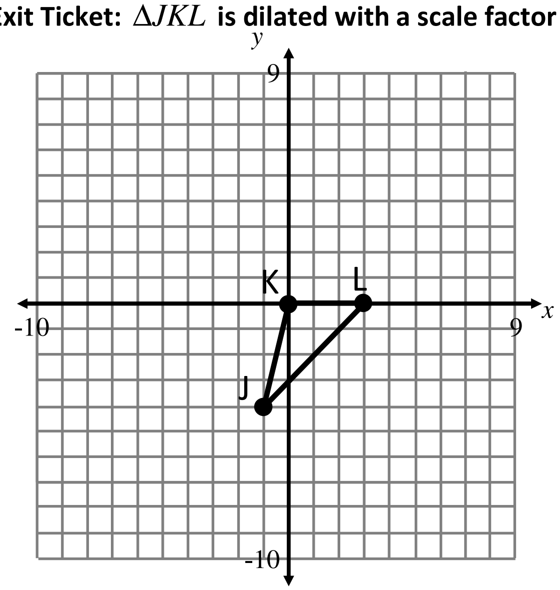
$Q'T' =$

Exit Ticket: ΔJKL **is dilated with a scale factor of 4 centered at** $(2,-2)$. **Circle all true statements.**



1. The dilation is classified as a reduction.
2. The coordinates of K' are $(-8,8)$.
3. The coordinates of J' are $(-10,-10)$.
4. $\overline{JK} \parallel \overline{J'K'}$
5. $K'L' = 13$
6. $J'L' = 4(JL)$

Exit Ticket: ΔJKL **is dilated with a scale factor of 4 centered at** $(2,-2)$. **Circle all true statements.**



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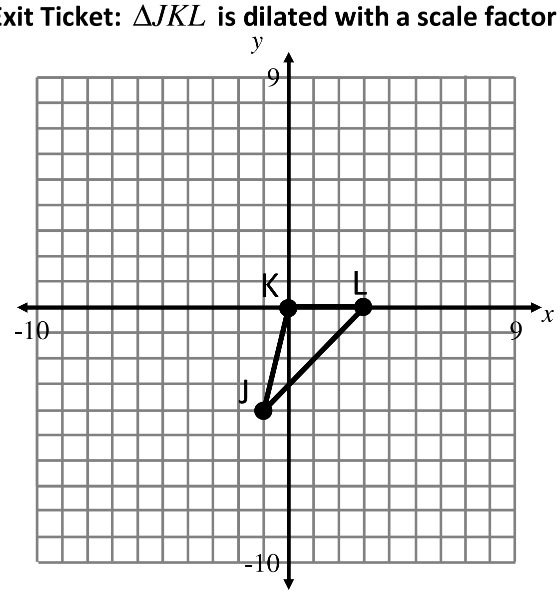

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