Grade Level/Course: Grade 8 and Geometry

Lesson/Unit Plan Name: Congruence through Transformations

Rationale/Lesson Abstract:
This lesson builds understanding of translations, reflections, and rotations using a pairs of congruent triangles on the coordinate plane. Students do hands on experiments with tracing paper to recreate the three transformations. It then provides students with examples of two congruent triangles in different locations on the coordinate plane and leads them through describing a sequence that demonstrates their congruence.

Timeframe: 2 days
Day 1-
There are a total of 8 examples, so you will need to move quickly through translations and reflections to have time to do well on rotations.
Day 2-
If students are comfortable with describing translations, reflections, and rotations, this lesson may be used separately.

Common Core Standard(s):
8.G.2 - Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

G.CO.5 –Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Instructional Resources/Materials:
Patty paper, protractors, copies of figures for students and extra homework examples (Day 1 pages 13-15, Day 2 pages 17-19,) pencils, (optional: glue or tape). Document camera for demonstration.

Note: Tracing paper is referred to as patty paper in this lesson. Patty paper is the paper that grocery stores sell to go in between hamburger patties to keep them from sticking together.

Note: A similar lesson can be done using computers and the GeoGebra program. The figures included in this lesson were created with GeoGebra.

Activity/Lesson:

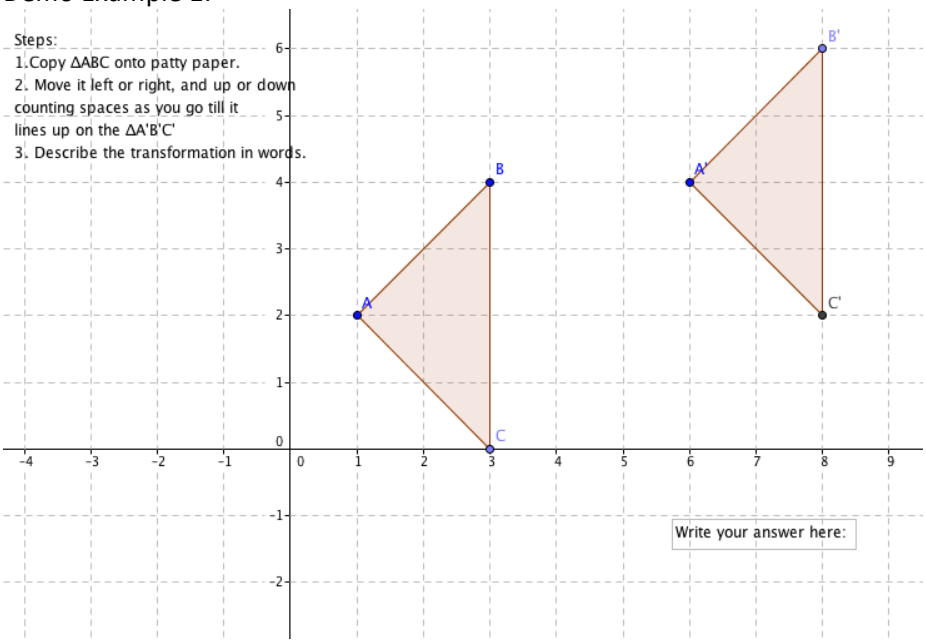
Day 1

Important Note: Most students will see that the triangles are congruent in all these examples. Even if they can see and describe without tracing, it is valuable to have them physically do the examples. The key is getting them to slow down, and use the mathematical practice of attending to precision particularly in their reasoning. The precise descriptions indicating how a figure is transformed from one location to another, is essentially the proof of why they are congruent. You can tell students, "Yes, they are congruent, but what if someone doesn't really believe you or see it for themself? When we use the patty paper to show that they map onto each other perfectly, that is the proof."

Introduction:

Show students the first example under the document camera. Demonstrate copying ΔABC by placing patty paper on top and tracing. Then, slowly move the patty paper first to the right 5 units, and then up 2 units, until it aligns with the image $\Delta A'B'C'$. Do it again.

Demo Example 1:



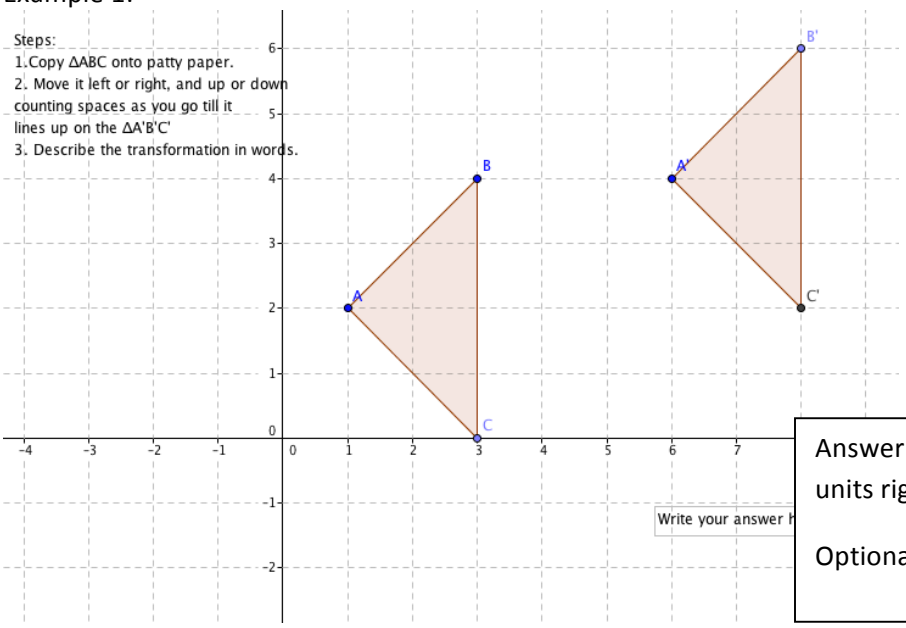
Have students open their notes. Write the notes with the students.

Title: Showing Congruence through Transformations Definition: Congruent - same size and shape - figures that can be carried onto each other through a series of rigid motions transformations; translations, reflections, and rotations

"First we are going to do translations."

Pass out patty paper to all the students and the first two examples. Guide students through example 1, even though you just showed them. Encourage them to repeat the process a few times. This should move fairly quickly.

Example 1:

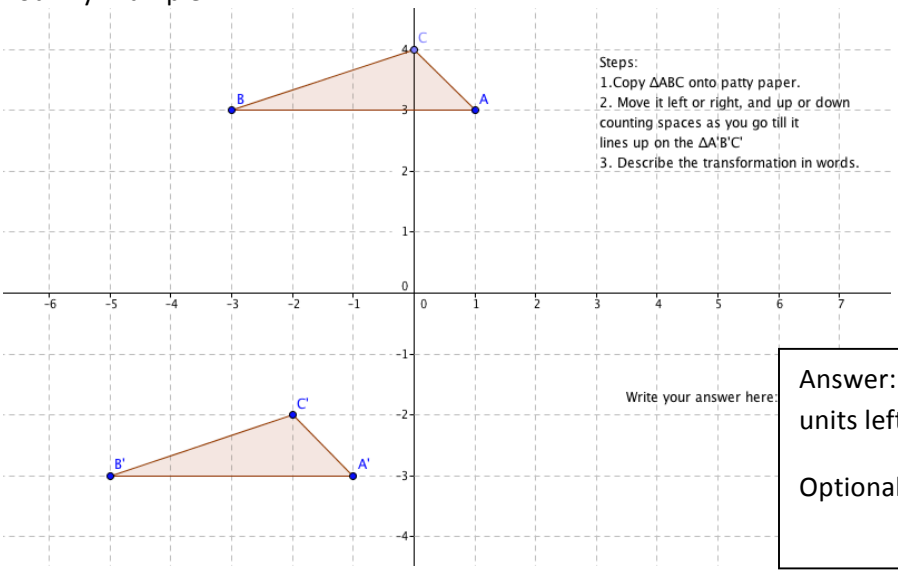


Answer: The triangle was translated 5 units right and 2 units up.

Optional Extension: $(x, y) \rightarrow (x + 5, y + 2)$

On the paper, write the answer together. Then, give students two minutes for example 2 as a you-try.

You-Try Example 2:



Answer: The triangle was translated 2 units left and 6 units down.

Optional Extension: $(x, y) \rightarrow (x - 2, y + 6)$

Quickly solicit answers from students, then, write the answer.

"How could I get the triangle to move to the left and up?"

"Translation notation shows the horizontal shift and the vertical shift with coordinates."

Translation- a transformation that slides a figure from one position to another

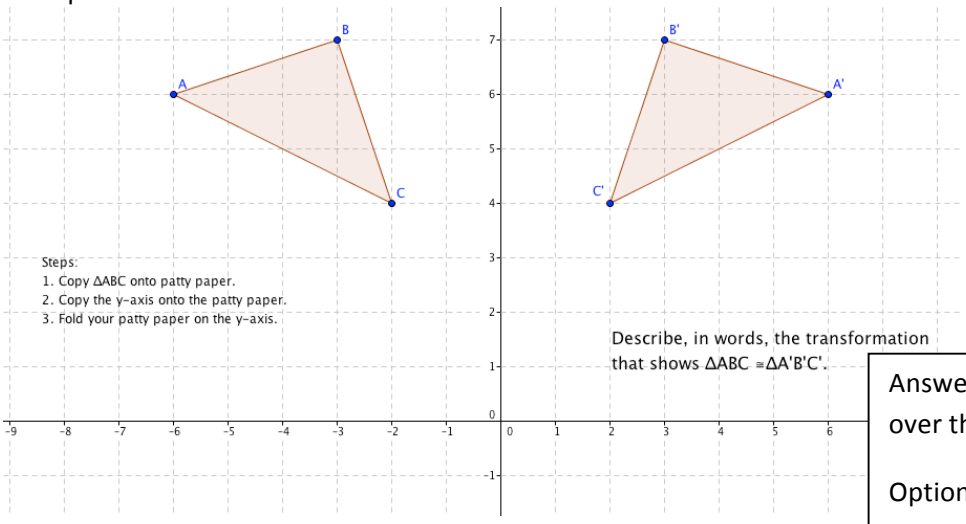
Math Notation: $(x, y) \rightarrow (x + a, y + b)$
a is the horizontal change left (-) or right (+), b is the vertical change up (+) or down (-).

Note: You may have students cut and paste the first two examples here.

"Another type of transformation also holds congruence. It is called reflection."

Pass out examples 3-5. Do example 3 together. Copy ΔABC by placing patty paper on top and tracing it. Without shifting the paper, copy the y-axis. Circulate to make sure students are doing this correctly. Together fold the paper on the y-axis.

Example 3:



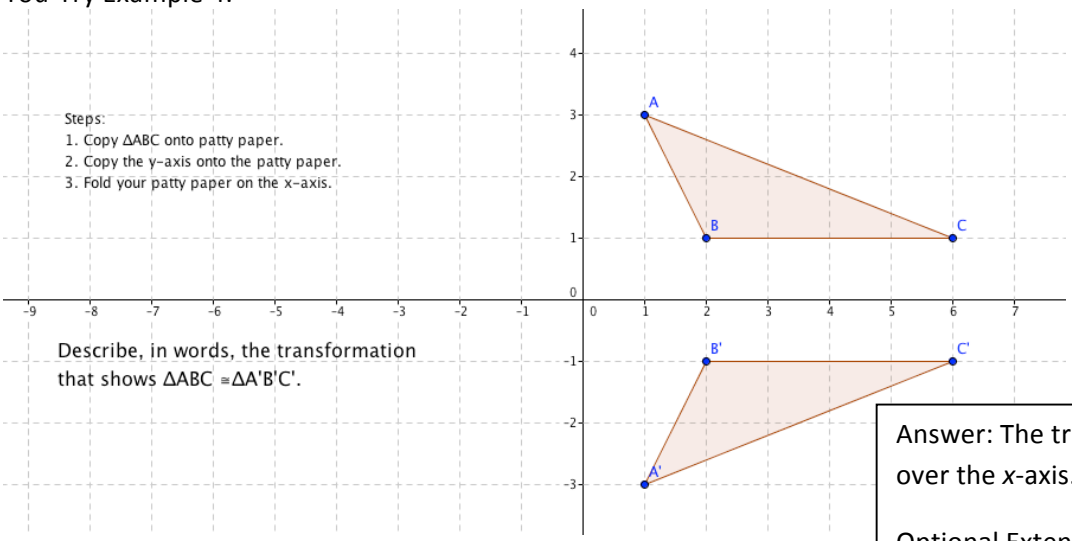
Answer: The triangle was reflected over the *y*-axis.

Optional Extension: $(x, y) \rightarrow (-x, y)$

Solicit answers from students, then, write the answer. You may have an optional discussion about the pre-image and image coordinates.

"What are we going to do differently in Example 4? You-try."

You-Try Example 4:



Answer: The triangle was reflected over the *x*-axis.

Optional Extension: $(x, y) \rightarrow (x, -y)$

Solicit answers from students, then, write the answer. Then, transition back to notes.

Reflection- a transformation where a figure is flipped over a line

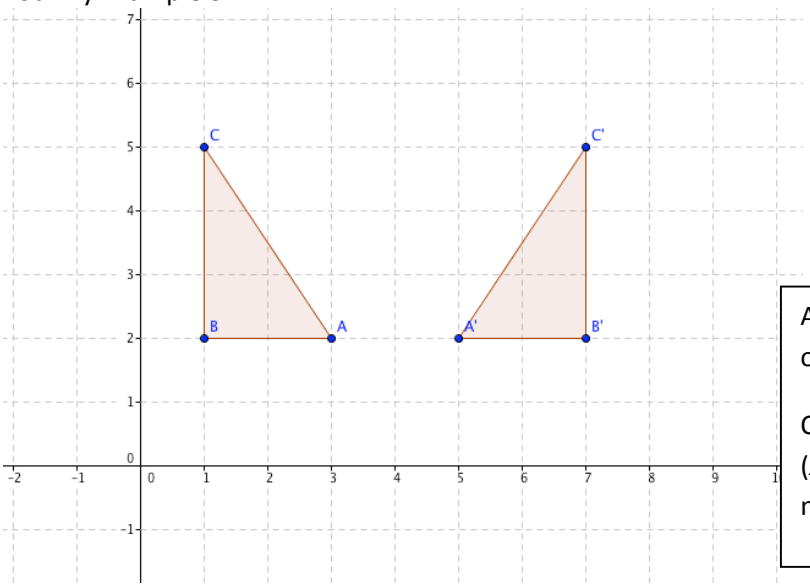
Math Notation: Reflection over the y-axis: $(x, y) \rightarrow (-x, y)$

Reflection over the x-axis: $(x, y) \rightarrow (x, -y)$

"Now let's look at example 5. What's going on here? There are no steps on this one. Where would the instructions tell you to fold the patty paper? Talk to your neighbor."

Call on a few students, and clarify that it is the line x=4. Then direct them to do the you-try and write the description of the transformation.

You-Try Example 5:



Answer: The triangle was reflected over the line $x = 4$

Optional Extension discussion:
$(x, y) \rightarrow (?, y)$ hmm… we would need more math…

Write the description of the transformation.

Note: You may have students cut and paste the next 3 examples here.

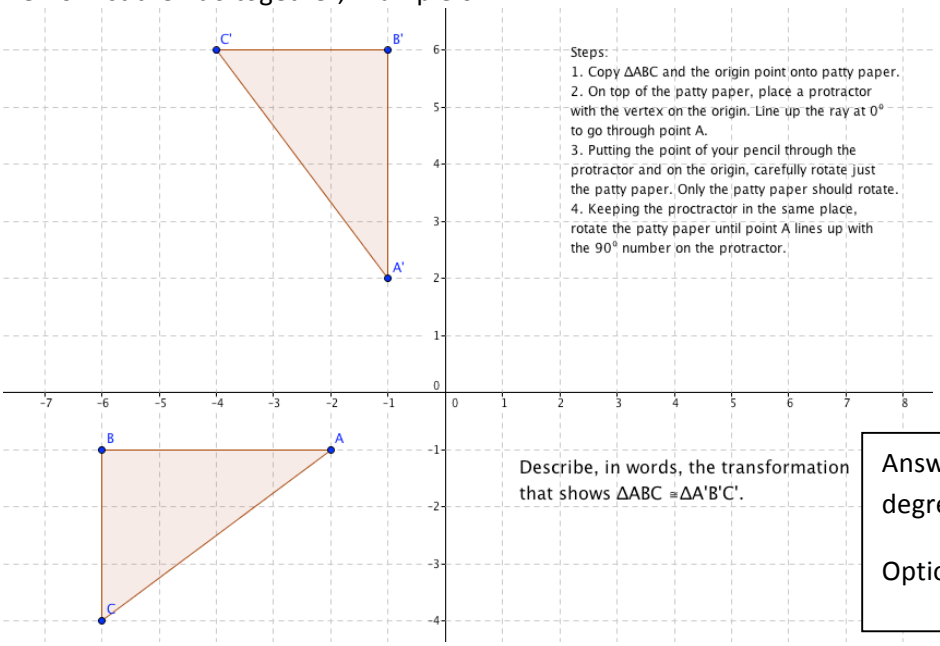
"There is one more transformation that can move the location of figures on the coordinate plane and keep congruence. If you know it, say it on the count of three. 1…2…3 _____." "Right, it's rotations."

Direct students to take notes.

Rotation- A transformation in which a figure is turned about a fixed point

Demonstrate example 6 before guiding students through. (These are definitely trickier and conceptually more confusing for students.) Have students watch. **Demonstrate copying ΔABC by placing patty paper on top and tracing. Then, trace the origin point. Next place the protractor on top of the patty paper, with its vertex on the origin and one ray through point A. Using the point of the pencil to hold the protractor in place, slowly rotate the patty paper 90 degrees clockwise until it aligns with the image $\Delta A'B'C'$. Do it again.** ***Then, pass out examples 6-8 and guide them through all three.***

Demo first then do together, Example 6:

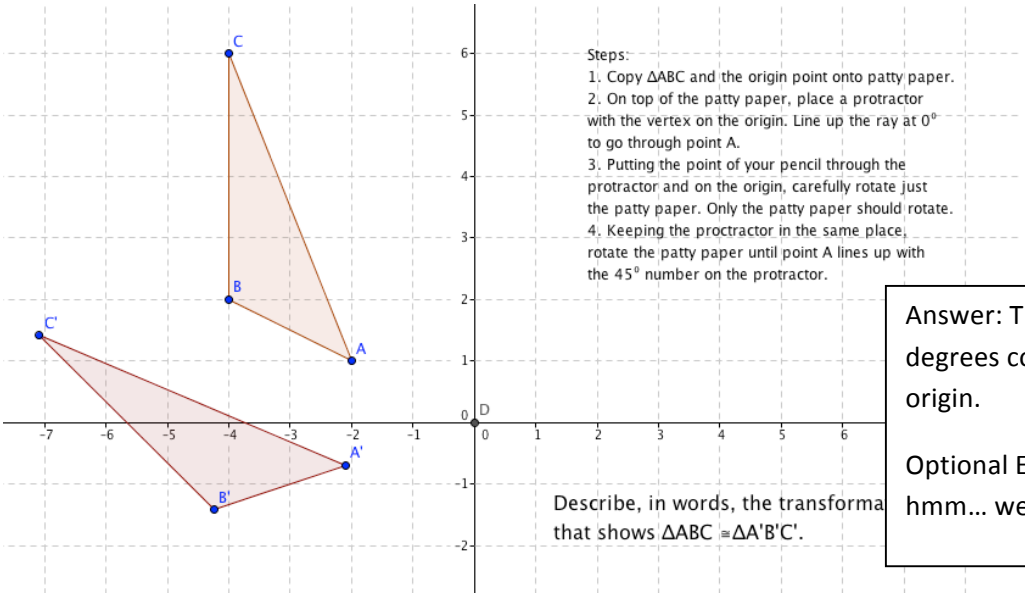


Answer: The triangle was rotated 90 degrees clockwise about the origin.

Optional Extension: $(x, y) \rightarrow (y, -x)$

Solicit answers from students, then, write the answer. You may have an optional discussion about the pre-image and image coordinates.

Example 7:

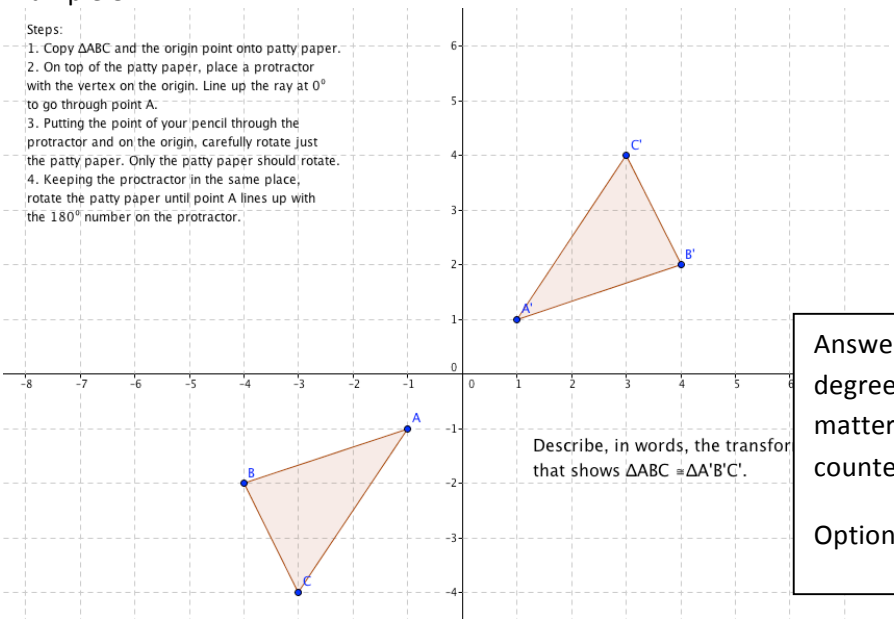


Answer: The triangle was rotated 45 degrees counter clockwise about the origin.

Optional Extension: $(x, y) \rightarrow (?,?)$
hmm… we would need more math

Solicit answers from students, then, write the answer. You may have an optional discussion about the pre-image and image coordinates.

Example 8:



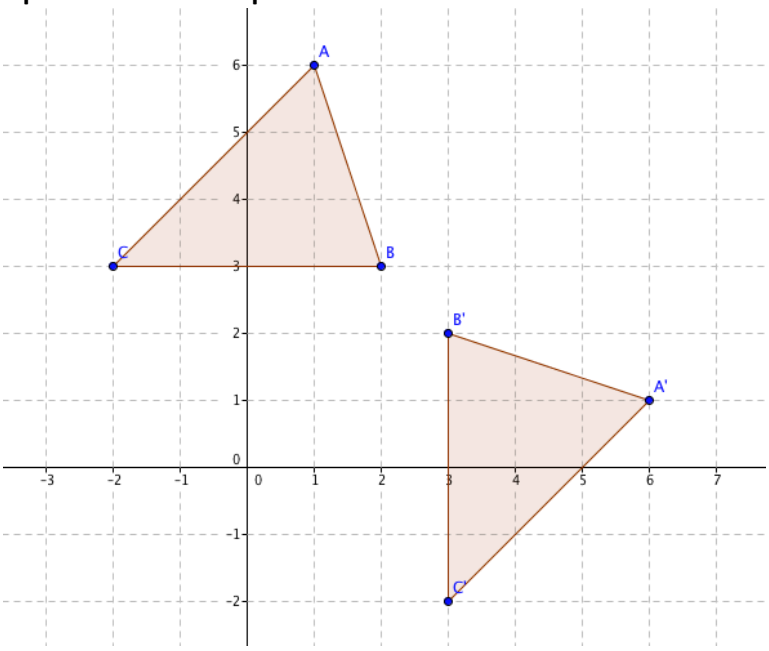
Answer: The triangle was rotated 180 degrees about the origin. It does not matter if it is rotated clockwise or counter clockwise.

Optional Extension: $(x, y) \rightarrow (-x, -y)$

Solicit answers from students, then, write the answer. You may have an optional discussion about the pre-image and image coordinates.

Note: You may have students cut and paste the next 3 examples here.

Optional Extra Example for Discussion: Reflection about the line *y* = *x*.



***An Exit Ticket and six additional Extra Examples for Practice or Homework and an answer key are included in the back.**

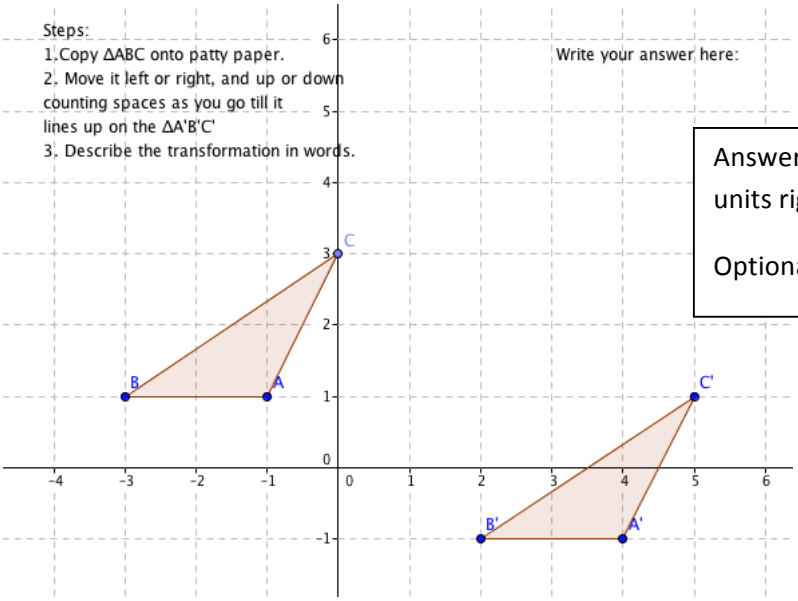
Answers to Assessment Day 1:

Exit ticket:

1. What are the three transformations that can be used to describe congruence?

Answer: Translations, Reflections, and Rotations

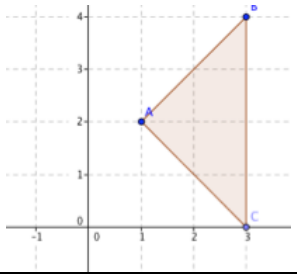
2. Describe, in words, the transformation that shows ΔABC is congruent to $\Delta A'B'C'$.



Answer: The triangle was translated 5 units right and 2 units down.

Optional Extension: $(x, y) \rightarrow (x + 5, y - 2)$

Answers to Warm-Up Day 1:

II. B	I. 
III. see student's drawings	IV. D

Activity/Lesson: Sequence of Transformations

Day 2

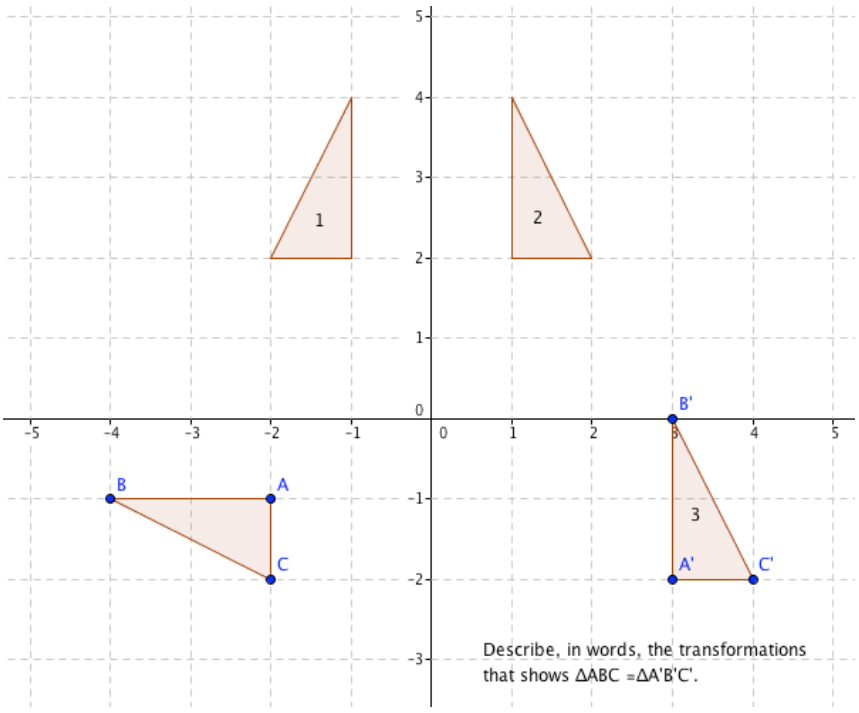
Introduction:

"Yesterday we used patty paper to recreate how one triangle was congruent to another through transformations. We practiced translations, reflections over the *x*-axis, *y*-axis, and over other lines, and rotations about the origin. Today we're going to put some transformations together to form a sequence that shows triangles are congruent."

Show students the first example under the document camera. Demonstrate copying ΔABC by placing patty paper on top and tracing. Then, randomly move the patty paper around until it aligns with the image $\Delta A'B'C'$. "See they are congruent." Then, do it again, but this time more slowly going first to the triangle with 1 in it, then 2, then 3. "Even though we know they are congruent, we need to describe why in words."

Pass out example 1-3 and patty paper. Guide students through the first sequence.

Demo first then do together Example 1:

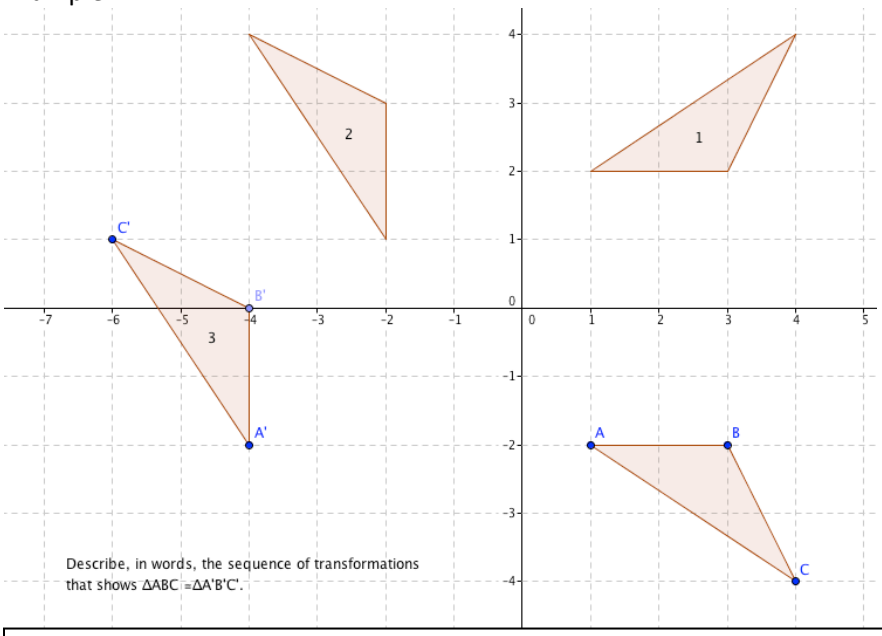


Answer: The triangle was rotated 90 degrees clockwise about the origin, reflected over the *y*-axis, and translated 2 units right and 4 units down.

Optional List the pre-image and image coordinate:
A (-2, -1) -> A' (3, -2); B (-4, -1) -> B' (3, 0); C (-2, -2) -> C' (4, -2)

Guide students through example 2. Then, solicit answers from the class, and write the answer.

Example 2:

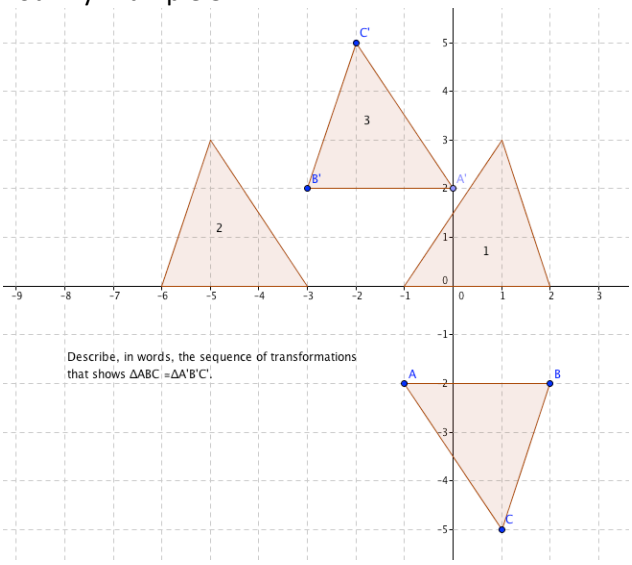


Answer: The triangle was reflected over the *x*-axis, rotated 90 degrees counter clockwise about the origin, and translated 2 units left and 3 units down.

Optional List the pre-image and image coordinate:
A (1, -2) -> A’ (-4, -2); B (3, -2) -> B’ (-4, 0); C (4, -4) -> C’ (-6, 1)

Direct students to do example 3 as a you-try. They may work together to check their answers. Then, solicit answers from the class, and write the answer.

You-Try Example 3:



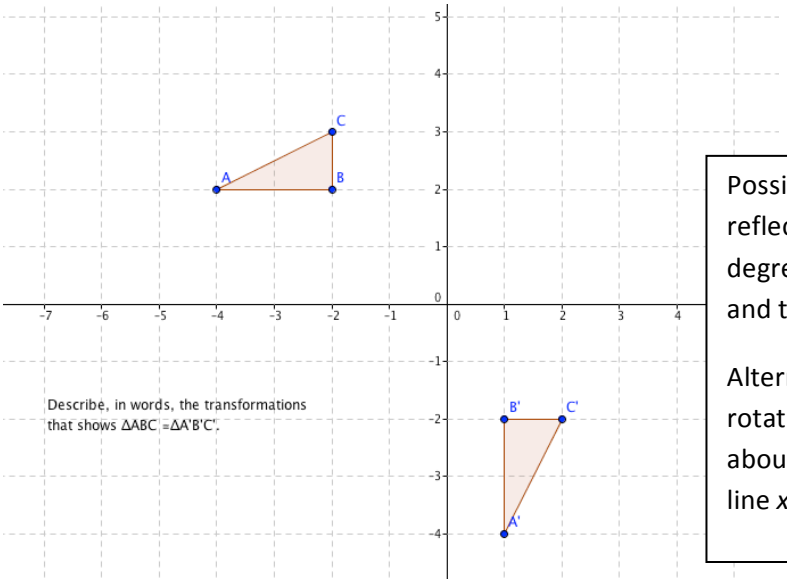
Answer: The triangle was reflected over the line *y* = -1, reflected over the line *x* = -2, and translated 3 units right and 2 units up.

Optional List the pre-image and image coordinate:
A (-1, -2) -> A’ (0, 2); B (2, -2) -> B’ (-3, 2); C (1, -5) -> C’ (-2, 5)

"Now, what if the steps are not shown?"

Have students watch you do example 4. Use patty paper to copy triangle ABC. Then slowly move the patty paper till it lines up on the image. "I ask myself, what am I doing with the patty paper? Do I have to flip it over? Do I have to shift it? Do I have to spin it? Can I get there a different way?" Repeat the process slowly a few times, so they can see. Then, pass it out and guide them through the process.

Example 4:

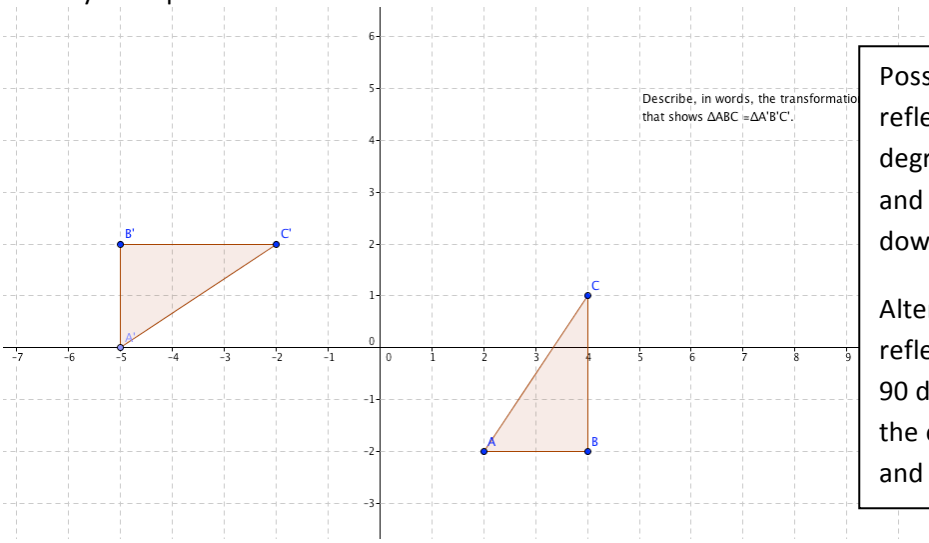


Possible Answer: The triangle was reflected over the *y*-axis, rotated 90 degrees clockwise about the origin, and translated 1 unit left.

Alternate answer: The triangle was rotated 90 degrees counterclockwise about the origin, reflected over the line $x = -1$, and translated 1 unit right.

Discuss the fact that there are many possible answers. However, the answer must include a rotation and a reflection. Write the answer.

You-Try Example 5:

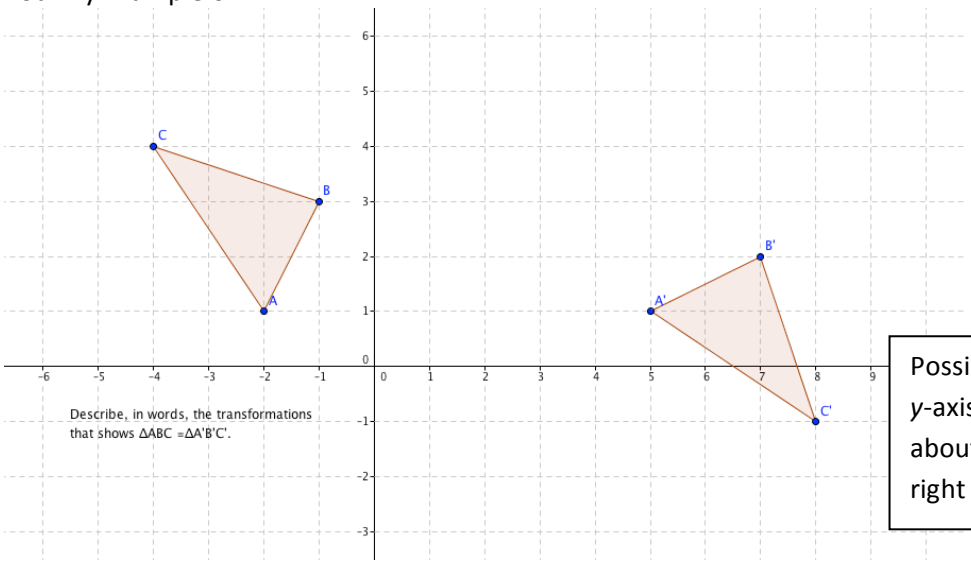


Possible Answer: The triangle was reflected over the *y*-axis, rotated 90 degrees clockwise about the origin, and translated 3 units left and 2 units down.

Alternate answer: The triangle was reflected over the line $y = 1$, rotated 90 degrees counterclockwise about the origin, and translated 1 unit left and 2 units down.

Again, discuss the fact that there are many possible answers. Ask a few students to describe their sequence. Have the class try each student's sequence to verify that it works. Write down at least one.

You-Try Example 6:



Possible Answer: Reflection over the *y*-axis, 90 degree rotation clockwise about the origin, translation 4 units right and 3 units up.

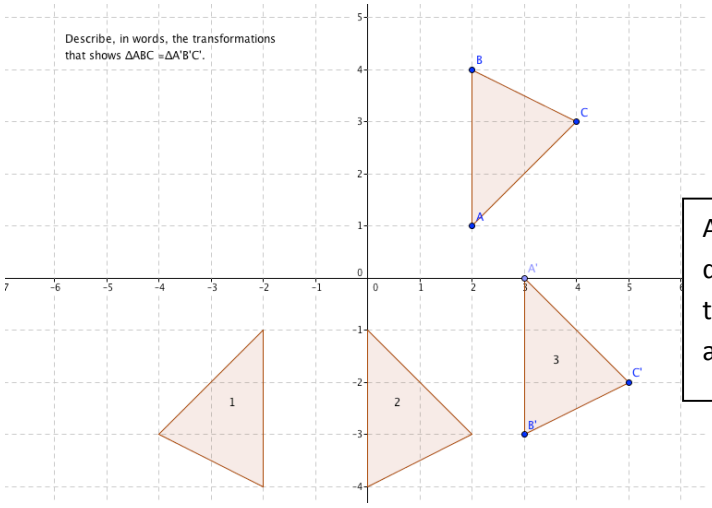
In partners have students explain their sequence to each other. Continue to discuss the fact that there are many possible answers. Ask a few students to describe their sequence. Write down at least one.

Summary: "If we are given two figures, we can show that they are congruent if we can describe a sequence of translations, rotations, and reflections that would carry a given figure onto another one."

***An Exit Ticket and four additional Extra Examples for Practice or Homework and an answer key are included in the back.**

Answers to Assessment Day 2:

Exit ticket:



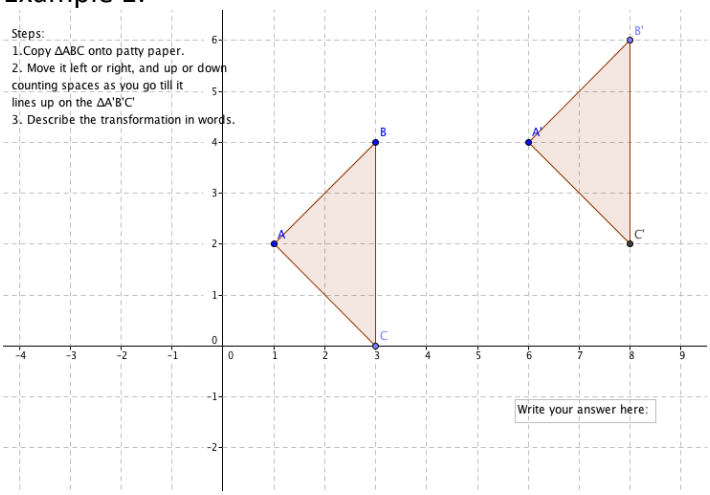
Answer: The triangle was rotated 180 degrees around the origin, reflected over the line $x = -1$, and translated 3 units right and 1 unit up.

Answers to Warm-Up Day 2:

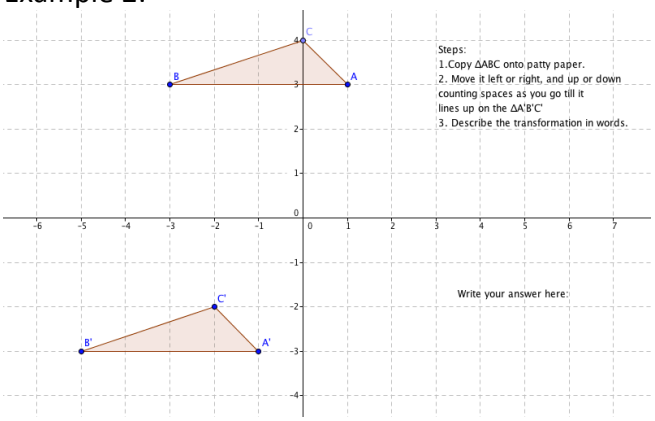
II. B	**I.** $x = -4$
III. Rotation 90 degrees clockwise around the origin.	**IV.** Reflection over the *y*- axis, rotation 90 degrees counter clockwise, and translation 2 units right.

Day 1 Examples Student Handouts

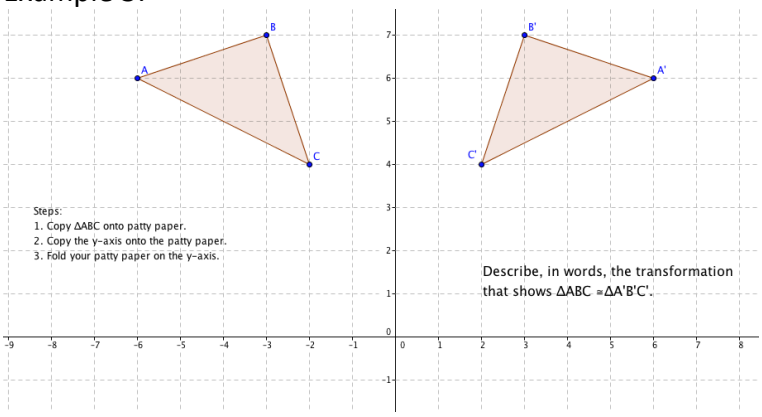
Example 1:



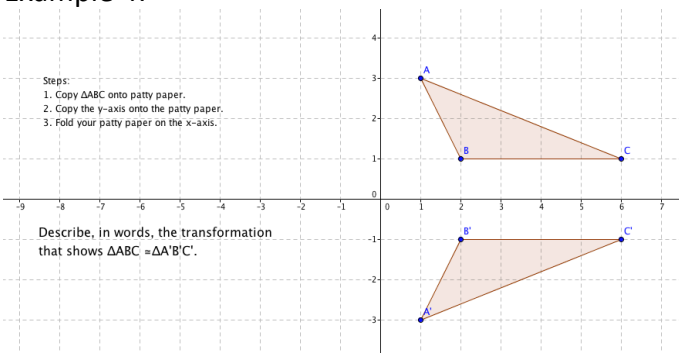
Example 2:



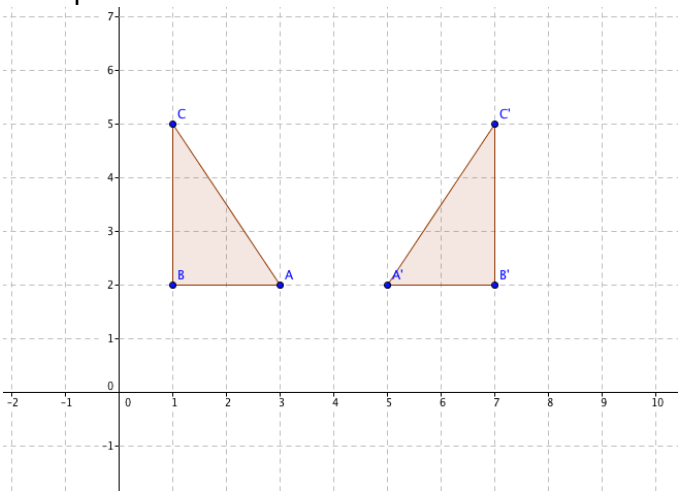
Example 3:



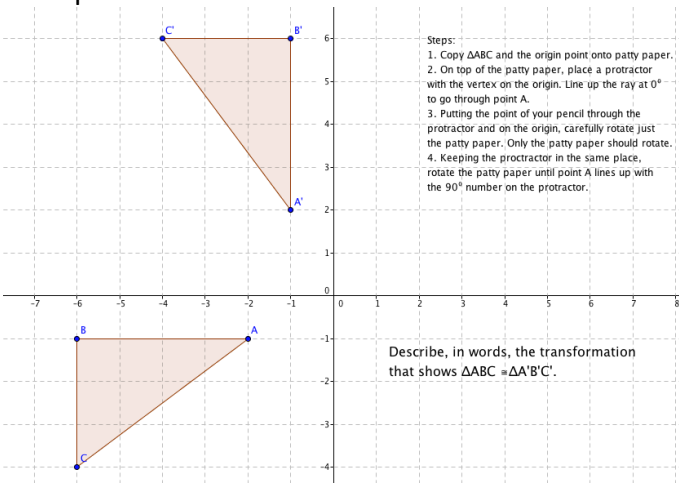
Example 4:



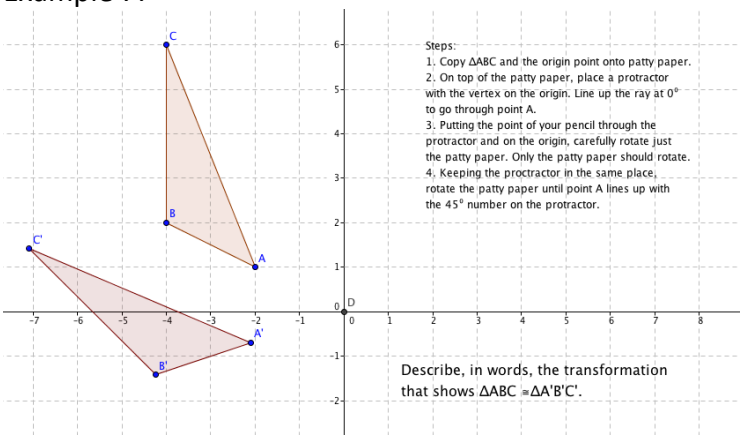
Example 5:



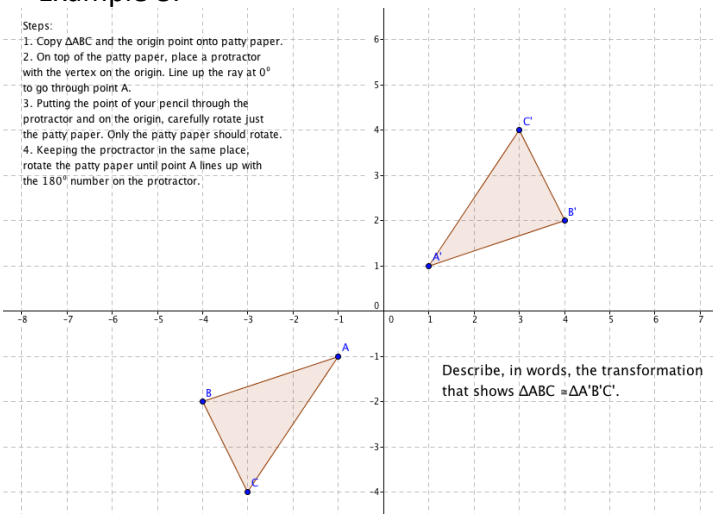
Example 6:



Example 7:



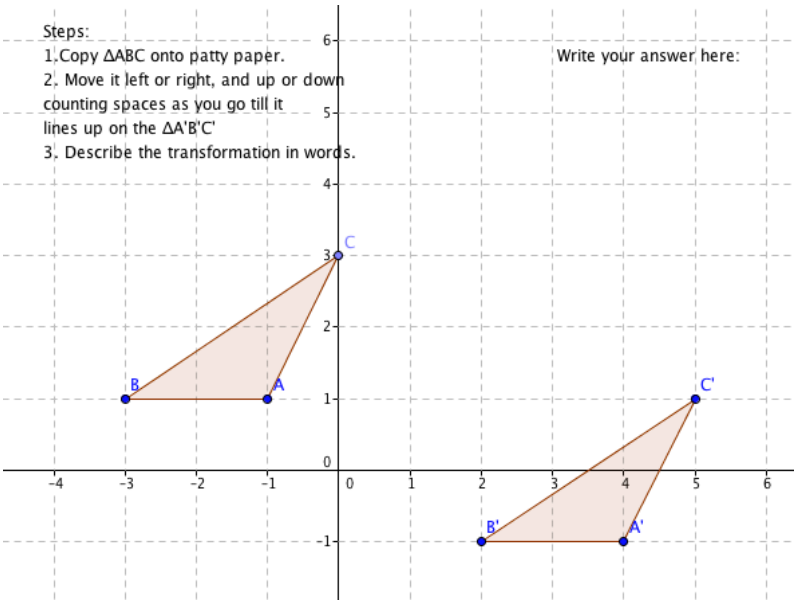
Example 8:



Exit Ticket Day 1:

1. What are the three transformations that can be used to describe congruence?

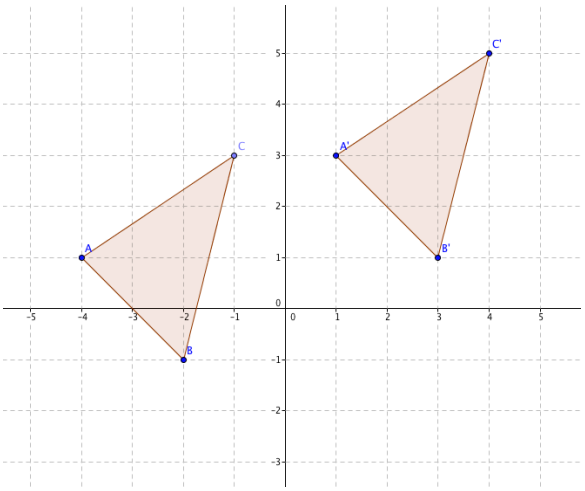
2. Describe, in words, the transformation that shows ΔABC is congruent to $\Delta A'B'C'$.



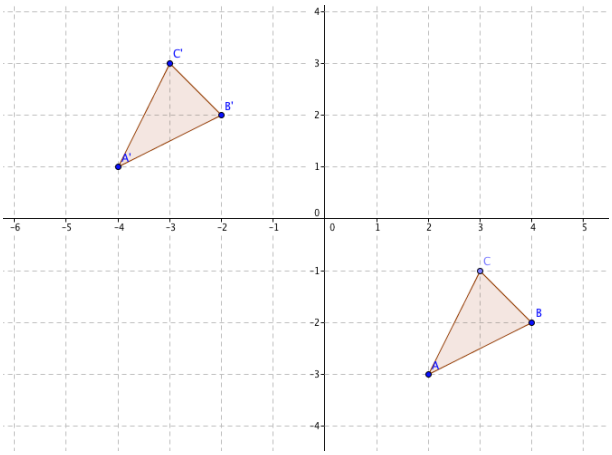
Day 1 Extra Examples for Practice or Homework:

Directions: Describe, in words, the transformation that shows ΔABC is congruent to $\Delta A'B'C'$.

1.



2.



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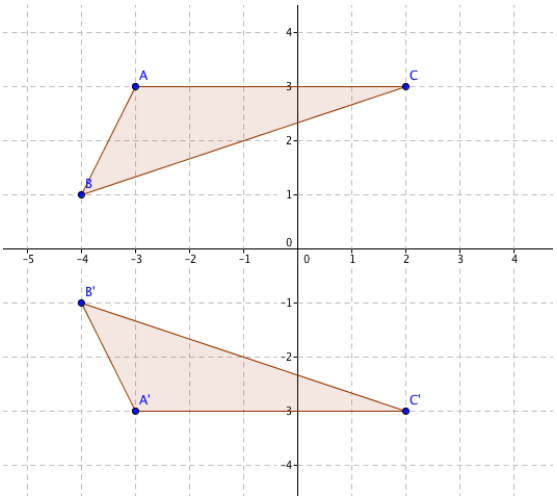

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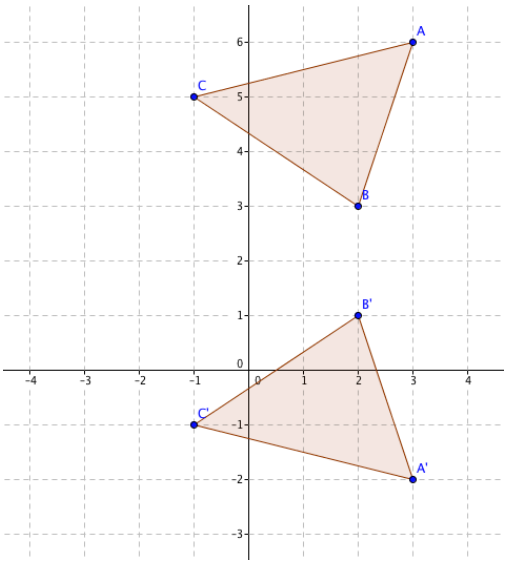

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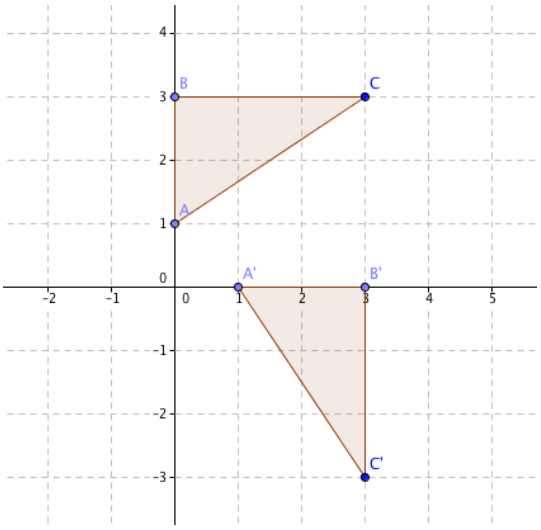

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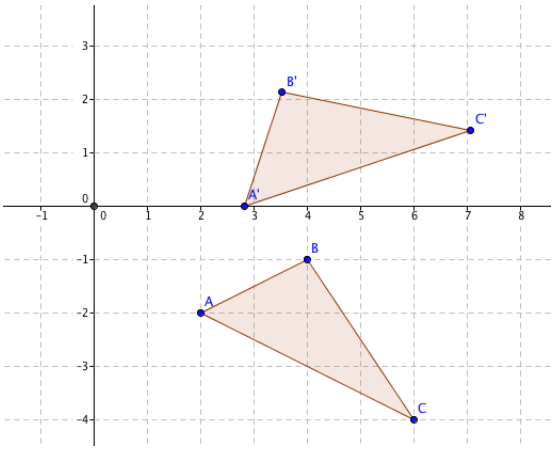
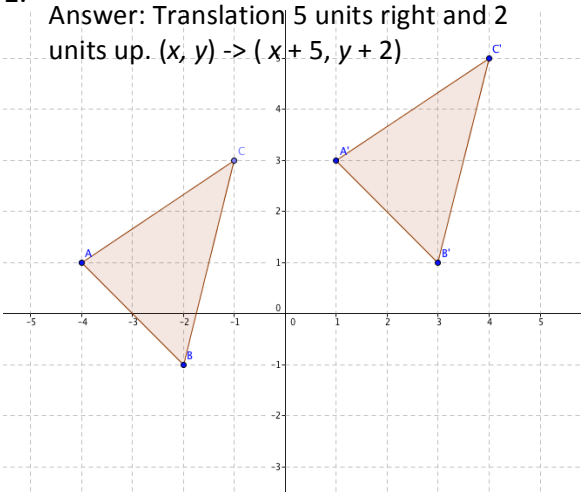


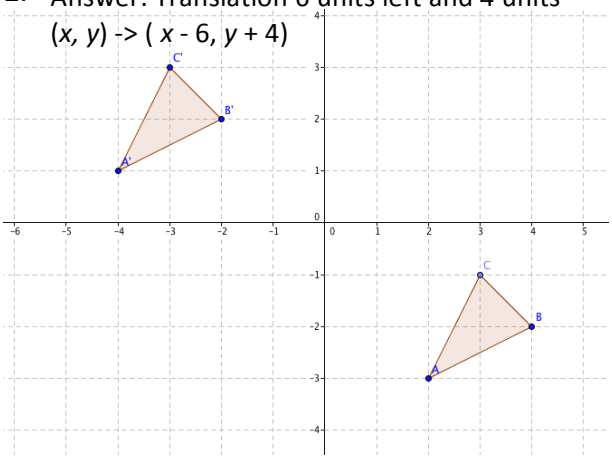
ANSWER KEY Day 1 Extra Examples for Practice or Homework:

Directions: Describe, in words, the transformation that shows ΔABC is congruent to $\Delta A'B'C'$.

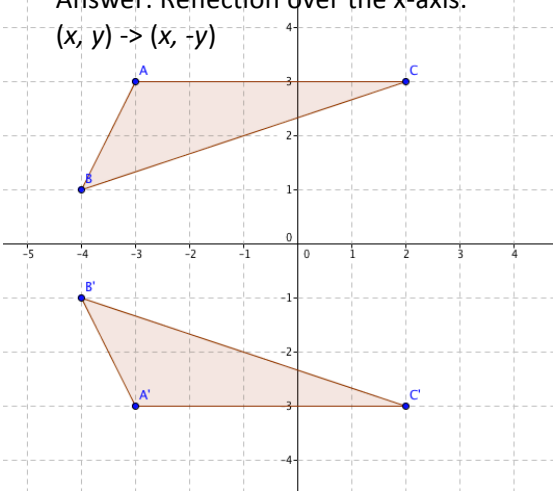
1. Answer: Translation 5 units right and 2 units up. (*x*, *y*) -> (*x* + 5, *y* + 2)



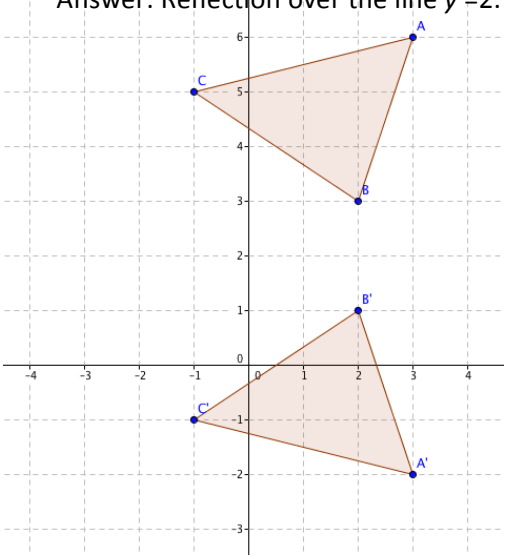
2. Answer: Translation 6 units left and 4 units (*x*, *y*) -> (*x* - 6, *y* + 4)



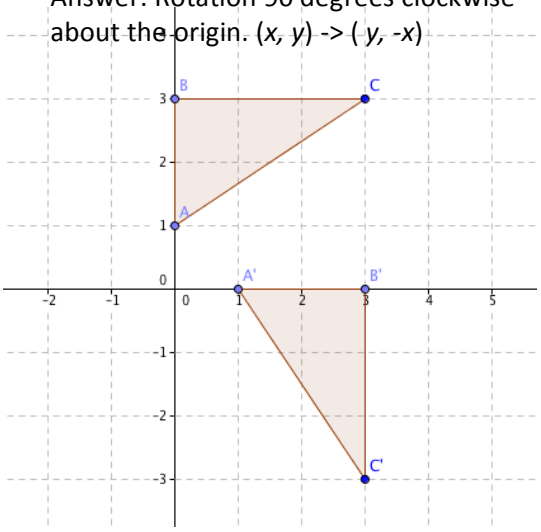
3. Answer: Reflection over the x-axis. (*x*, *y*) -> (*x*, -*y*)



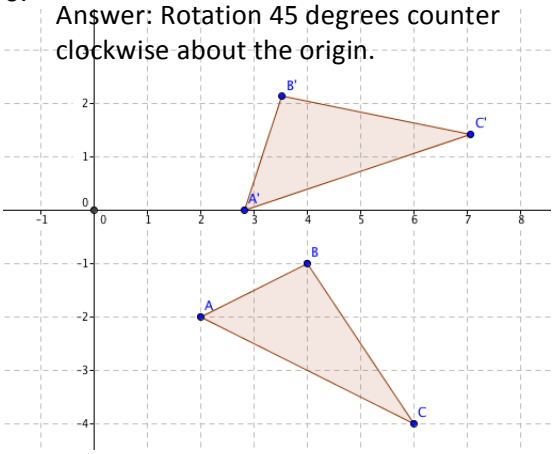
4. Answer: Reflection over the line *y* =2.



5. Answer: Rotation 90 degrees clockwise about the origin. (*x*, *y*) -> (*y*, -*x*)

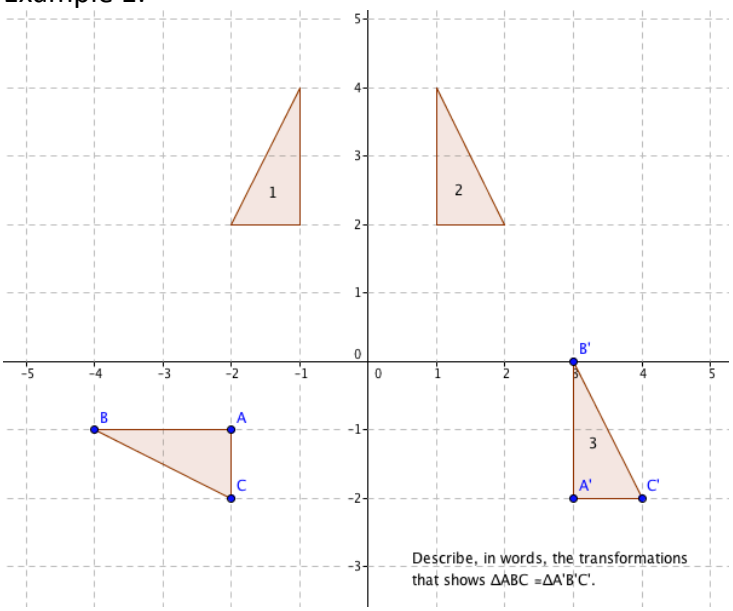


6. Answer: Rotation 45 degrees counter clockwise about the origin.

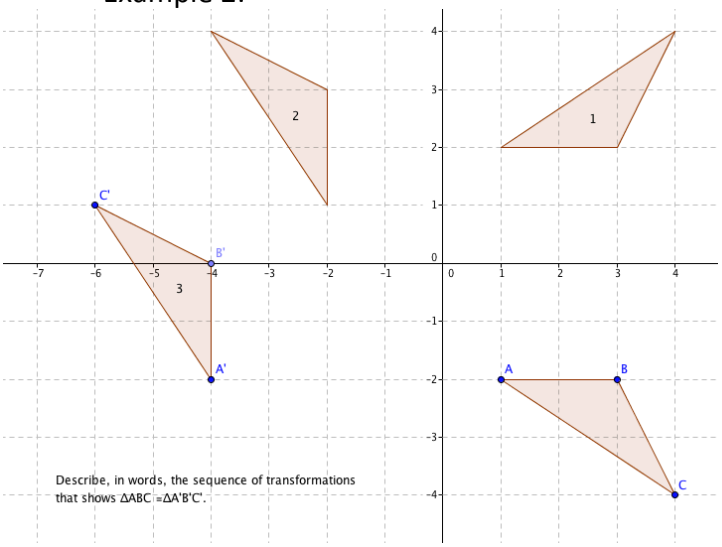


Day 2 Examples Student Handouts

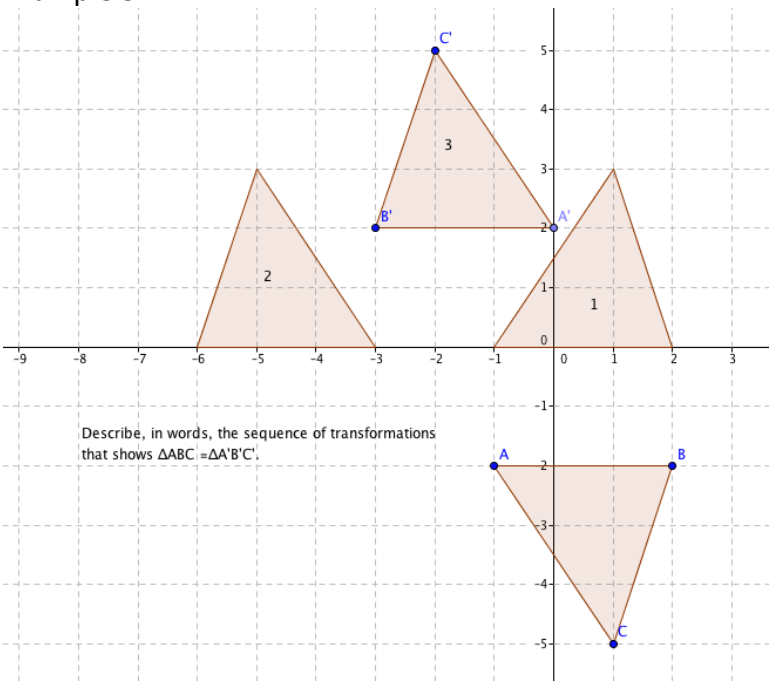
Example 1:



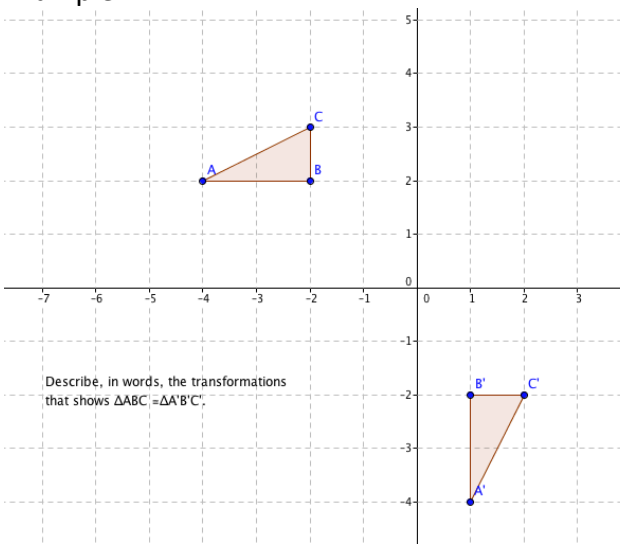
Example 2:



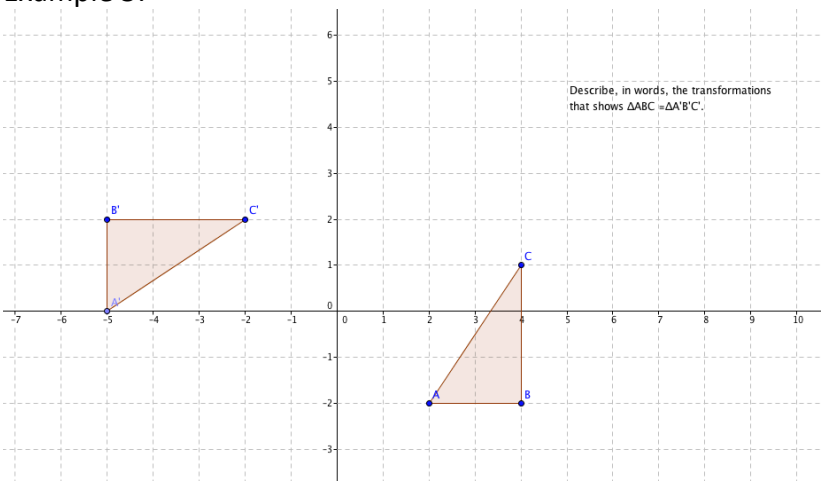
Example 3:



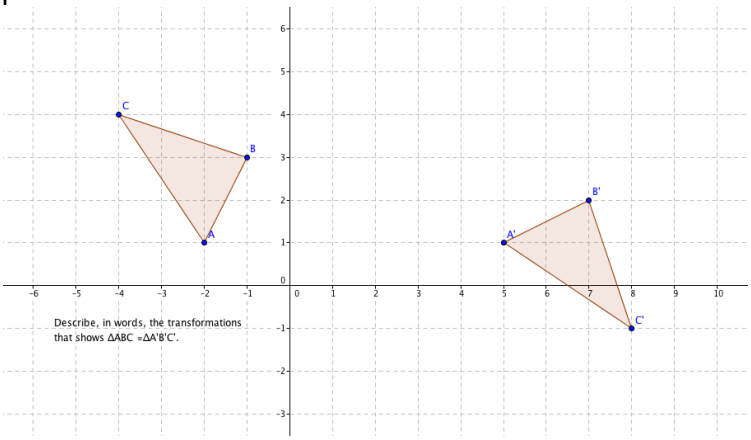
Example 4:



Example 5:

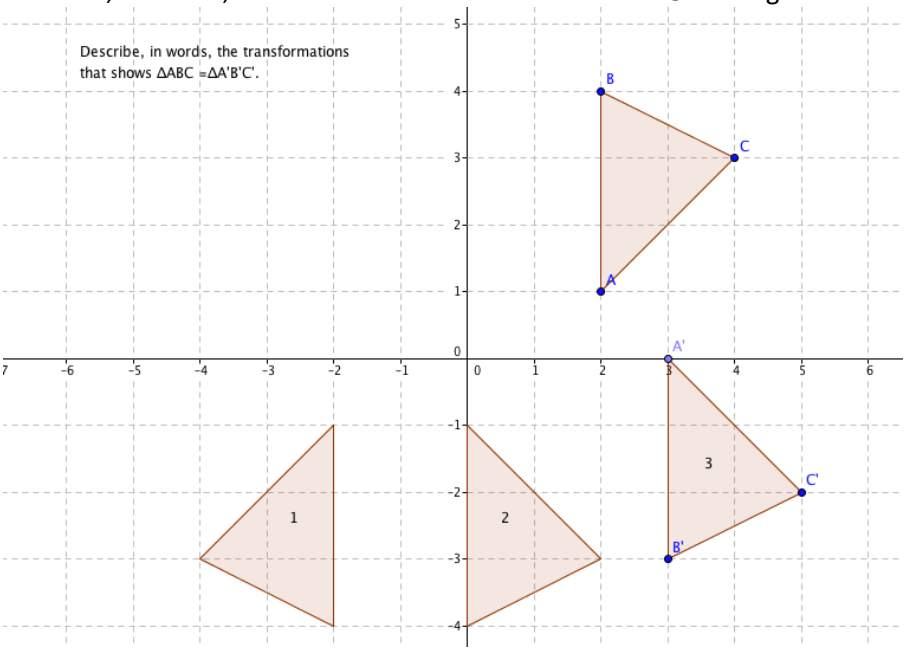


Example 6:



Exit Ticket Day 2:

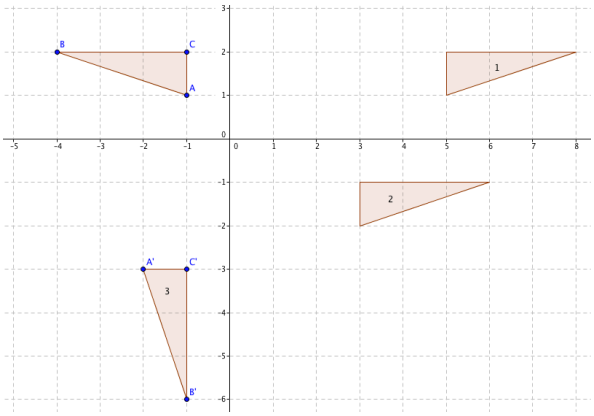
Describe, in words, the transformation that shows ΔABC is congruent to $\Delta A'B'C'$.



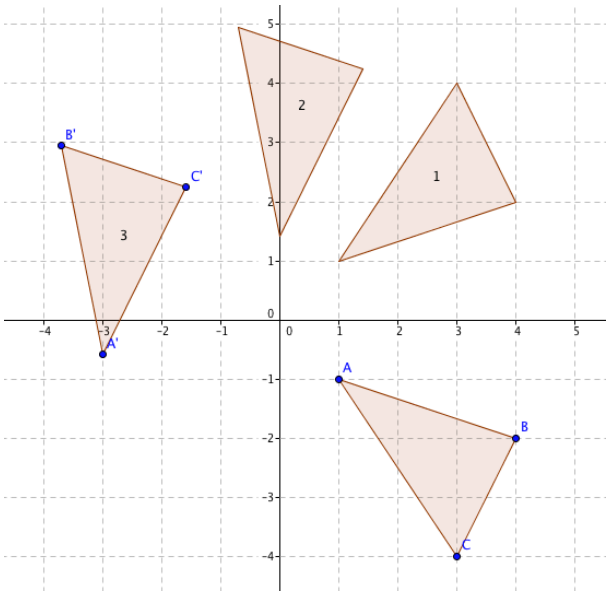
Day 2 Extra Examples for Practice or Homework:

Directions: Describe, in words, the transformation that shows ΔABC is congruent to $\Delta A'B'C'$.

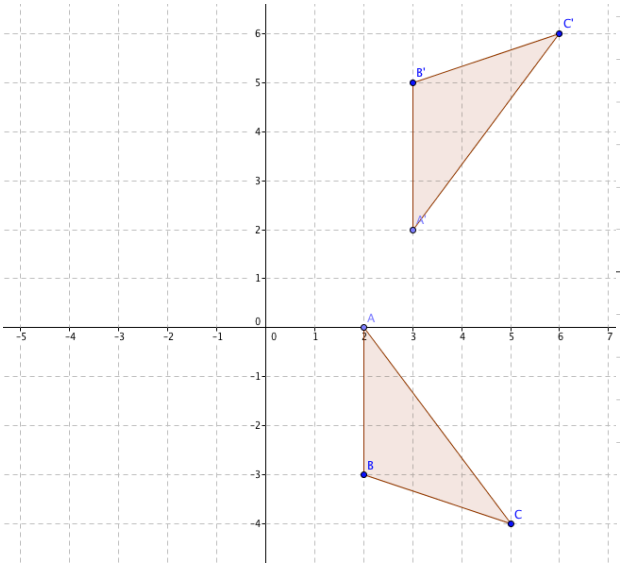
1.



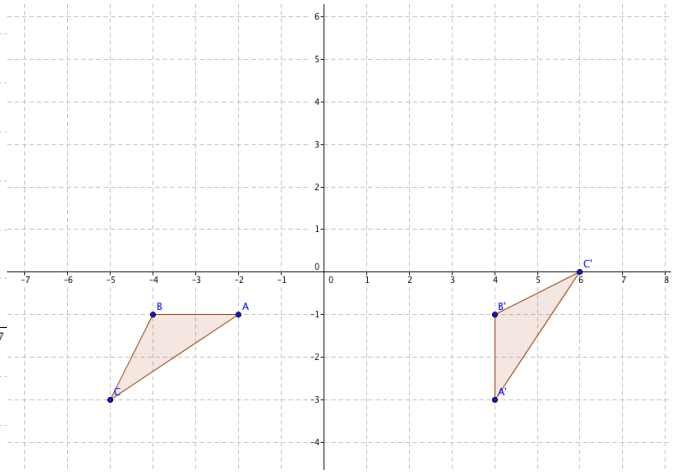
2.



3.



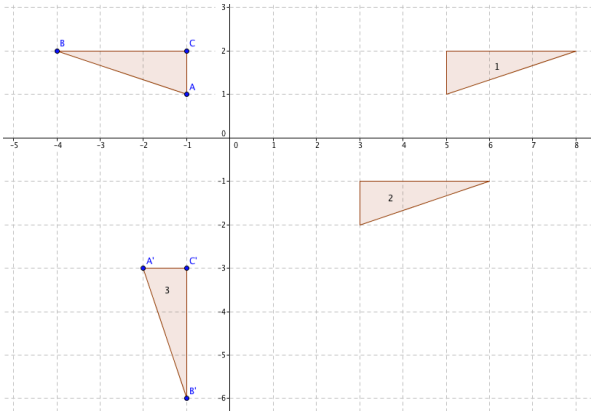
4.



ANSWER KEY Day 2 Extra Examples for Practice or Homework:

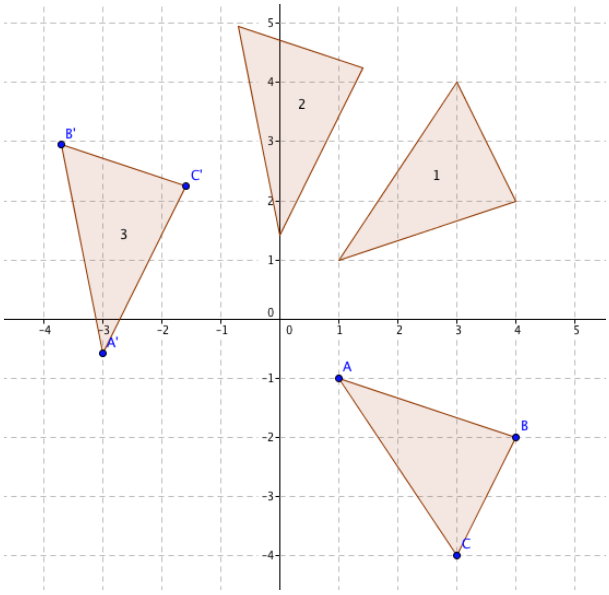
Directions: Describe, in words, the transformation that shows ΔABC is congruent to $\Delta A'B'C'$.

1.



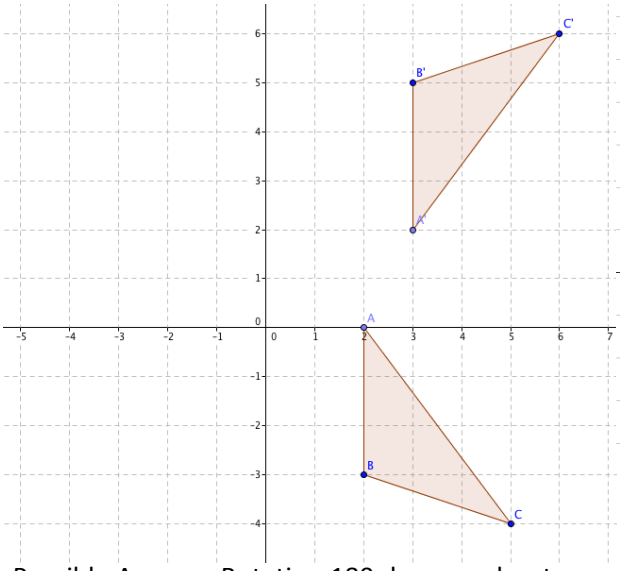
Answer: Reflection across the line $x = 2$, translation 2 units left and 2 units down, rotation 90 degrees clockwise about the origin.

2.



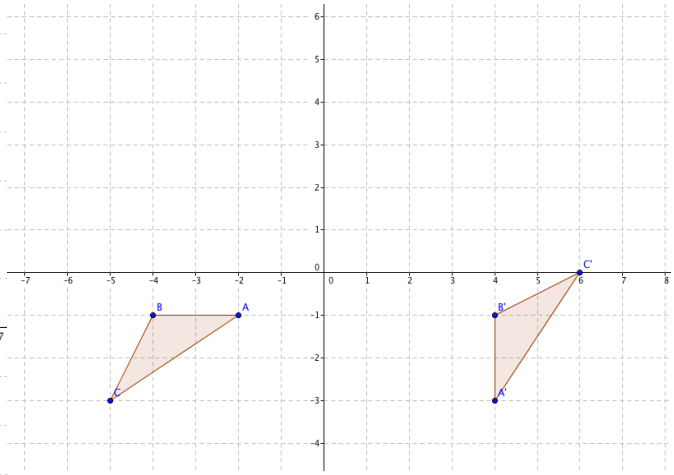
Answer: Reflection over the *x*-axis, rotation 45 degrees counter clockwise about the origin, translation 3 units left and 2 units down.

3.



Possible Answer: Rotation 180 degrees about the origin, translation 3 units right and 2 units up, reflection over the line $x = 2$.

4.

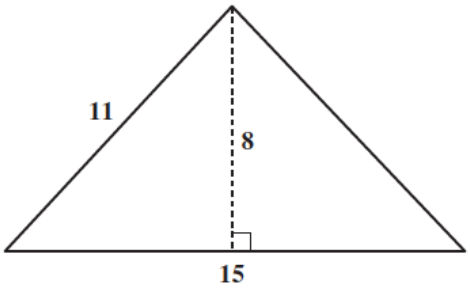


Possible Answer: Reflection over the *x*-axis, rotation 90 degrees clockwise about the origin, translation 3 units right and 5 units down.

Warm-Up Day 1

CAHSEE:

II.



What is the area of the triangle shown above?

A 44 square units

B 60 square units

C 88 square units

D 120 square units

Review: 5.G.1

I. Draw a coordinate plane, graph, and label the points *A*(1, 2); *B*(2, 4); *C*(2, 0).

y

x

Current: 8.G.2

III. Draw a picture of a mountain being reflected in a lake.

SBAC: 8.G.1

IV. Segment *FG* begins at point *F*(-2, 4) and ends at point *G*(-2, -3). The segment is translated <*x* - 3, *y* +2 > then reflected across the *y*-axis to form *F'G'*.

How many units long is segment *F'G'*?

Ⓐ 0

Ⓑ 2

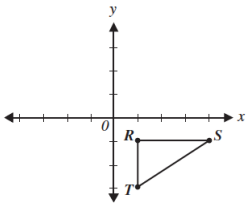
Ⓒ 3

Ⓓ 7

Warm-Up Day 2

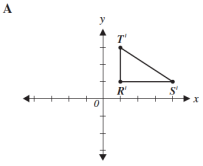
CAHSEE:

II.

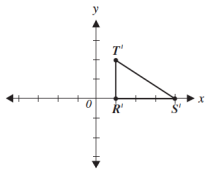


Which of the following triangles R'S'T' is the image of triangle RST that results from reflecting triangle RST across the y-axis?

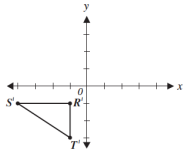
A



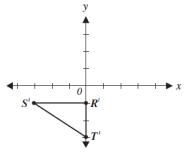
C



B



D



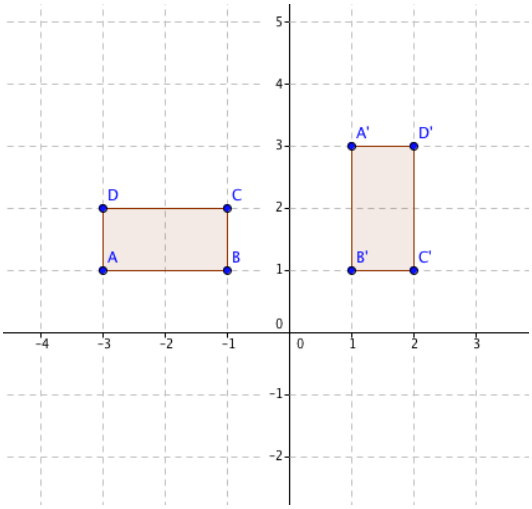
Review: 8.EE.7

I. Solve and check:

$3(x - 4) = 5x - 4$

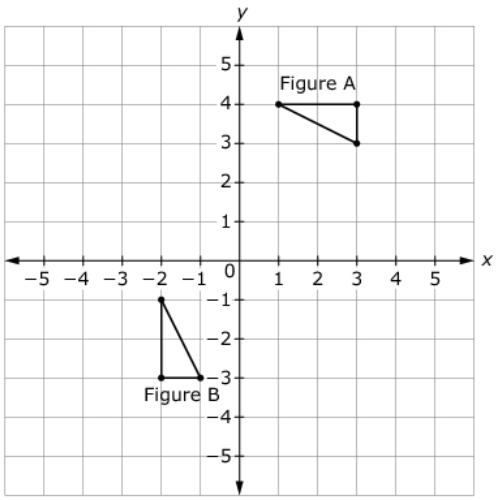
Current:

III. Describe in words why the figure ABCD is congruent to the figure A'B'C'D'.



SBAC:

IV. Two figures are shown on the coordinate grid.



Show that Figure A and Figure B are congruent by describing a sequence of basic transformations that maps Figure A onto Figure B. In your response, be sure to identify the transformations in the order they are performed.