Warm-Up

CST/CAHSEE: Algebra

What is the solution for this equation?

$$|2x - 3| = 5$$

A. $x = -4$ or $x = 4$
B. $x = -4$ or $x = 3$
C. $x = -1$ or $x = 4$
D. $x = -1$ or $x = 3$

Review: Algebra

What is the solution for this equation?

$$|x + 10| = 12$$

- Show two ways to solve this equation.

Current: Algebra

Absolute Value is defined as:

A. A number that is always positive.
B. A number that is always negative.
C. A number’s distance from zero.
D. A number’s value no matter what.

Other: Algebra

What is the solution of the system graphed below?

A. $(-2, 3)$
B. $(2, 3)$
C. $(3, 2)$
D. No Solution

- Write the equation of each line.
- Which answer choices could you eliminate immediately and why?

- True or False: An absolute value equation always has two solutions. Explain or provide an example.
### Warm-Up Solutions

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<td>[</td>
<td>2x - 3</td>
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<td>2x - 3 = 5</td>
<td>x + 10 = 12</td>
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<td>2x - 3 + 3 = 5 + 3</td>
<td>x + 10 - 10 = 12 - 10</td>
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<td>2x = 8</td>
<td>x = 2</td>
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<td>[ \frac{2x}{2} = \frac{8}{2} ]</td>
<td>[ \frac{2x}{2} = \frac{-2}{2} ]</td>
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<tr>
<td>x = 4</td>
<td>x = -1</td>
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**Answer Choice C:** Absolute Value is defined as a number’s distance from zero.

**False:** An absolute value equation has only one answer when the absolute value expression is equal to zero.

Example: \[ |3x - 12| = 0 \]

The only value of \( x \) that satisfies this equation is \( x = 4 \).

Similarly, an absolute value expression that is equal to a negative number has no solution. A number’s distance form zero cannot be negative.

Example: \[ |3x - 12| = -5 \]

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**Answer Choice C:** The solution of the system is the point of intersection of the two lines. That point is \( (3, 2) \).

We may immediately eliminate answer choices A and D. The ordered pair listed in Choice A is in Quadrant II and our point of intersection is clearly in Quadrant I. Answer Choice D states that there is no solution, which would indicate a system of parallel lines. Our lines are clearly intersecting.

Equations:

\[ y = 3 \]
\[ y = 2x + 8 \]
All About Absolute Value Functions

Graphing Absolute Value Functions:

Graph \( y = |x| \). We will call this the “mother function” for absolute value functions.

Graph each function. Explain how it compares to the mother function.

1) \( y = |x| + 3 \)

The graph is translated up 3 units.

2) \( y = |x| - 2 \)

The graph is translated down 2 units.

3) \( y = |x + 4| \)

The graph is translated left 4 units.

4) \( y = |x - 1| \)

The graph is translated right 1 unit.
5) \( y = |x + 3| - 4 \)

The graph is translated left 3 units and down 4 units.

Summarize the function behavior you observed from the previous examples:

- \( y = |x + a| \) translates left \( a \) units
- \( y = |x - a| \) translates right \( a \) units
- \( y = |x| + b \) translates up \( b \) units
- \( y = |x| - b \) translates down \( b \) units

Explain how the graph for each of the following functions compares to the mother function. Do this first without graphing, then graph to check your answer.

6) \( y = |x + 2| \)
   The graph is translated left 2 units.

7) \( y = |x| - 4 \)
   The graph is translated down 4 units.

8) \( y = |x - 5| - 2 \)
   The graph is translated right 5 units and down 2 units.

NOTE: Stop here and debrief warm-up, CST and review items.
Solving Absolute Value Equations in One Variable:
Any equation in one variable can be rewritten as a system of two equations in two variables. We will explore solving absolute value equations by rewriting them as systems of equations.

Examples:

9) Solve $|x| = 3$

Rewrite as a system of equations:

$y = |x|$

$y = 3$

Graph to find the solutions for the system of equations:

The points of intersection are $(3, 3)$ and $(-3, 3)$.
The solutions to $|x| = 3$ are the $x$ coordinates of the points of intersection.
Solutions: $x = -3, \ x = 3$

10) Solve $|x + 2| = 5$

Rewrite as a system of equations:

$y = |x + 2|$

$y = 5$

Graph to find the solutions for the system of equations:

The points of intersection are $(3, 5)$ and $(-7, 5)$.
The solutions to $|x + 2| = 5$ are the $x$ coordinates of the points of intersection.
Solutions: $x = -7, \ x = 3$
11) **You Try:**

Solve \(|x - 3| = 2\)

Rewrite as a system of equations:

\[
\begin{align*}
y &= |x - 3| \\
y &= 2
\end{align*}
\]

Graph to find the solutions for the system of equations:

The points of intersection are (5, 2) and (1, 2).

The solutions to \(|x - 3| = 2\) are the \(x\) coordinates of the points of intersection.

Solutions: \(x = 1\), \(x = 5\)

**This method can be extended to solve absolute value inequalities as well:**

12) Our graph from example 9 showed us the solutions for \(|x| = 3\). It also shows the solutions to \(|x| < 3\) and \(|x| > 3\)

Solutions for \(|x| = 3\)

Solutions for \(|x| < 3\)

Solutions for \(|x| > 3\)
13) Use the graph from example 10 to find the solutions for $|x+2| \leq 5$ and $|x+2| \geq 5$

![Graph showing solutions for $|x+2|=5$, $|x+2| \leq 5$, and $|x+2| \geq 5$.]

Solutions for $|x+2|=5$ → $x=-7$ or $x=3$

Solutions for $|x+2| \leq 5$ → $-7 \leq x \leq 3$

Solutions for $|x+2| \geq 5$ → $x \leq -7$ or $x \geq 3$

14) **You Try:** Use your graph from example 11 to solve $|x-3| \geq 2$.

![Graph showing solutions for $|x-3|=2$, $|x-3| \leq 2$, and $|x-3| \geq 2$.]

Solutions for $|x-3|=2$ → $x=1$ or $x=5$

Solutions for $|x-3| \leq 2$ → $1 \leq x \leq 5$

Solutions for $|x-3| \geq 2$ → $x \leq 1$ or $x \geq 5$

The solutions for $|x-3| \geq 2$ are $x \leq 1$ or $x \geq 5$. 
### Part 1: The Mother Function
Graph the function below. Describe the graph. Write a sentence explaining why it takes the shape it does.

\[ y = |x| \]

### Part 2: Related Functions
Graph each function. Explain how it compares to the graph of the mother function.

1. \[ y = |x| + 3 \]

2. \[ y = |x| - 2 \]
Summarize the function behavior you observed from the previous examples.

*y* = |*x* + *a*|: ______________________________

*y* = |*x* − *a*|: ______________________________

*y* = |*x* + *b*|: ______________________________

*y* = |*x* − *b*|: ______________________________
Part 3: Predictions
Predict how the graph for each of the following functions compares to the mother function. Do this first without graphing; use your transparency to show your prediction to a neighbor. Then, graph to check your answer.

6. \( y = x + 2 \)
   Prediction:

7. \( y = |x| - 4 \)
   Prediction:

8. \( y = |x - 5| - 2 \)
   Prediction:
Part 4: Solving Absolute Value Equations as a System of Equations
Any equation in one variable can be written as a system of equations in two variables.
Solve the equations by re-writing each as a system. Graph to solve. (Use transparencies to predict first!)

9. Solve: \(|x| = 3\)

10. Solve: \(|x + 2| = 5\)

11. Solve: \(|x - 3| = 2\)
Part 5: Extensions
This method may also be used to solve absolute value inequalities.

|   | Solve: $|x| < 3$ and $|x| > 3$ |
|---|--------------------------------|
| 12. | ![Graph of $|x| < 3$ and $|x| > 3$] |

|   | Solve: $|x + 2| \leq 5$ and $|x + 2| \geq 5$ |
|---|---------------------------------------------|
| 13. | ![Graph of $|x + 2| \leq 5$ and $|x + 2| \geq 5$] |

|   | Solve: $|x - 3| \leq 2$ and $|x - 3| \geq 2$ |
|---|---------------------------------------------|
| 14. | ![Graph of $|x - 3| \leq 2$ and $|x - 3| \geq 2$] |
**Backline Masters:** Photocopy onto transparencies and cut out. Each student should be given one of each.

| Mother Function: $y = |x|$ | Line: $y = x$ |
|---------------------------|-------------|
| ![Graph of y = |x|] | ![Graph of y = x] |
| ![Graph of y = |x|] | ![Graph of y = x] |
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