

Grades 6-8, Claim 4

**Example Item 4A.1a (Grade 6)**

Primary Target 4A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 4B, Quaternary Target 1A (CCSS 6.RP.A)

Juan has  $7\frac{3}{4}$  cups of nuts. He wants to make either banana nut muffins or carrot muffins. The table shows how many cups of nuts are needed for each batch.

**Amount of Nuts Needed Per Batch of Muffins**

Muffin Type	Amount of Nuts per Batch
Banana nut	$\frac{1}{2}$ cup
Carrot	$\frac{5}{8}$ cup

Juan decided to make only carrot muffins. What is the maximum number of whole batches of carrot muffins Juan can make with  $7\frac{3}{4}$  cups of nuts?

Enter your answer in the response box.

**Rubric:** (1 point) Student enters the correct number (12).

**Response Type:** Equation/Numeric

**Commentary:** The task could also ask about banana nut muffins, or about both for a 2-point item. A more cognitively demanding version of the task could ask how many whole batches can be made if he wants to make half banana nut and half carrot.

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**Example Item 4A.1a (Grade 6)**

Primary Target 4A (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 4B, Quaternary Target 4F

Hummingbirds drink nectar from flowers and sugar water from bird feeders.

- Sugar water is made by mixing 50 grams of sugar with 200 grams of water.
- A hummingbird's favorite flower nectar is 21% sugar by mass.

The amount of food a hummingbird eats at one time is always the same whether it eats sugar water or flower nectar.

**Part A**

Will the hummingbird get more sugar from a meal of sugar water made according to the recipe, or from an equal-sized meal of flower nectar? [Drop down choices: sugar water, flower nectar]

**Part B**

How much more sugar, in grams, would a hummingbird get from 4 grams of the [fills in with student's choice for the more sugary food type from part A] than from 4 grams of the [fills in with student's choice for the less sugary food type from part A]?

**Interaction:** Once the student selects the more sugary food type in part A, part B populates with the student's choice. The student can go back and change the choice in part A, in which case the statement of part B changes as well. Title the response box in Part B "Grams of sugar."

**Rubric:** (2 points) The student selects the more sugary food item (flower nectar) and identifies the additional amount of sugar correctly (0.04).

(1 point) The student identifies the food made by the recipe and enters the difference as 0.16, which corresponds to assuming the recipe is 25% sugar by weight (a likely mistake) but then correctly computing the difference.

**Response Type:** Drop Down Menu<sup>5</sup> and Equation/Numeric

**Note:** Functionality for this item type does not currently exist, although the item could be modified to work with current technology by making Part A a hot Spot (choose between "Recipe" and "Flower Nectar") and by wording Part B, "How much more sugar, in grams, would a hummingbird get from 4 grams of the option you chose in Part A than from 4 grams of the other option?"

<sup>5</sup> Drop-Down Menu response type is not currently available, but is a planned enhancement to the test-authoring tool by 2017.

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**Example Item 4A1.b (Grade 7)**

Primary Target 4A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 4B, Quaternary Target 4D  
[Adapted from Illustrative Mathematics task 50]

Alice, Raul, and Maria are baking cookies together.

They need  $\frac{3}{4}$  cup of flour and  $\frac{1}{3}$  cup of butter to make one batch of cookies.

They each brought the ingredients they had at home.

- Alice brought 2 cups of flour and  $\frac{1}{4}$  cup of butter
- Raul brought 1 cup of flour and  $\frac{1}{2}$  cup of butter
- Maria brought  $1\frac{1}{4}$  cups of flour and  $\frac{3}{4}$  cups of butter.

Assume the students have plenty of the other ingredients (sugar, salt, baking soda, etc.) they need to make the cookies.

What is the maximum number of whole batches of cookies they can make with the ingredients they brought from home?

Enter your answer in the second response box.

**Response Type:** Equation/Numeric

**Commentary:** Difficulty and grade level can be varied by varying the complexity of the numbers used. Item aligns with 4D because students must choose which fraction division limits the number of batches that can be made.

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**Example Item 4A.1c (Grade 6)**

Primary Target 4A (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 4B, Quaternary Target 4F

Adapted from <https://www.illustrativemathematics.org/illustrations/985>

- Mrs. Jonas, her son Cody, and her daughter Laura drove from home to Cody’s tennis practice.
- Mrs. Jonas then drove Laura to her soccer game and stayed to watch.
- After the game, mother and daughter picked up Cody from the tennis courts on the way home.
- Once home, Mrs. Jonas saw that they had driven 15 miles that day.

Mrs. Jonas took the shortest routes to and from each destination.

The figure shows the location of the Jonas family home, the tennis courts, and the soccer field. The gridlines in the figure represent the streets, and all distances between cross streets are approximately the same.



**Part A:**

Write an equation that can be used to find the distance,  $d$ , between the tennis courts and home.  
Enter your answer in the first response box.

**Part B:**

What is the distance, in miles, between home and the tennis courts?  
Enter your answer in the second response box.

**Rubric:** (2 points) Student correctly answers both parts ( $10d = 15$ , or  $d + 4d + 4d + d = 15$  or equivalent equation for Part A; 1.5 or  $1 \frac{1}{2}$  for Part B)

(1 point) Student correctly answers only one part.

**Response Type:** Equation/Numeric (Note: Label the two response boxes “Part A” and “Part B.”)

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## **Task Model 4A.2**

### **Task Expectations**

- The student solves a problem involving ratios, proportional relationships, or linear functions.
- The student identifies needed information and chooses the ratio, proportional relationship, or linear function required to complete the problem. The problem should require the student to do one of the following:
  - ignore irrelevant information,
  - request or conduct research to find missing information,
  - identify constraints that are not explicitly stated, or
  - make an estimate for one or more quantities and use that estimate to solve the problem.

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**Example Item 4A.2a (Grade 8)**

Primary Target 4A (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C), Tertiary Target 4D, Quaternary Target 4F

This table represents the cost of renting a truck from Moving Company X and Moving Company Y. Each company charges a one-time rental fee plus a charge for each mile driven.

Moving Company	One-time Rental Fee	Charge per Mile
X	\$150	\$0.25
Y	\$ 50	\$0.75

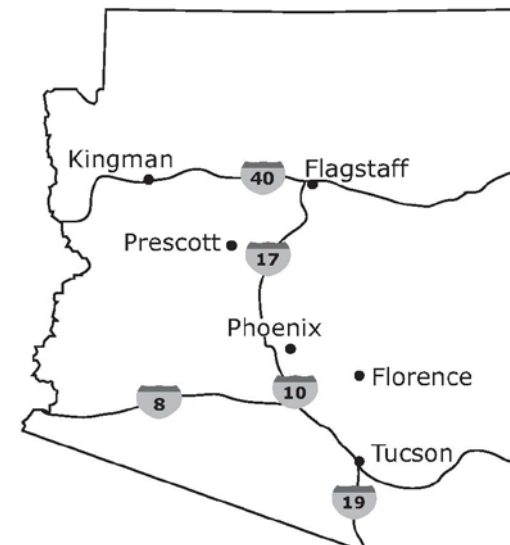
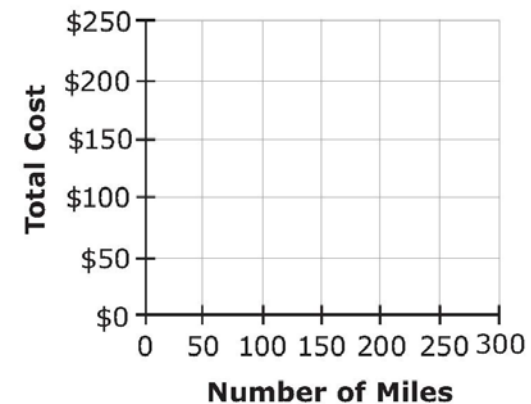
**Part A**

Use the Add Arrow tool to graph two linear equations that represent the cost of using each moving company given a number of miles driven.

**Part B**

Select the moving company that will be the **least** expensive to move between the given cities. Refer to the map shown to determine the distances.

Cities	Company A	Company B
Tucson to Phoenix		
Phoenix to Flagstaff		
Tucson to Flagstaff		



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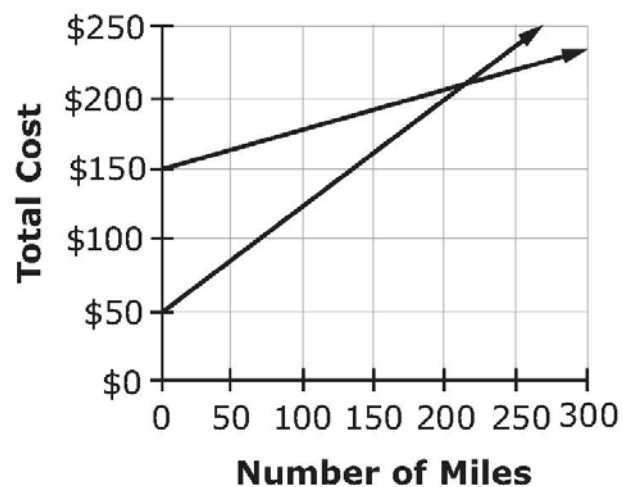
**Interaction:** The student can use the ruler tool to measure distances on the map.

**Rubric:** Each part of this item is scored independently for a total of 2 points.

Part A (1 point) The student correctly graphs both functions.

Part B (1 point) The student selects the correct cells in the table.

**Exemplar:**



Cities	Company A	Company B
Tucson to Phoenix		
Phoenix to Flagstaff		
Tucson to Flagstaff		

**Interaction:** The Add Arrow tool will be available (with one arrow) to graph the lines, as well as Hot Spot to select the correct cells in the table. Also, the ruler tool needs to be active.

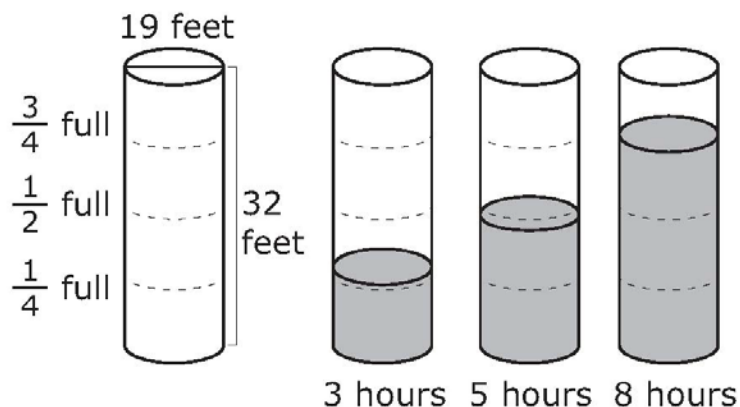
**Response Type:** Graphing and Hot Spot

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**Example Item 4A.2b (Grade 8)**

Primary Target 4A (Content Domain G), Secondary Target 1I (CCSS 8.G.C), Tertiary Target 1A (CCSS 7.RP.A), Quaternary Target 4B

An empty tank in the shape of a cylinder is being filled with water. The tank is filled at a constant rate for a total of 10 hours. The figure shows the height of water in the tank at the given number of hours after filling started.



Enter the **percent** of the tank that is filled with water at 10 hours.

**Rubric:** (2 points) The student enters the correct numerical value for the percent (93.75–94).

(1 point) The student gives the height of water in the tank after 10 hours (30–30.1) OR the volume of water in the tank 10 hours (8500–8532), but forgets to find the percentage.

**Response Type:** Equation/Numeric (label the response box with %)

**Commentary:** The task can be done knowing only the information from the third picture (the height is 24 feet after 8 hours), so students who ignore extraneous information are rewarded. Notice that it is not necessary to compute the volume to find the percent, since it can be found by computing the ratio of the heights. Although it is not expected that many students will notice this, the task thus also rewards students with good modeling sense and geometric insight.



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**Target 4B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.**

Items that require the student to make decisions about the solution path needed to solve a problem are aligned with Target 4B. Note that Target 4B is never the primary target for an item, but is frequently a Tertiary or Quaternary Target for an item with primary alignment to other targets; see, for example, items in Task Models for 4A, 4C, and 4E.

**General Task Model Expectations for Target 4B**

- The student is presented with a multi-step problem with little or no scaffolding, or
- The student must make estimates or choose between different reasonable assumptions in order to solve the problem.

**Target 4C: State logical assumptions being used.**

**General Task Model Expectations for Target 4C**

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - identifies information or assumptions needed to solve the problem,
  - researches to provide information needed to solve the problem, or
  - provides a reasoned estimate of a quantity needed to solve the problem.

It is not necessary that a student constructs a complete solution to the problem for this target.

- Tasks in this model generally have either more information than is needed solve the problem (and students must choose) or not enough information (and students must make a reasoned estimate).
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks for this target may also assess Target 4F.
- Tasks have DOK Level 1 or 2

**Task Model 4C.1**

**Task Expectations:**

- Student chooses from a list of possible assumptions, or makes an estimate, and then solves a problem using the assumption or estimate.

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**Example Item 4C.1a (Grade 7)**

Primary Target 4C (Content Domain SP), Secondary Target 1I (CCSS 7.SP.C), Tertiary Target 4B, Quaternary Target 4D

Ramos flips a coin 100 times and records the results in a table.

**Results of 100 Coin Flips**

Outcome of Flip	Number of Times
Heads	74
Tails	26

**Part A**

Select an assumption about the outcome of a single flip of this coin [heads and tails are equally likely; heads are 3 times as likely as tails]

**Part B**

Based on your assumption, which would be the most likely outcome for the next **2** flips?

- |                          |
|--------------------------|
| A. two heads             |
| B. two tails             |
| C. one head and one tail |

**Interaction:** The student must first select from the drop-down menu to make an assumption, and then select a correct option based on that assumption.

**Rubric:** (1 point) Student makes correct choice based on the assumption they choose (C for the first assumption, A for the second assumption).

**Response Type:** Drop-down Menu; Hotspot

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### Task Model 4C.2

#### Task Expectations:

- The student is given a problem with insufficient information and must indicate what information is needed to complete the solution to a problem.

#### Example Item 4C.2a (Grade 7)

Primary Target 4C (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 4F  
[Adapted from Illustrative Mathematics task 1564.]

Chichén Itzá was a Mayan city in what is now Mexico. The picture shows El Castillo, also known as the pyramid of Kukulcán, which is located in the ruins of Chichén Itzá.



The pyramid is approximately 30 meters tall, and there are 91 steps leading up to a temple at the top.

What additional information do you need to know to estimate the height above the ground, in meters, of the 50th step? Select **all** that apply.

- A. Each of the steps has approximately the same height.
- B. The base of the pyramid is about 55 meters wide.
- C. The height of the temple is about 6 meters.
- D. The base of the pyramid is a square.

**Rubric:** (1 point) The student selects the correct options (A and C).

**Response Type:** Multiple Choice, multiple correct response

## **Target 4D: Interpret results in the context of a situation.**

Target 4D identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems that require students to find a numerical answer as well as many problems where students construct an equation or a graph.

### **General Task Model Expectations for Target 4D**

- The student is presented with a problem situation in everyday life, society, or the workplace or a mathematical model of such a situation. The student interprets the solution to the problem in terms of the context, in terms of the model, or compares the results of the model with the real-world data it represents.
  - Item types with a primary alignment to 4D focus on interpreting results in terms of the model or comparing the results of the model with the real-world data it represents.
  - It is not necessary for a student to generate a complete solution for problems with a primary alignment to this target.
- Tasks in Targets 4A, 4C, 4E, and 4F frequently have this target as a tertiary or quaternary alignment because students must interpret their results in terms of the context.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 2 or 3.

### **Task Model 4D.1**

- The student is presented with a mathematical model of real-world data.
- The student interprets the solution to the problem in terms of the model or compares the results of the model with the real-world data it represents.

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**Example Item 4D.1a (Grade 8)**

Primary Target 4D (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4C

This graph shows the average number of words a child can say from birth to 36 months.



Which statement is the **most accurate** description of the growth in the number of words a child speaks based on the graph shown?

- A. Children learn to say new words at a steady rate starting about 12 months of age.
- B. Children are constantly learning to say new words from the moment they are born.
- C. Children learn to say new words more slowly during their second year than during their third year.
- D. Children begin learning to say words around 24 months and stop learning to say new words at 36 months.

**Rubric:** (1 point) The student chooses the best interpretation of the graph (C).

Note: To distinguish from Claim 1 items, interpretations should extend beyond simply looking at the graph and should help to evaluate whether students understand which interpretations are defensible. Item authors should be careful with language not to “overstate” a particular conclusion since all data based interpretations are subject to some error.

**Response Type:** Multiple Choice, single correct response

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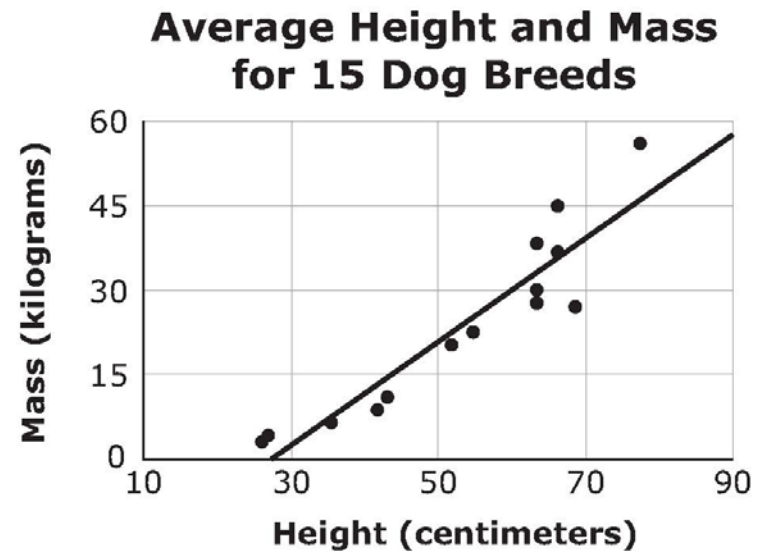
**Example Item 4D.1b (Grade 8)**

Primary Target 4D (Content Domain SP), Secondary Target 1J (CCSS 8.SP.A), Tertiary Target 4E

This scatter plot and line of best fit show the relationship between the height and mass of 15 different dog breeds.

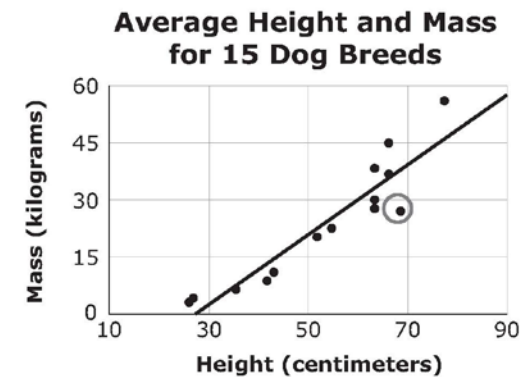
The mass of the Afghan Hound is less than would be predicted by the line of best fit, and the difference between the predicted mass and the actual mass is greater than for any other breed.

Click on the point in the scatterplot that corresponds to the Afghan Hound.



**Rubric:** (1 point) The student clicks the point that below and farthest away from the graph (see figure).

**Response Type:** Hot Spot



**Target 4E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.**

**General Task Model Expectations for Target 4E**

- The student is presented with a problem arising in everyday life, society, or the workplace. The student either
  - Chooses between competing mathematical models to solve the problem (which may depend on different interpretations of the problem)
  - Evaluates a partial or complete (possibly incorrect) solution to the problem
  - Constructs a mathematical model to solve the problem

It is not necessary that a student to generate a complete solution for problems in this target.

- Tasks in this model can also assess Target 4B (Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem). Thus some tasks should plausibly entail a chain of reasoning to complete the task (not just a single step). For example, it might be necessary for the student to construct a two-step arithmetic expression to evaluate a model or solution, or to try out a geometric shape and then perform a calculation to see if it satisfies the requirements.
- The student is often required to draw upon knowledge from different domains, including knowledge from earlier grade-levels.
- Tasks have DOK Level 3 or 4

**Task Model 4E.1**

**Task Expectations:**

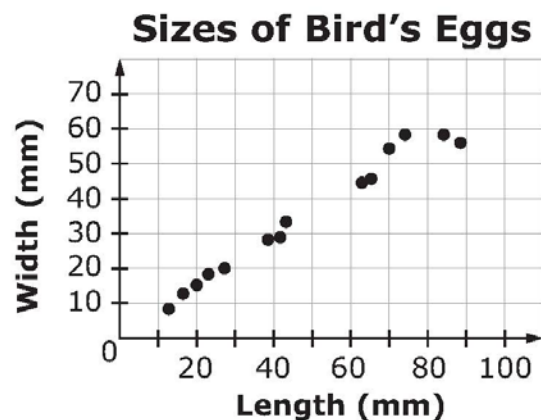
- Students construct an expression, equation, proportional relationship, linear function, or geometric figure that models a given problem.
- Models can be represented in symbolic or graphical form.
- The model is not explicitly given, but should be inferred from the situation.
- Students are expected to reason autonomously from a context to the model.

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**Example Item 4E.1a (Grade 8)**

Primary Target 4E (Content Domain SP), Secondary Target 1J (CCSS 8.SP.A), Tertiary Target 4D, Quaternary 4B

This scatter plot shows the lengths and the widths (in millimetres) of the eggs of some American birds.



Use the information in the scatter plot to support each answer.

**Part A**

The scatter plot shows an association between the length of a bird egg and its width. Describe that association.

**Part B**

Fossils show that dinosaur eggs closely resemble the shape of bird eggs. One type of dinosaur (sauropods) grew from eggs that were 180 millimeters in length.

Assume that sauropod eggs were the same shape as bird eggs. What is the approximate width, in millimeters, of sauropod eggs? Explain how you determined your answer.

**Rubric:** (2 points) The student is able to answer both parts correctly and provide sufficient explanation/support for the answer to *Part B*.

(1 point) The student only answers one part correctly.



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**Exemplar<sup>6</sup>:**

Part A: Typically, the greater the length of the egg, the greater the width.

Part B: The width is approximately 126 mm (accept values between 115 and 135 mm).

“I multiplied the length by about 0.7” or “The width is a little less than the length” or “I doubled the width of the egg that is 90 mm long.”

**Response Type:** Short Text (handscored)

**Example Item 4E.1b (Grade 8)**

Primary Target 4E (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4F, Quaternary Target 4D

Cory is buying copper for a construction project. He pays \$1.85 per pound of copper for the first 100 pounds. He pays \$1.75 per pound of copper for every pound over 100 pounds. Cory calculated that it would cost \$228.75 to purchase 125 pounds of copper. He wrote an equation that allows him to determine the cost of copper for any number of pounds of copper over 100 pounds.

His equation is in the form  $y = n(x - 100) + p$  where  $y$  is the amount of money, in dollars, Cory pays for  $x$  total pounds of copper when  $x$  is greater than 100. What are his values for  $n$  and  $p$ ?

Enter the value of  $n$  in the first response box.

Enter the value of  $p$  in the second response box.

**Rubric:** (1 point) The student enters the correct values for  $n$  and  $p$  (1.75 and 185).

**Response Type:** Equation/Numeric (Note: Label each response box  $n = [\text{box}]$ ,  $p = [\text{box}]$ )

<sup>6</sup> An exemplar response represents only one possible solution. Typically, many other solutions/responses may receive full credit. The full range of acceptable responses is determined during rangefinding and/or scoring validation.

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### Task Model 4E.2

#### Task Expectations:

- The student chooses between two or more different models to solve a given problem, between two or more problems that fit a given model, or between two or more different solutions to a given problem.
- Different models or solutions can depend on different (possibly incorrect) interpretations of the problem, but do not have to.
- The student assesses the fit of a particular model being used.

#### Example Item 4E.2a (Grade 8)

Primary Target 4E (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4F, Quaternary Target 4D  
(Source: Adapted from Illustrative Mathematics 8-F Modeling with a Linear Function)

Select **all** situations that can be modeled by the linear equation  $y = 2x + 5$ .

- A. There are initially 5 rabbits on a farm. Each month thereafter the number of rabbits is 2 times the number in the month before. How many rabbits are there after  $x$  months?
- B. Joe earns \$2 for each magazine sale. He also earns \$5 for each hour he spends trying to sell magazines. How much money will he earn after selling magazines for  $x$  hours?
- C. Sandy charges \$2 an hour for babysitting. Parents are charged \$5 if they arrive home later than scheduled. Assuming the parents arrived home late, how much money does she earn for  $x$  hours?
- D. The Reader's Club is a members-only audio book rental store. There is a \$2 sign-up fee and a \$5 per audio book rental fee. How much would Laney owe on her first visit if she becomes a member and rents  $x$  audio books?
- E. Andre is saving money for a new CD player. He began saving with a \$5 gift and will continue to save \$2 each week. How much money will he have saved at the end of  $x$  weeks?

**Rubric:** (1 point) The student identifies all situations modeled by the equation (C and E).

**Response Type:** Multiple Choice, multiple correct response

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**Example Item 4E.2b (Grade 8)**

Primary Target 4E (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4D

The table shows the relationship between the average number of hours students studied for a mathematics test and their average grade.

Hours Studied	Average Grade
0	62
1	78
2	85
5	74

Which type of function is most likely to model these data?

- A. linear function with positive rate of change
- B. linear function with negative rate of change
- C. non-linear function that decreases then increases
- D. non-linear function that increases then decreases

**Rubric:** (1 point) The student recognized the function most likely to model the data (D).

**Response Type:** Multiple Choice, single correct response

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**Target 4F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).**

Target 4F identifies a key step in the modeling cycle, and is thus present in the majority of modeling problems.

**Task Model 4F.1**

**Task Model Expectations**

- Students are presented with a mathematical problem in a real-world context where the quantities of interest are not named explicitly, are named but represented in different ways, or the relationship between the quantities is not immediately clear.
- The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains.

**Example Item 4F.1a (Grade 7)**

Primary Target 4E (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 4F, Quaternary Target 4D

Megan has \$2500. She spends money on the following:

- \$800 on rent
- \$400 on food
- \$200 on utility services
- \$250 on loan payments
- \$ $x$  on other expenses

Let  $y$  represent the amount of money in dollars Megan has left. Write an equation that represents the relationship between the amount of money Megan spends on other expenses and the amount of money Megan has left.

**Rubric:** (1 point) The student computes Megan's spending and represents the remaining money with an equation ( $y = 850 - x$ , or equivalent).

**Response Type:** Equation/Numeric

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**Example Item 4F.1b (Grade 6)**

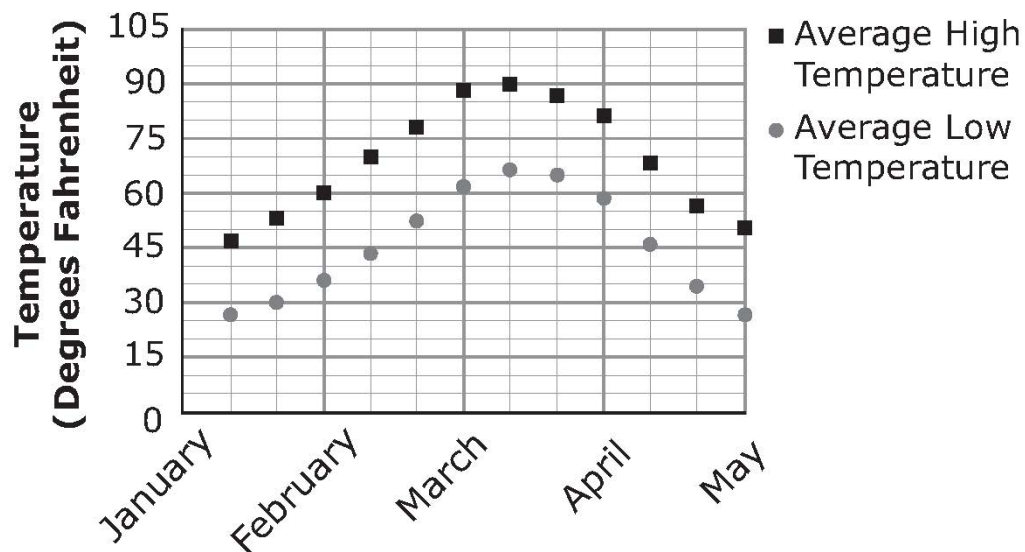
Primary Target 4F (Content Domain EE), Secondary Target 1G (CCSS 6.EE.C), Tertiary Target 4D

**Part A**

If you were going to plan a picnic, what temperature would you hope to have for the picnic? Enter the temperature, in degrees Fahrenheit, you think would be best in the first response box. You may change your answer later if you wish.

**Part B**

The average monthly high and low temperatures for a town are shown in the graph below.



Select a month from the drop down menu where the temperature you chose would fall between the high and low temperatures for that month. [January, February,... December, no month will work]

**Interaction:** The student enters a temperature for a theoretical picnic in the first response box, then answers Part B with a drop down menu. The student can change his or her preferred temperature. The temperature a student chooses does not affect his or her score for the item except that the next choice must be consistent with it. When the student mouses over the points in the graph, the corresponding value appears (alternatively, there is a table of values as well).

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	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Average high in °F:	47	53	60	69	79	88	90	87	81	69	56	46
Average low in °F:	26	30	36	43	53	62	66	65	58	46	34	26

**Rubric:** (1 point) The student selects a month where the temperature he or she chose falls between the high and low temperatures for that (e.g., if the student selects 80, then they choose either June, July, August, or September).

**Response Type:** Equation/Numeric and Drop-down

**Note:** Functionality for this item type does not currently exist, but is planned for future enhancements to the item authoring tool in 2017.

**Example Item 4F.1c (Grade 8)**

Primary Target 4F (Content Domain F), Secondary Target 1F (CCSS 8.F.B), Tertiary Target 4D

The relationship between Jack’s distance from home and the time since he left home is linear, as shown in the table.

Time (hrs)	Distance (mi)
0	7.5
2	17.5
4	27.5

Based on the values in the table, determine whether each statement is true. Select True or False for each statement.

Statement	True	False
Jack’s initial distance from home is 7.5 miles.		
Jack’s distance increases by 5 miles every 1 hour.		
Jack’s distance from home at 3 hours is 23.5 miles.		

**Rubric:** (1 point) Student determines each statement as being either true or false (TTF).

**Response Type:** Matching Table