Target 3A: Test propositions or conjectures with specific examples.

General Task Model Expectations for Target 3A

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- In response to a claim or conjecture, the student should:
  - Find a counterexample if the claim is false,
  - Find examples and non-examples if the claim is sometimes true, or
  - Provide supporting examples for a claim that is always true without concluding that the examples establish that truth, unless there are only a finite number of cases and all of them are established one-by-one. The main role for using specific examples in this case is for students to develop a hypothesis that the conjecture or claim is true, setting students up for work described in Claim 3B.
- False or partially true claims that students are asked to find counterexamples for should draw upon frequently held mathematical misconceptions whenever possible.
- Note: When asking students for a single example, take care to avoid mathematical language that suggests a single example proves a conjecture.
- Tasks have DOK Level 2.

Task Model 3A.1

- The student is presented with a proposition or conjecture and asked to give
  - a counterexample if the claim is false,
  - examples and non-examples if the claim is sometimes true, or
  - one or more supporting examples for a claim that is always true without concluding that the example(s) establish that truth.

Example Item 3A.1a (Grade 6)

Primary Target 3A (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C), Tertiary Target 3G

> Linh said, "The opposite of 5 is -5. The opposite of $\frac{2}{3}$ is $-\frac{2}{3}$. I think the opposite of a number is always negative."
>
> Linh's claim is **not** true. Give an example of a number whose opposite is **not** a negative number.
>
> Enter your answer in the response box.

Rubric: (1 point) The student enters a negative number or 0 in the response box.

Response Type: Equation/Numeric

Example Item 3A.1b (Grade 7)
Primary Target 3A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 3G

When you divide 100 by a positive whole number, the result is always less than or equal to 100. This is not always true when you divide by a positive fraction. Give an example of a fraction $\frac{a}{b}$ where $100 \div \frac{a}{b} < 100$ Enter your fraction in the first response box. Give an example of a fraction $\frac{c}{d}$ where $100 \div \frac{c}{d} > 100$ Enter your fraction in the second response box.

Rubric: (1 point) The student enters appropriate fractions in the response boxes ($\frac{a}{b} > 1$ and $\frac{c}{d} < 1$)

Response Type: Equation/Numeric

Task Model 3A.2

- The student is presented with one or more propositions or conjectures and several examples and asked which examples support or refute one or more of the propositions.
- Items in this task model should cover all cases and not be unintentionally misleading about the truth status of a particular proposition or conjecture.

Example Item 3A.2a (Grade 6)
Primary Target 3A (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C), Tertiary Target 3G

Gina said, "For every possible value of n, we know that $|-n| = n$."

Marco said, "Sometimes $|-n| = -n$."

Who is correct?

A. Gina
B. Marco

Select **all** the values for n shown below that support the correct claim.

B. $n = 12$
C. $n = 4.5$
D. $n = \frac{1}{2}$
E. $n = -4.5$
F. $n = -100$

Rubric: (1 point) The student selects the correct student (B, Marco) and all of the correct values that support Marco's claim (E and F).

Response Type: Multiple Choice, multiple select response

Example Item 3A.2b (Grade 8)
Primary Target 3A (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3G

Franco said that for any values a, b, and c the equation $a^2 + b^2 = c^2$ is always true. Mary disagrees.

Which of the following values for a, b, and c support Mary's claim? Select **all** that apply.

A. $a = 6$, $b = 8$, $c = 10$
B. $a = 2$, $b = 4$, $c = 6$
C. $a = b = c = 0$
D. $a = -2$, $b = 2$, $c = 0$

Rubric: (1 point) The student selects all of the correct values that support Mary's claim (B, D).

Response Type: Multiple choice, multiple select response

Target 3B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

General Task Model Expectations for Target 3B

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8 with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- Items for this target can probe a key mathematical structure such as that found in expressions and equations, ratios and proportional relationships, and the rational number system.
- Items for this target can require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context. The difference between items for Claim 2A and Claim 3B is that the focus in 3B is on communicating the reasoning process in addition to getting the correct answer.
- Note that in grades 6-8, items provide less structure than items for earlier grades to focus on justifying or refuting a proposition or conjecture.
- Many machine-scorable items for these task models can be adapted to increase the autonomy of student's reasoning process but would require hand-scoring.
- Tasks have DOK Level 3, 4.

Task Model 3B.1

- The student is presented with a proposition or conjecture. The student is asked to identify or construct reasoning that justifies or refutes the proposition or conjecture.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.1a (Grade 6)
Primary Target 3B (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C, 4.G.A), Tertiary Target 3C

Lola said, "If n is a positive number, then the points $P = (n, n)$, $Q = (-n, n)$, $R = (-n, -n)$, and $S = (n, -n)$ are the vertices of a square in the coordinate plane."

Select **all** of the statements that support Lola's claim that the figure is a square.

A. The number n is a whole number.
B. The angles at P, Q, R and S, are all 90 degrees.
C. The distances between P and Q, Q and R, R and S, and S and P are n units.
D. The distances between P and Q, Q and R, R and S, and S and P are $2n$ units.

Rubric: (1 point) The student selects all of the statements that support Lola's claim (B and D).

Response Type: Multiple Choice, multiple select response

Example Item 3B.1b (Grade 8)
Primary Target 3B (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3C

The numbers *a*, *b*, and *c* are **not** zero and $a \bullet b = c$.

Part A
Click on the equation below that **must** also be true.

A. $-a \bullet b = c$
B. $a \bullet -b = c$
C. $-a \bullet -b = c$
D. $-a \bullet -b = -c$

Part B
Choose **four** statements that support your claim.

A. $-a = (-1) \bullet a$
B. $-b = (-1) \bullet b$
C. $-c = (-1) \bullet c$
D. $(-1) \bullet (-1) = 1$
E. $(-1) \bullet (1) = -1$
F. You can multiply numbers in any order.

Rubric: (2 point) The student selects the correct equation (C) and selects four statements that support the claim (A, B, D, and F).
(1 point) The student does one or the other.

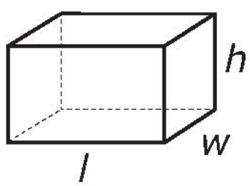
Response Type: Multiple choice, single correct response and multiple choice, multiple select response

Task Model 3B.2

- The student is asked a mathematical question and is asked to identify or construct reasoning that justifies his or her answer.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.2a (Grade 6)
Primary Target 3B (Content Domain G), Secondary Target 1H (CCSS 6.G.A), Tertiary Target 3A

A right rectangular prism has a height of 5 centimeters. Is it possible that the volume of the prism is 42 cubic centimeters?



(Not drawn to scale)

<u>If it is possible:</u>

Enter a possible length and width, in cm, of a prism with a height of 5 cm in two response boxes.

<u>If it is **not** possible:</u>

Enter a possible volume (in cubic centimeters) and the corresponding length and width (in centimeters) in the response boxes.

Rubric: (1 point) The student enters dimensions that are possible (e.g., any two numbers whose product is 8.4).

Response Type: Equation/Numeric (2 response boxes)

Commentary: This item addresses the misconception that the side-lengths of a right rectangular prism must be whole numbers or the related misconception that if the product of two numbers is a whole number then each factor must also be a whole number. Sixth grade is the year where students address the key related concepts most directly.

Example Item 3B.2b (Grade 7)
Primary Target 3B (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A)

A robot moves at a constant speed. It travels *n* miles in *t* minutes. The robot's pace is the number of minutes it takes to travel one mile.

Part A
A. What is the robot's speed in miles per minute?
B. What is the robot's pace in minutes per mile?

Part B
If the robot's speed is greater than 1, then the pace is

A. Greater than 1.
B. Equal to 1.
C. Less than 1.
D. Cannot be determined.

Explain your reasoning.

Rubric: (2 points) The student enters the correct speed (n/t) in the first response box and the correct pace (t/n) in the second response box and selects the correct statement about the pace (C) and enters a correct explanation (see Examples below).
(1 point) The student gets Part A right or Part B right, but not both.

<u>Example 1</u>
If the speed a/b is greater than 1, then the pace b/a must be less than one. The speed and the pace are reciprocals. If a number is greater than 1, then its reciprocal is less than one and vice-versa.

<u>Example 2</u>
The speed is greater than 1, so a/b>1. If we multiply both sides by b we get a>b. If we divide both sides by a, we get 1>b/a, which is the pace. So the pace is less than 1.

Response Type: Equation/numeric, multiple choice single correct response, hand-scored text box.

Note: Functionality for this item type does not currently exist, but the item could be implemented with a single text box.

Example Item 3B.2c (Grade 8)
Primary Target 3B (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C), Tertiary Target 3F, Quaternary Target 3G

Part A
Is it possible for three linear equations in *x* and *y* to have a solution common to all three? [drop-down choices: yes, no]

Part B
[If "yes" is selected] Use the Arrow tool to draw the graphs of three equations that have a common solution. Add a point that represents the common solution.

[If "no" is selected] Explain why this is not possible in the response box.

Interaction: The student has to select yes or no before seeing Part B. If the student selects "yes" then he/she sees the graphing tools and is asked to graph the system. If he/she selects "no" there is a text box that asks for an explanation as to it is not possible. The student can change his/her mind.

Rubric: (1 point) The student selects "yes" and draws three lines that intersect in a single point and places a point at the intersection of the three lines (it is allowable for the lines to coincide, but they have to draw three graphs).

Response Type: Drop-Down Menu[6] and Graphing/Short-Text

Note: Functionality for this item type does not currently exist but it could be implemented by showing Parts A and B simultaneously. When possible, the point of having a student try to explain his or her incorrect reasoning is that in the process of trying to construct an argument, he or she may self-correct.

Task Model 3B.3

- Items for this target require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context.
- The difference between Claim 2 task models and this task model is that the student needs to provide some evidence of his/her reasoning. The difference between Claim 4 task models and this task model is that the problem is completely well posed and no extraneous information is given.

[6] Drop-Down Menu response type is not yet available in the Smarter Balanced item authoring tool, but it is a scheduled enhancement by 2017.

Example Item 3B.3a (Grade 6)
Primary Target 3B (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 3C

Clark biked 4 miles in 20 minutes. How far can he go in 2 hours if he bikes at this rate?

Enter your answer in the first response box.

Show how you would solve this problem with a table or an equation (choose one option).

Option 1: Table

Enter values in the table so that it shows the number of miles, *m*, Clark can bike in 2 hours at this rate.

Miles (*m*)							
Minutes							
Hours							

Option 2: Equation

Enter an equation that can be solved to find the number of miles, *m*, Clark can bike in 2 hours at this rate in the second response box.

Rubric: (2 points) The student enters the correct number of miles (24) and fills in the table with at least two columns, one of which contains the correct answer, or enters an equation that can be solved to find the answer (see examples below of each).
(1 point) The student does one of these parts correctly.

Example for Option 1

Miles	4	8	12	16	20	24	
Minutes	20	40	60	80	100	120	
Hours	1/3	2/3	1	4/3	5/3	2	

Example for Option 2

$2 \cdot 3 \cdot 4 = m$ *or* $4/20 = m/120$ or equivalent equation.

Response Type: Equation/Numeric and Fill-in Table

Note: The functionality for this kind of combination of item types does not currently exist, but is a scheduled enhancement for 2017.

Example Item 3B.3b (Grade 7)
Primary Target 3B (Content Domain EE), Secondary Target 1D (7.EE.B), Tertiary Target 3C

In February, the price of a gallon of gasoline increased by 23% from the price in January. In March, the price decreased by 11% from the price in February. In March, gas cost $2.63 per gallon.

How much did a gallon of gasoline cost in January, in dollars? Round your answer to the nearest cent. Enter your answer in the response box.

Which equation shown can be solved to find x, the cost of gas in January?

A. $(0.11)(0.23)x = 2.63$
B. $(1.11)(1.23)x = 2.63$
C. $(0.89)(1.23)x = 2.63$
D. $(1.11)(0.77)x = 2.63$

Rubric: (2 points) The student enters the correct cost of a gallon of gas (2.40) and selects the correct equation (C).
(1 point) The student does one of these parts correctly.

Response Type: Equation/numeric and multiple choice, single correct response

Note: Current functionality doesn't allow for mixing equation/numeric and multiple choice, so in the meantime the first part could be made multiple choice.

Example Item 3B.3c (Grade 8)
Primary Target 3B (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 4F

A car is traveling at a constant speed and drove 75 miles in 1.5 hours. One mile is approximately 1.6 kilometers. Approximately how fast is the car traveling in kilometers per hour?

Explain or show clear steps for how you determined your answer.

Rubric: (2 points) The student includes the correct numeric value in the response (80) and provides a coherent, complete explanation or sequence of computations that shows where this comes from (see Examples).
(1 point) The student enters the correct numeric value but does not provide a coherent explanation OR the student provides an incorrect speed and includes an explanation that shows an understanding of how the answer could be found, but with some computational errors or a small misstep in reasoning.

Example 1

Going 75 miles in 1.5 hours is the same as going 50 miles per hour.
50 miles is 50*1.6=80 km.
A car driving 50 miles per hour is driving 80 kilometers per hour.

Example 2

75 miles in 1.5 hours is 75/1.5 = 50 mi/hr.
50 mi/hr * 1.6 km/mi =80 km/hr.
The car is traveling at 80 kilometers per hour.

Response Type: Short Text (handscored)

Target 3C: State logical assumptions being used.

General Task Model Expectations for Target 3C

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- For some items, the student must explicitly identify assumptions that
  - Make a problem well-posed, or
  - Make a particular solution method viable.
- When possible, items in this target should focus on assumptions that are commonly made implicitly and can cause confusion when left implicit.
- For some items, the student will be given a definition and be asked to reason from that definition.
- Tasks are DOK Level 2, 3.

Task Model 3C.1

- The student is asked to identify an unstated assumption that would make the problem well-posed or allow them to solve a problem using a given method.

Example Item 3C.1a (Grade 6)
Primary Target 3C (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 3G

Lyla flew her radio-controlled airplane 500 feet in 20 seconds. She claims that the speed of her airplane was 25 feet per second during the flight. What assumption must Lyla make for her claim to be true?

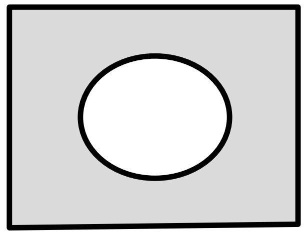
A. The airplane flew in a circle.
B. The airplane flew in a straight line.
C. The airplane flew at a constant speed.
D. The airplane flew faster at the end of the flight than at the beginning.

Rubric: (1 point) The student selects the correct statement (C).

Response Type: Multiple Choice, single correct response

Example Item 3C.1b (Grade 7)
Primary Target 3C (Content Domain G), Secondary Target 1F (CCSS 7.G.B), Tertiary Target 3G

Glenn saw the figure below and said,
"If I find the length (*l*), width (*w*), and radius (*r*), then the area (*A*) of the shaded region is $A = l \cdot w - \pi r^2$."



Which assumptions must Glenn be making in order for his equation to give the correct area of the shaded region? Select **all** that apply.

A. The quadrilateral is a rhombus.
B. The quadrilateral is a rectangle.
C. The curved figure in the center is a circle.
D. The curved figure in the center is a sphere.

Rubric: (1 point) The student selects the correct assumptions (B and C).

Response Type: Multiple Choice, single correct response

Task Model 3C.2

- The student will be given one or more definitions or assumptions and be asked to reason from that set of definitions and assumptions.

Example Item 3C.2a (Grade 7)
Primary Target 3C (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3C

A **perfect square** is a number *s* that is the product of an integer, *n*, and itself, so that $s = n^2$.

Examples of perfect squares include 25 because it is equal to 5^2 and 81 because it is equal 9^2.

Can a perfect square be negative?

A. Yes; an example is –25.
B. No; a square of any integer is always positive.
C. Sometimes Yes, sometimes No; it depends on the value of *n*.
D. There is not enough information to tell.

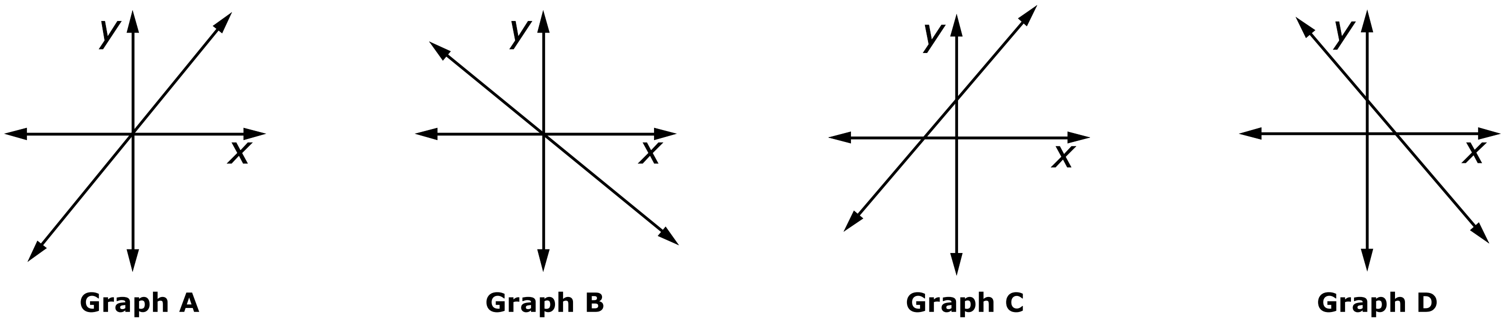
Rubric: (1 point) The student selects the correct statement (B).

Response Type: Multiple Choice, single correct response

Example Item 3C.2b (Grade 8)
Primary Target 3C (Content Domain EE), Secondary Target 1C (CCSS 8.EE.B), Tertiary Target 3F

A proportional relationship between *x* and *y* is one that can be represented by the equation $y = k \bullet x$, where k is a positive number.

Consider these four graphs.



Based on this definition, identify whether or not each graph could represent a proportional relationship. Answer "Yes" if it does represent a proportional relationship and "No" if it does not.

	Yes	**No**
Graph A		
Graph B		
Graph C		
Graph D		

Rubric: (1 point) The student identifies the correct graphs (YNNN).

Response Type: Matching Table

Target 3D: Use the technique of breaking an argument into cases.

General Task Model Expectations for Target 3D

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- The student is given
  - a problem that has a finite number of possible solutions, some of which work and some of which don't, or
  - a proposition that is true in some cases but not others.
- Items for Claim 3 Target D should either present an exhaustive set of cases to consider or expect students to consider all possible cases in turn in order to distinguish it from items in other targets.
- Items have DOK Level 2, 3.

Task Model 3D.1

- The student is given a problem that has a finite number of possible solutions, some of which work and some of which don't.

Example Item 3D.1a (Grade 7)

Primary Target 3D (Content Domain RP), Second Target 1A (CCSS 7.RP.A), Tertiary Target 3G

Green paint can be made by mixing yellow paint with blue paint. Two mixtures make the same shade of green if the ratio of yellow to blue is the same. Assume n is a positive number.

Identify **one or more** of the mixtures below that will make the same shade of paint as a mixture of 10 liters of yellow paint and 15 liters of blue paint. Answer "Yes" if it will make the same shade of paint, answer "No" if it will not.

	Liters of Yellow Paint	**Liters of Blue Paint**	**Yes**	**No**
Mixture 1	$2n$	3		
Mixture 2	2	$3n$		
Mixture 3	$2n$	$3n$		

Rubric: (1 point) The student identifies the correct mixture (NNY).

Response Type: Matching Table

Note: A drag-and-drop version of this could allow students to determine the equivalent mixtures themselves.

Example Item 3D.1b (Grade 8)
Primary Target 3D (Content Domain G), Secondary Target 1G (CCSS 8.G.A), Tertiary Target 3G

Select **all** of the following situations that show that Figure *P* is congruent to Figure *Q.*

A. There is a translation that takes Figure *P* to Figure *Q*.
B. There is a rotation that takes Figure *P* to Figure *Q*.
C. There is a reflection that takes Figure *P* to Figure *Q*.
D. There is a dilation that takes Figure *P* to Figure *Q*.

Rubric: (1 point) The student selects the correct transformations (A, B, and C).

Response Type: Multiple choice, multiple selection response

Task Model 3D.2

- The student is given a proposition and asked to determine in which cases the proposition is true.

Example Item 3D.2a (Grade 7)
Primary Target 3D (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3C

Given x and y are rational numbers, when is $|x + y| = |x| + |y|$ true?

A. This is never true.
B. This is always true.
C. This is true when x and y have opposite signs.
D. This is true when x and y have the same sign.

Rubric: (1 point) The student selects the correct statement (D).

Response Type: Multiple Choice, single correct response

Example Item 3D.2b (Grade 8)
Primary Target 3D (Content Domain EE), Secondary Target 1B (CCSS 8.EE.A), Tertiary Target 3C

Maggie claims that when you raise a whole number to a power, the result is always a greater number.
That is, $s^n > s$. For example:

$4^3 > 4$
$5^4 > 5$
$10^9 > 10$

Maggie's claim is **not** true for all values of *n* and *s*. For what values of *n* and *s* is Maggie's claim true? Complete the inequalities.

s > [ ]

n > [ ]

Rubric: (1 point) The student enters the correct values in the response boxes (1 and 1).

Response Type: Equation/Numeric (two response boxes, label the boxes with *s* > and *n* >, respectively.)

Target 3E: Distinguish correct reasoning from flawed reasoning

General Task Model Expectations for Target 3E

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- The student is presented with valid or invalid reasoning and told it is flawed or asked to determine its validity. If the reasoning is flawed, the student identifies, explains, and/or corrects the error or flaw.
- The error should be more than just a computational error or an error in counting, and should reflect an actual error in reasoning.
- Analyzing faulty algorithms is acceptable so long as the algorithm is internally consistent and it isn't just a mechanical mistake executing a standard algorithm.
- Items have DOK Level 2, 3, 4.

Task Model 3E.1

- Some flawed reasoning or student work is presented and the student identifies and/or corrects the error or flaw.
- The student is presented with valid or invalid reasoning and asked to determine its validity. If the reasoning is flawed, the student will explain or correct the flaw.

Example Item 3E.1a (Grade 6)
Primary Target 3E (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 3C

Emma was solving the equation $t - 4 = 16$. She said, "I'm looking for a number *t* that is 4 less than 16. So $t = 12$."

Which statement best describes the flaw in Emma's reasoning?

A. Emma's answer is right but she should just subtract 4 from both sides of the equation.
B. Emma's answer is wrong but she thought about the equation correctly.
C. Emma is confused about which number the 4 is being subtracted from.
D. Emma should subtract the 16 from the 4 instead of 4 from the 16.

Rubric: (1 point) The student selects the correct analysis of the flaw in reasoning (C).

Response Type: Multiple choice, single correct response

Example Item 3E.1b (Grade 7)
Primary Target 3E (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 3C

Dena is trying to solve this problem:

> A store has a sale where every item has a sale price that is 20% less than the regular price. Write an expression that represents the sale price of an item if the regular price is *p* dollars.

Dena said, "To find 20% of a number, I should multiply by 0.20. So the sale price of an item will be $0.20p$."

Which statement best describes Dena's reasoning?

A. Dena is correct.
B. Dena needs to subtract 0.20p from the regular price, *p.*
C. Dena should calculate the sale price as $20p$ and then divide by 100.
D. Dena is trying to solve an impossible problem because it doesn't say what the regular price is.

Rubric: (1 point) The student selects the statement that represents correct reasoning (B).

Response Type: Multiple choice, single correct response

Task Model 3E.2

- Two or more approaches or chains of reasoning are given and the student is asked to identify the correct method and justification OR identify the incorrect method/reasoning and the justification.

Example Item 3E.2a (Grade 7)
Primary Target 3E (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 3C

Clyde and Lily were solve the equation $\frac{8}{9} \div \frac{1}{2} = x$.

Clyde said, "I can think of this division problem as a multiplication problem." Then he wrote:		Lily said, "You need to invert and multiply." Then she wrote:	
Step 1.	$\frac{8}{9} \div \frac{1}{2} = x$	Step 1.	$\frac{8}{9} \div \frac{1}{2} = x$
Step 2.	$\frac{1}{2}x = \frac{8}{9}$	Step 2.	$\frac{8}{9} = 2 \bullet x$
Step 3.	$2\left(\frac{1}{2}x\right) = 2\left(\frac{8}{9}\right)$	Step 3.	$\frac{1}{2}(2x) = \left(\frac{1}{2}\right) \bullet \left(\frac{8}{9}\right)$
Step 4.	$x = \frac{16}{9}$	Step 4.	$x = \frac{8}{18}$

Who solved the problem correctly?

A. Only Clyde solved the equation correctly.
B. Only Lily solved the equation correctly.
C. They both solved the equation correctly.
D. Neither one solved the equation correctly.

Rubric: (1 point) The student selects the correct characterization of these two approaches (A).

Response Type: Multiple choice, single correct response

Example Item 3E.2b (Grade 8)
Primary Target 3E (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C), Tertiary Target 3C, Quaternary Target 3F

The students in Mr. Martin's class are learning about linear equations. Kenny made a claim and two supporting claims about the possible number of solutions to a system of linear equations. Rhonda made a different claim with two supporting claims.

Indicate whether each claim is valid or not valid.

Kenny's Claims	Valid	Not Valid
Claim 1. A system of two linear equations can only have zero solutions or one solution.		
Claim 1a. If the corresponding lines are distinct and parallel, then there are no solutions.		
Claim 1b. If the corresponding lines are distinct and intersect, then there is one solution.		

Rhonda's Claims	Valid	Not Valid
Claim 2. A system of two linear equations can have more than one solution.		
Claim 2a. If the corresponding lines intersect in exactly two places, then there will be exactly two solutions.		
Claim 2b. If the corresponding lines completely coincide, then there are an infinite number of solutions.		

Rubric: (1 point) The student selects the correct claims (NVV, VNV).

Response Type: Matching Table

Target 3F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions

Task Model 3F.1

- The student uses concrete referents to help justify or refute an argument.
- In grade 6, items in this task model should focus on the use of number lines. In grade 7, they should focus on number lines and graphs of proportional relationships. In grade 8, they should focus on graphs of linear equations and systems of linear equations and geometric contexts related to transformations of the plane or the Pythagorean Theorem.
- Items have DOK Level 2, 3.

Example Item 3F.1a (Grade 7)
Primary Target 3F (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C), Tertiary Target 3D

P and T are numbers and $P + T = 0$.

Select **all** of the statements about P and Q that could be true.

A. $P = 0$ and $T = 0$

B. $P = 0$ or $T = 0$, but not both.

C. P can be any positive number and T can be any negative number.

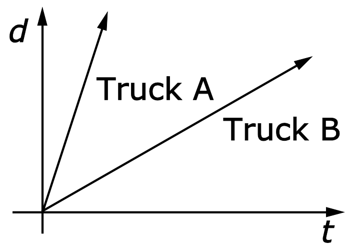
D. P and T are on opposite sides of zero and equally distant from zero on the number line.

Rubric: (1 point) The student selects the correct statements (A, D).

Response Type: Multiple Choice, multiple correct response

Example Item 3F.1b (Grade 7)
Primary Target 3F (Content Domain NS), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 3D

Two trucks are traveling on a highway at a constant speed. The graphs of their distances, *d*, over time, *t*, are shown.



Which truck is traveling faster, and how do you know?

Truck [drop-down menu choices: A, B] is traveling faster because the graph is [drop-down menu choices: steeper, less steep, longer, shorter].

Rubric: (1 point) The student chooses the correct truck (A) and the correct reason (steeper).

Response Type: Drop-down menu

Note: Functionality for this item type does not currently exist, but could be implemented with two-part multiple choice.

Example Item 3F.1c (Grade 8)
Primary Target 3F (Content Domain G), Secondary Target 1H (CCSS 8.G.B), Tertiary Target 3B

The Pythagorean Theorem states that if a right triangle has legs of length *a* and *b* and hypotenuse of length *c*, then $a^2 + b^2 = c^2$.

Figures 1 and 2 represent the key ideas in a proof of the Pythagorean Theorem.

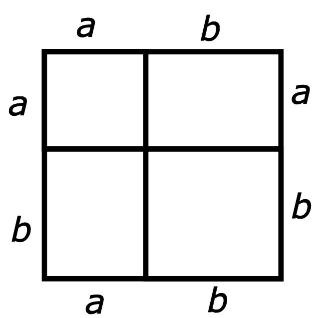


Figure 1

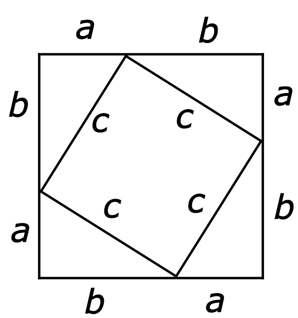


Figure 2

Create an outline a proof for the Pythagorean Theorem based on Figures 1 and 2, by dragging the seven statements shown into a logical sequence.

A right triangle has legs of length *a* and *b* and hypotenuse of length *c*.

1.
2.
3.
4.
5.
6.
7.

Thus, $a^2 + b^2 = c^2$

Subdivide the large square in Figure 1 into a square with side-length *a*, a square with side-length *b*, and two rectangles with side-lengths *a* and *b*.
Subdivide the large square in Figure 2 into four right triangles with legs *a* and *b* and a square with side-length *c*.
The total area of the large square in Figure 1 is $a^2 + b^2 + ab + ab$.
The total area of the large square in Figure 2 is $c^2 + 4(\frac{1}{2}ab)$.
Start with two large squares with sides of length $a + b$.
$a^2 + b^2 + ab + ab = c^2 + 4(\frac{1}{2}ab)$
The two large squares have the same area because they are congruent.

Rubric: (2 points) The student drags the steps of the proof into a logical order. Note that 1 must be first and 7 must be last and 2 must precede 5 and 3 must precede 6, but any other permutations are allowed as long as they are consistent with these constraints).
(1 point) The student gets the steps in an order consistent with the constraints described above, but has at most one step out of order.

Exemplar (more solutions are possible as noted above)

1. Start with two large squares with sides of length $a + b$.
2. Subdivide the large square in Figure 1 into a square with side-length a, a square with side length b, and two rectangles with side-lengths a and b.
3. Subdivide the large square in Figure 2 into four right triangles with legs a and b and a square with side-length c.
4. The two large squares have the same area because they are congruent.
5. The total area of the large square in Figure 1 is $a^2 + b^2 + ab + ab$.
6. The total area of the large square in Figure 2 is $c^2 + 4(\frac{1}{2}ab)$.
7. $a^2 + b^2 + ab + ab = c^2 + 4(\frac{1}{2}ab)$.

Response Type: Drag and Drop

Target 3G: Determine conditions under which an argument does and does not apply

Target 3G is a closely related extension of the expectations in Targets 3A, 3B, 3C, and 3D, and as with those targets, is often a tertiary alignment for items in those targets. Students often test propositions and conjectures with specific examples (as described in Target 3A) for the purpose of formulating conjectures about the conditions under which an argument does and does not apply. Students then must explicitly describe those conditions (as in Target 3C). Expectations for Target 3D include determining conditions under which an argument is true given cases—the next step is articulating those cases autonomously (Target 3B).