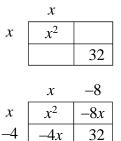
Benchmark 3 Study Guide

1

Find the roots (zeros, x-intercepts) of $f(x) = x^2 - 12x + 32$.

We are looking for the solutions to $0 = x^2 - 12x + 32$

Method 1: Factoring Using Guess & Check



Using an Area Model, fill in a Generic Rectangle with the first and last terms. Given x^2 is the area (product), the dimensions (factors) have to be x and x.

We GUESS which factors will give us a product of 32 and when combined as the middle term will give us -12x.

$$\therefore 0 = (x-4)(x-8)$$

 $0 = x-4$ or $0 = x-8$
 $x = 4$ or $x = 8$

Method 2: Completing the square

 $x^2 - 12x + 32 = 0$

$x^{2} - 12x + 32 - 32 = 0 - 32$ $x^{2} - 12x + = -32 + $	Move constant term to other side Leave space to complete the square
$x^2 - 12x + (-6)^2 = -32 + (-6)^2$	Complete the square with $\left(\frac{-12}{2}\right)^2$
$x^2 - 12x + 36 = -32 + 36$	Simplify
$(x-6)^2 = 4$	Simplify and factor
$\sqrt{\left(x-6\right)^2} = \sqrt{4}$	Inverse operations : square root
$x-6=\pm 2$	
x = -2 + 6 or	x = 2 + 6
x = 4 or	<i>x</i> = 8

Method 3: Quadratic formula

$x^2 - 12x + 32 = 0 \qquad a = 1, b$	p = -12, and $c = 32$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$x = \frac{-(-12)\pm\sqrt{(-12)^2 - 4(1)(32)}}{2(1)}$	Substitute
$x = \frac{12 \pm \sqrt{144 - 128}}{2(1)}$	Multiply within the radical
$x = \frac{12 \pm \sqrt{16}}{2}$	Add within the radical
$x = \frac{12 \pm 4}{2}$	Simplify the radical
$x = 6 \pm 2$	Divide and simplify
x = 4 or $x = 8$	A-REI.4b

1a′

You Try:

Find the x-intercepts of $f(x) = 25x^2 - 1$



You try:

Which of the following could be part of the process of finding the roots of $y = x^2 - 4x - 60$?

A)
$$0 = (x+10)(x-6)$$

B)
$$0 = (x - 10)(x + 6)$$

C)
$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-60)}}{2(1)}$$

D)
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-60)}}{2(1)}$$

E)
$$x^2 - 4x + (-4)^2 = 60 + (-4)^2$$

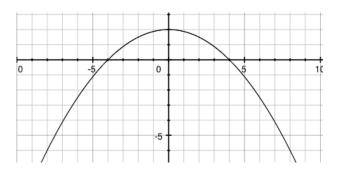
F)
$$x^2 - 4x + (-2)^2 = 60 + (-2)^2$$

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Benchmark 3 Study Guide

2

Answer the following questions about the graph of the function g(x) shown below:



a) What is the *y*-intercept?

The *y*-intercept is where the graph crosses the *y*-axis, in this case at the point (0, 2) or y = 2.

b) What are the *x*-intercept(s)?

The *x*-intercept is where the graph crosses the *x*-axis, in this case at the points (-4, 0) and (4, 0) or x = -4 or 4.

c) How has this graph transformed from the mother function, $f(x) = x^2$?

g(x) is wider than the graph of $f(x) = x^2$ (which means the quadratic coefficient has an absolute value less than 1). g(x) is concave down (which means the quadratic coefficient is negative), while $f(x) = x^2$ is concave up. g(x) is also shifted 2 units up from $f(x) = x^2$ (which means the constant term is +2).

d) Is the average rate of change from x = 0 to x = 4 negative, positive, zero, or undefined?

If we look only at the portion of the graph between x = 0 and x = 4, we can see that the graph is decreasing. This means that the average rate of change on this interval is negative.

e) What is the approximate value of g(-8)?

We are looking at the graph where x = -8 and trying to determine what the *y*-value is at that point. We can approximate that $g(-8) \approx -6$.

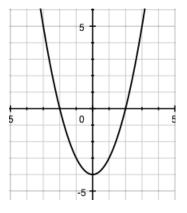
f) Is g(x) increasing or decreasing on $-4 \le x \le 0$?

If we look only at the portion of the graph between x = -4 and x = 0, we can see that the graph is increasing.



You try:

Answer the following questions about the graph of the function g(x) shown below:



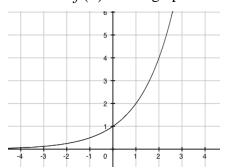
- **a**) What are the *y*-intercept(s)?
- **b**) What are the *x*-intercept(s)?
- c) How has this graph transformed from the mother function, $f(x) = x^2$?

- **d**) Is the average rate of change from x = 0 to x = 2 negative, positive, zero, or undefined?
- e) What is the approximate value of g(-3)?

f) Is g(x) increasing or decreasing on $-2 \le x \le 0$?

3

The function $f(x) = 2^x$ is graphed below.



Using this graph as the parent function f(x), sketch a graph of each of the following functions.

A) $g(x) = 2^x + 3$

g(x) is of the form f(x) + 3. The constant is being added to the function and not to the function's domain. This means the parent function will shift up (since 3 is positive) three units.

To check this, I could also create

x	-1	0	1
g(x)	3.5	4	5

B) $g(x) = 2^{-x}$

a table.

g(x) is of the form f(-x). The domain of x is being transformed to -x. This means the parent function will reflect over the y-axis.

To check this, I could also create

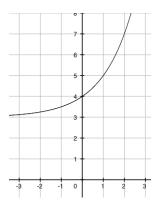
a table.	x	-1	0	1
	g(x)	2	1	0.5

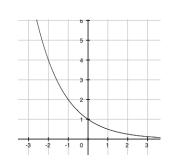


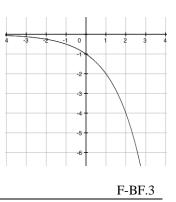
g(x) is of the form -f(x). The range of the function is being transformed to its opposites. This means the parent function will reflect over the *x*-axis.

To check	k this, I	could a	also ci	eate
. 11				
a table.		1		1

e.	x	-1	0	1	
	g(x)	-0.5	-1	-2	









3 cont'd

a table.

D)
$$g(x) = 2^{x+3}$$

g(x) is of the form f(x + 3). The constant is being added to the function's domain. This means the parent function will shift to the left or right. Since the constant is positive, it will shift to the left three units.

To check this, I could also create

-4

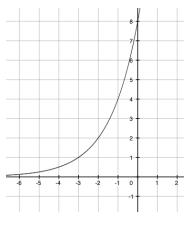
0.5

-3

1

x

g(x)





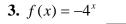
You try: Match the function with its graph.
1.
$$f(x) = 4^x$$
 _____ **4.** $f(x) = 4^x - 2$

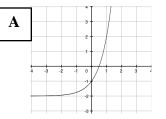
-2

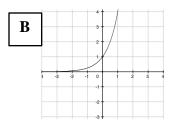
2

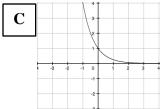
2.
$$f(x) = 4^{-x}$$

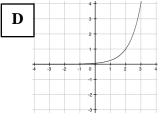
5. $f(x) = 4^{x-2}$

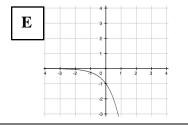












Benchmark 3 Study Guide

The Average Rate of Change of a function f(x) from x = a to x = b is

 $\frac{f(b) - f(a)}{b - a} \quad \frac{\text{change in function values}}{\text{change in } x}$

A) Given the function, f(x) = 2x-1 find the average rate of change from x = 0 to x = 1.

To do this, I need to evaluate the function at x = 0 and x = 1. f(0) = 2(0) - 1 f(1) = 2(1) - 1

$J(\mathbf{e}) = (\mathbf{e})^{-1}$	J (-) _(-) -
f(0) = 0 - 1	f(1) = 2 - 1
f(0) = -1	f(1) = 1

Now, I can substitute those values into the formula. f(b) - f(a)

 $\frac{f(x) - f(x)}{b - a}$ $= \frac{1 - (-1)}{1 - 0}$ $= \frac{1 + 1}{1}$ The average rate of change of f(x) from x = 0 to x = 1 is 2. = 2

B) Given the function, $g(x) = 2^x + 1$ find the average rate of change from x = 0 to x = 1.

To do this, I need to evaluate the function at x = 0 and x = 1.

$g(0) = 2^0 + 1$	$g(1) = 2^1 + 1$
g(0) = 1 + 1	g(1) = 2 + 1
g(0) = 2	g(1) = 3

Now, I can substitute those values into the formula.

 $\frac{g(b) - g(a)}{b - a}$ $= \frac{3 - 2}{1 - 0}$ $= \frac{1}{1}$ The average rate of change of g(x) from x = 0 to x = 1 is 1.

C) From x = 0 to x = 1, what do we notice about f(x) and g(x)?

On the interval from x = 0 to x = 1, f(x) is increasing at a greater rate than g(x). Given what we know about linear and exponential functions, they are both increasing everywhere.

F-IF.6

4a´ You try:

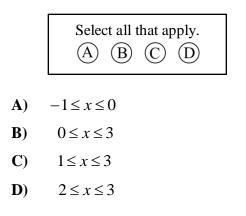
1) Given the function, $f(x) = -x^2 + 4x - 1$ find the average rate of change from x = 1 to x = 5.

2) Given the function, $g(x) = 3^x$ find the average rate of change from x = 1 to x = 3.

4b' You try:

 $f(x) = 3x - 1 \qquad g(x) = 2^x$

Given the functions above, select *all* of the following intervals on which the average rate of change of f(x) is **greater** than the average rate of change of g(x).

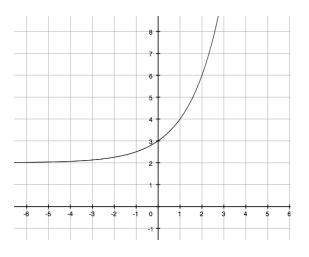


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Benchmark 3 Study Guide

5

Answer the following questions about the graph of the function g(x) shown below:



a) What are the *y*-intercept(s)?

The *y*-intercept is where the graph crosses the *y*-axis, in this case at the point (0, 3) or y = 3.

b) What are the *x*-intercept(s)?

The *x*-intercept is where the graph crosses the *x*-axis. In this case, there is no *x*-intercept, because the exponential function has an asymptote at the line y = 2.

c) What is the average rate of change between x = 0and x = 2?

First, if I look at the portion of the graph between x = 0and x = 2, I see that it is increasing, so I know my average rate of change will be positive. I know that I need to evaluate the function at the boundary points. g(0) = 3 and g(2) = 6. Therefore, I can use the formula for rate of change.

average rate of change =
$$\frac{g(b) - g(a)}{b - a}$$

= $\frac{6 - 3}{2 - 0}$
= $\frac{3}{2}$

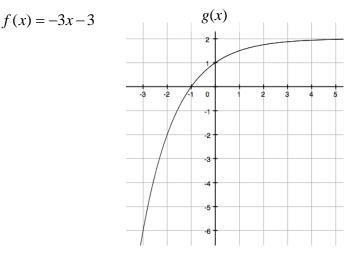
d) Is the function increasing, decreasing, both, or neither?

I can see that the function is increasing everywhere. F-IF.4 **5** You Try:

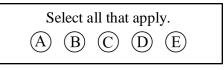
Two functions, f(x) and g(x) are represented below.

Function 1:

Function 2:



Select *all* of the following statements that are TRUE about the functions above.



- A) f(x) and g(x) have the same x-intercept(s).
- **B**) f(x) and g(x) have the same y-intercept(s).
- C) f(x) and g(x) have the same average rate of change between x = -3 and x = -1.
- **D**) Both functions are decreasing everywhere.
- **E**) Both functions have a maximum at (0, 2)

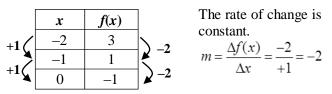
F-IF.9

Benchmark 3 Study Guide



<u>Linear functions</u>: f(x) = mx + bYou can tell a function is linear if its rate of change is constant.

Linear function in a table:



<u>Linear functions in proportional situations</u>: With phrases like:

"a decrease of \$20 per month"

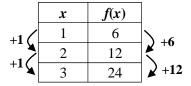
"60 miles an hour"

"3 bananas for every \$5"

Exponential functions: $f(x) = a \cdot b^x$

You can tell a function is exponential when its values are changing at a constant multiplicative rate over constant intervals, or a constant percent rate.

Exponential function in a table:



Here you see that f(x) is doubling (multiplying by 2). This is a 100% increase.

Exponential functions in situations: With phrases like: "bacteria triples every minute" "a 30% increase per month" "decays by half of its population each hour"

F-LE.1&2

6a´

You Try:

Match each situation to the type of function that would be used to model it.

- A Linear model
- **B**) Exponential model
- **C**) Neither
- 1) The amount of grain in a silo is 15 lbs and gains 10 lbs every 3 minutes.
- 2) A soccer player kicks a ball off a cliff and it hits the ground 5 seconds later.
- **3)** The amount of fish in a lake starts at 132 and triples every 2 years.

6b′ You Try:

Match each example with the function that could be used to model its behavior.

- $(A) \quad f(t) = a + 1,000t$
- (**B** $) \quad f(t) = b(5)^t$

$$f(t) = 5t + c$$

$$(D) \quad f(t) = d(1.05)^t$$

- The number of people in line for a rollercoaster increases by 5 people every minute.
- 2) A car's value increases by 5% every year.
- The number of cells in a petri dish multiplies five times every hour.
- A teacher's salary increases \$1,000 for every year they teach.

Benchmark 3 Study Guide

7

The two types of sequences studied in this course are arithmetic and geometric.

Arithmetic Sequences $a_1, a_1 + d, a_1 + 2d, a_1 + 3d,$	Geometric Sequences $a_1, a_1r, a_1r^2, a_1r^3, \dots$
The <u>common difference</u> of an arithmetic sequence can be found using $d = a_n - a_{n-1}$.	The <u>common ratio</u> of a geometric sequence can be found using $r = \frac{a_n}{a_{n-1}}$.
The <u>explicit formula</u> for an arithmetic sequence is $a_n = a_1 + (n-1)d$.	The <u>explicit formula</u> for a geometric sequence is $a_n = a_1 r^{n-1}$.

For each sequence below, state whether it is arithmetic, geometric, or neither. If it is an arithmetic or geometric sequence, write its explicit formula.

Example 1: 19, 15, 11, 7, ...

This sequence is arithmetic because the terms have a common difference.

19, 15, 11, 7,	OR	$d = a_n - a_{n-1}$	$d = a_n - a_{n-1}$
		$d = a_2 - a_1$	$d = a_3 - a_2$
-4 -4 -4		d = 15 - 19	d = 11 - 15
		d = -4	d = -4

To find the explicit formula:

$a_n = a_1 + (n-1)d$	Write explicit formula
$a_n = 19 + (n-1)(-4)$	Substitute $a_1 = 19$ and $d = -4$
$a_n = 19 - 4n + 4$	Distribute
$a_n = -4n + 23$	Combine like terms

Example 2: 5, -10, 20, -40, ...

This sequence is geometric because the terms have a common ratio.

5,
$$-10$$
, 20, -40 , ... OR
•(-2) •(-2) •(-2)

 $r = \frac{a_n}{a_{n-1}}$ $r = \frac{a_2}{a_1}$ $r = \frac{-10}{5} = -2$

To find the explicit formula:

 $a_n = a_1 r^{n-1}$ Write explicit formula $a_n = 5(-2)^{n-1}$ Substitute $a_1 = 5$ and r = -2

F-BF.2

7a´ You try:

Determine whether each sequence below is arithmetic, geometric, or neither. If possible, identify the common difference or ratio.

- **1.** 7, 10, 13, 16, ...
- **2.** 1, 4, 9, 16, ...
- **3.** 4, 40, 400, …

5.
$$\frac{3}{4}$$
, $\frac{3}{2}$, 3, 6, 12,...

7b You Try: Match each sequence to its explicit formula.

 (A)
 $a_n = -3n + 24$

 (B)
 $a_n = 3(2)^{n-1}$

 (C)
 $a_n = 3^{n-1}$

 (D)
 $a_n = 3n$

 1)
 3, 6, 9, 12, ...

 2)
 3, 6, 12, 24, ...

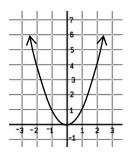
 (A)
 21, 18, 15, 12, ...

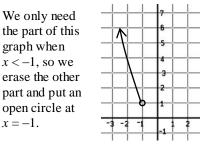
8

Graph the following piecewise-defined function.

$$f(x) = \begin{cases} x^2, & \text{for } x < -1 \\ 3x - 1, & \text{for } x \ge -1 \end{cases}$$

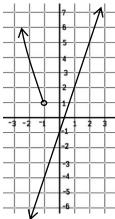
First, we will sketch the first piece of the graph, $f(x) = x^2$.

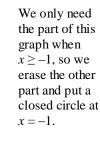


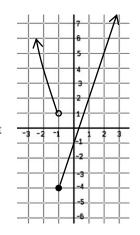


Then, we will sketch the second piece of the graph, f(x) = 3x - 1.

x = -1.







a) What is f(-1)? Why?

This is the boundary point so you have to pay close attention to which piece of the function includes x = -1. I can see from the graph that f(x) has a closed circle at the point (-1, -4), therefore, f(-1) = -4. I could also evaluate the function using the second piece at -1 to show that it would be -4.

> **b**) Is f(x) increasing, decreasing, or neither between x = -1 and x = 0? Explain your answer.

We can look at the piece of the graph from x = -1 to x = 0 and see that the graph is increasing.

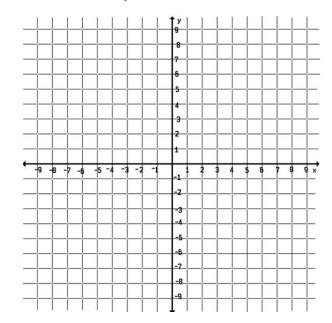
F.IF.7a

Benchmark 3 Study Guide

You try: 8′

Graph the following piecewise-defined function.

$$p(x) = \begin{cases} x+2, & \text{for } x < -2\\ x^2, & \text{for } x \ge -2 \end{cases}$$



a) What is p(-2)? Why?

b) Is p(x) increasing, decreasing, or neither between x = -2 and x = 0? Explain your answer.

End of Study Guide

Benchmark 3 Study Guide

You Try Solutions:



You try: Find the *x*-intercepts of $f(x) = 25x^2 - 1$

The binomial can also be thought of as the trinomial $Ax^2 + Bx + C$, where Bx = 0x and B = 0.

Method 1: Factoring Using Guess & Check

To find the *x*-intercepts, we solve for *x* when f(x) = 0.

 $0 = 25x^{2} + 0x - 1$ Write original expression 0 = (-1)(+1) The last terms have to be -1 and 1.

$$0 = (5x - 1)(5x + 1)$$

The first terms are either 25x and xor 5x and 5x. Use intuition to try one of these pairs and check if the outer and inner terms add to 0x. It checks: 5x + (-5x) = 0x

$$\therefore 0 = (5x-1)(5x+1)$$

$$5x-1 = 0 \quad \text{or} \quad 5x+1 = 0$$

$$5x = 1 \quad 5x = -1$$

$$x = \frac{1}{5} \quad x = -\frac{1}{5}$$

Method 2: Using square roots

2

To find the *x*-intercepts, we solve for *x* when f(x) = 0. $25x^2 - 1 = 0$

$$5x^{2}-1+1 = 0+1$$

$$25x^{2} = 1$$

$$\frac{25x^{2}}{25} = \frac{1}{25}$$

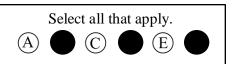
$$x^{2} = \frac{1}{25}$$

$$\sqrt{x^{2}} = \pm \sqrt{\frac{1}{25}}$$

$$x = \pm \frac{1}{5}$$

1b' You try:

Which of the following could be part of the process of finding the roots of $y = x^2 - 4x - 60$?



A)
$$0 = (x+10)(x-6)$$

B)
$$0 = (x - 10)(x + 6)$$

C)
$$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-60)}}{2(1)}$$

D)
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-60)}}{2(1)}$$

E)
$$x^2 - 4x + (-4)^2 = 60 + (-4)^2$$

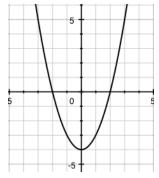
F)
$$x^2 - 4x + (-2)^2 = 60 + (-2)^2$$

Benchmark 3 Study Guide

2′

You try:

Answer the following questions about the graph of the function g(x) shown below:



a) What are the y-intercept(s)?

The *y*-intercept is where the graph crosses the *y*-axis, in this case at the point (0, -4) or y = -4.

b) What are the *x*-intercept(s)?

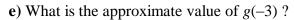
The x-intercept is where the graph crosses the x-axis, in this case at the points (-2, 0) and (2, 0) or x = -2 or 2.

c) How has this graph transformed from the mother function, $f(x) = x^2$?

g(x) is shifted 4 units down from $f(x) = x^2$ (which means the constant term is -4).

d) Is the average rate of change from x = 0 to x = 2negative, positive, zero, or undefined?

If we look only at the portion of the graph between x = 0 and x = 2, we can see that the graph is increasing. This means that the average rate of change on this interval is positive.

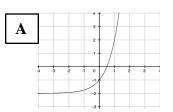


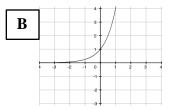
We are looking at the graph where x = -3 and trying to determine what the y-value is at that point. We can approximate that $g(-3) \approx 5$.

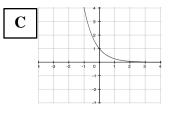
f) Is g(x) increasing or decreasing on $-2 \le x \le 0$?

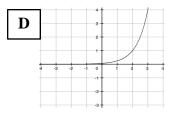
If we look only at the portion of the graph between x = -2 and x = 0, we can see that the graph is decreasing.

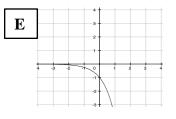
3 You try: Match the function with its graph.
1.
$$f(x) = 4^{x}$$
 B **4.** $f(x) = 4^{x} - 2$ A
2. $f(x) = 4^{-x}$ C **5.** $f(x) = 4^{x-2}$ D
3. $f(x) = -4^{x}$ E













You try:

1) Given the function, $f(x) = -x^2 + 4x - 1$ find the average rate of change from x = 1 to x = 5.

To do this, I need to evaluate the function at
$$x = 1$$
 and $x = 5$.

$$f(1) = -(1)^{2} + 4(1) - 1 \qquad f(5) = -(5)^{2} + 4(5) - 1$$

$$f(1) = -1 + 4 - 1 \qquad f(5) = -25 + 20 - 1$$

$$f(1) = 2 \qquad f(5) = -6$$

Now, I can substitute those values into the formula.

$$\frac{f(b) - f(a)}{b - a}$$

$$= \frac{(-6) - (2)}{5 - 1}$$
The average rate of change of $f(x)$

$$= \frac{-8}{4}$$
from $x = 1$ to $x = 5$ is -2 .

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4a´ ^{You try:} cont'd

2) Given the function, g(x) = 3^x find the average rate of change from x = 1 to x = 3.
To do this, I need to evaluate the function at x = 1 and x = 3.

 $g(1) = 3^1$ $g(3) = 3^3 = 3 \times 3 \times 3$

0()	8
g(1) = 3	g(3) = 27

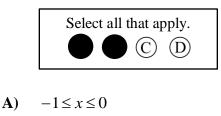
Now, I can substitute those values into the formula.

 $\frac{g(b) - g(a)}{b - a}$ $= \frac{27 - 3}{3 - 1}$ $= \frac{24}{2}$ The average rate of change of g(x)from x = 1 to x = 3 is 12.

4b[′] You try:

$$f(x) = 3x - 1 \qquad g(x) = 2^x$$

Given the functions above, select *all* of the following intervals on which the average rate of change of f(x) is **greater** than the average rate of change of g(x).



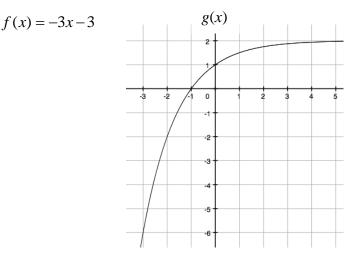
- **B**) $0 \le x \le 3$
- $\mathbf{C}) \qquad 1 \le x \le 3$
- $\mathbf{D}) \qquad 2 \le x \le 3$

5 You Try:

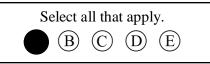
Two functions, f(x) and g(x) are represented below.

Function 1:

Function 2:

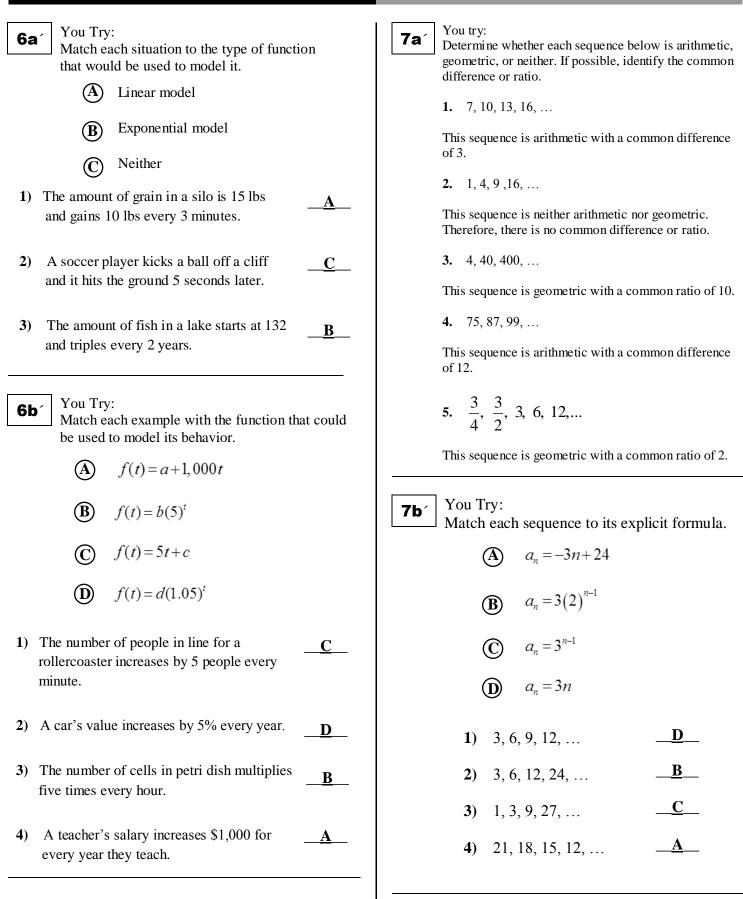


Select *all* of the following statements that are TRUE about the functions above.



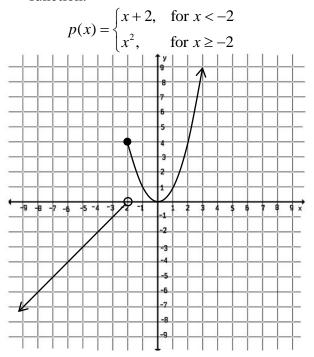
- A) f(x) and g(x) have the same x-intercept(s).
- **B**) f(x) and g(x) have the same y-intercept(s).
- C) f(x) and g(x) have the same average rate of change between x = -3 and x = -1.
- **D**) Both functions are decreasing everywhere.
- **E**) Both functions have a maximum at (0, 2)

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You try: Graph the following piecewise-defined function.



a) What is p(-2)? Why? This is the boundary point so you have to pay close attention to which piece of the function includes x = -2. I can see from the graph that p(x) has a closed circle at the point (-2, 4), therefore, p(-2) = 4. I could also evaluate the function at -2 to show that it would be 4.

b) Is p(x) increasing, decreasing, or neither between x = -2 and x = 0? Explain your

We call to the piece of the graph from x = -2 to x = 0 and see that the graph is decreasing.

