Grade Level/Course: Grades 6-7
Lesson/Unit Plan Name: Solving One-Step Linear Equations Multiple Ways
Rationale/Lesson Abstract: Semi-Concrete pictures and number lines are used to foster a deeper conceptual understanding of solving one-step equations.
Timeframe: 60 min.
Common Core Standard(s): 6.EE.6- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; Understand that a variable can represent an unknown number. 6.EE.7- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which $p,q,$ and x are all nonnegative rational numbers.

<div align="center">Warm-Up: SR Practice Item</div>

Grade 6 Selected Response   *6.EE.3*

For answer choices A-E, bubble in "Yes" if the choice represents the expression $2(x+4)$ and "No" if it does not.

A) $(x+4)(x+4)$ ◯ Yes ◯ No

B)   2
$$\overset{x \quad + \quad 4}{\boxed{|\!|\!|\!|}}$$
                                                      ◯ Yes ◯ No

C) $2x+4$ ◯ Yes ◯ No

D)   ▭ ▢▢▢▢   ▭ ▢▢▢▢   ◯ Yes ◯ No

E) $2x+8$ ◯ Yes ◯ No

Expressions and Equations   **6.EE**
Apply and extend previous understandings of arithmetic to algebraic expressions.   3. Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.

Warm-Up Scoring and Distractor Analysis:

Scoring:

2 Pts = Student selects "Yes" for B, D, E **and** "No" for A,C
1 Pt. = Student selects "Yes" for B,D or B,E or D,E **and** "No" for A,C
0 Pts= Student selected "Yes" for A or C along with any other correct responses

Key and Distractor Analysis:
A. Demonstrates misunderstanding of the distributive property of multiplication and/or property of exponents as the repeated multiplication of a base.

B. **Key:** Demonstrates understanding of distributive property of multiplication when applied to an area model.

C. Demonstrates misunderstanding of the distributive property of multiplication.

D. **Key:** Demonstrates understanding of multiplication as repeated addition of terms when applied to algebra tiles.

E. **Key:** Demonstrates understanding of distributive property of multiplication.
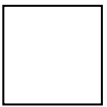
Instructional Resources/Materials: Pencil, Paper, Copies of page 9

Activity/Lesson:

* In all "I Do" examples, visual approaches will be shown twice to highlight the progression of the approach. Students may use one picture or number line to show each visual approach.
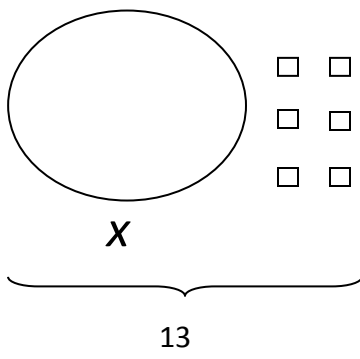
Example 1 (I Do):

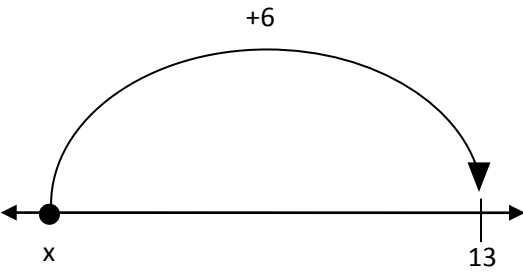| **Draw a Picture**	**Number Line**	**Traditional**
□ = 1 ▯ = 10 ▭ = 100	$x + 6 = 13$	$x + 6 = 13$
$x + 6 = 13$	"We are adding a positive value to x so we are increasing in value. We move right on the number line".	$x + 6 = 13$ $x + 6 - 6 = 13 - 6$ $x + 0 = 7$ $x = 7$
"A quantity x plus three is thirteen. Let's draw a picture."	(Begin on left)	"How do we isolate the variable x?"
	+6	(Subtract 6)
X		"By using the additive inverse of 6, we can create a zero pair".
13	x 13	"How do we balance the equation?"
*Final Picture on Page 3	"Where do we end up when we add 6 to x?" (13)	(subtract 6 from 13)

Activity/Lesson continued: Example 1 (I Do- Continued)

Draw a Picture

$\square$ =1   ▯ =10   ▢ =100
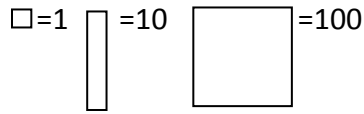
"Remember, x is the difference between 13 and 6. How can we figure out what x is?"

(subtract 6 from 13)
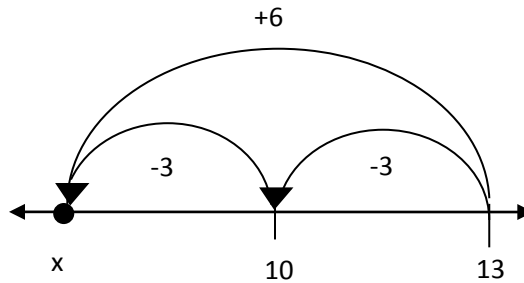
X
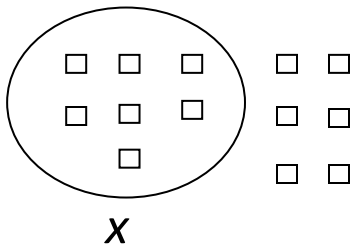
13

$\therefore \; x = 7$

Number Line

$x + 6 = 13$

+6

-3   -3

x   10   13

"If we added 6 to x to get to 13, how do we get back to x?"

(Move left 6 or subtract 6 from 13)

"Where do we end up?"

(7)

$\therefore \; x = 7$

Traditional

$x + 6 = 13$
$x + 6 - 6 = 13 - 6$
$x + 0 = 7$
$x = 7$

"What is the value of the variable x?"

(7)

Substitute and check:

$x + 6 = 13$
$(7) + 6 = 13$

"The value x = 7 makes this statement true."

Example 2 (We Do)

Draw a Picture

$x + 132 = 453$

100 100 100

100

100

X
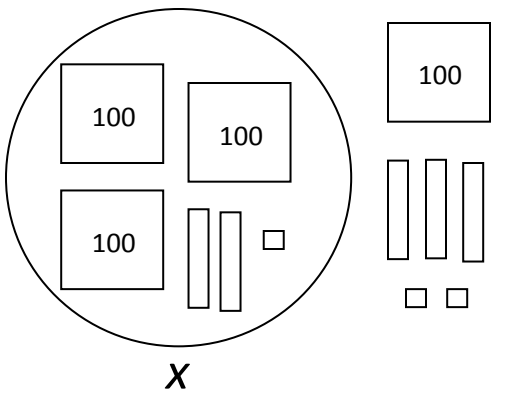
453

$\therefore \; x = 321$

Number Line

$x + 132 = 453$

+132

-2   -30   -100

x   323   353   453
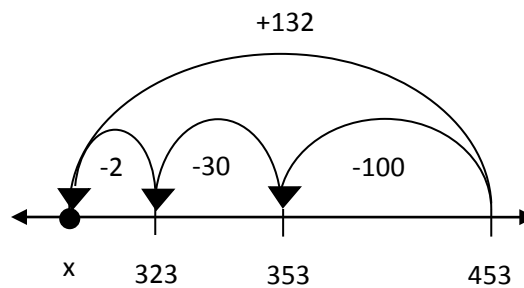
$\therefore \; x = 321$

Traditional

$x + 132 = 453$
$x + 132 - 132 = 453 - 132$
$x + 0 = 321$
$x = 321$

Substitute and check:

$x + 132 = 453$
$(321) + 132 = 453$

"The value x = 321 makes this statement true."
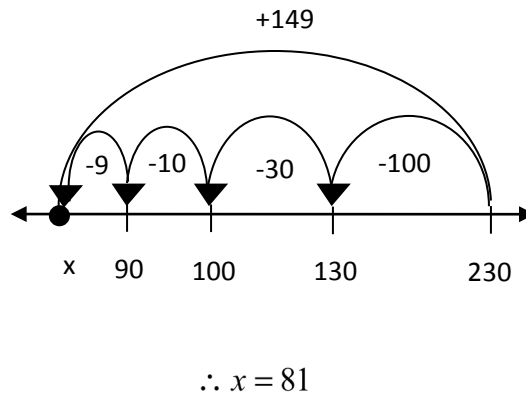
Activity/Lesson continued: Example 3 (You Do)

Draw a Picture	Number Line	Traditional

Draw a Picture

$$x + 149 = 230$$

$$230$$
$$\therefore x = 81$$
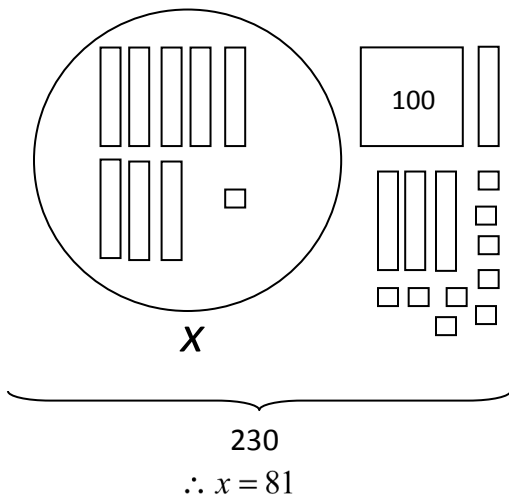
Number Line

$$x + 149 = 230$$

+149

-9 -10 -30 -100

x   90   100   130   230

$$\therefore x = 81$$

Traditional

$$x + 149 = 230$$
$$x + 149 - 149 = 230 - 149$$
$$x + 0 = 81$$
$$x = 81$$

Substitute and check:

$$x + 149 = 230$$
$$(81) + 149 = 230$$
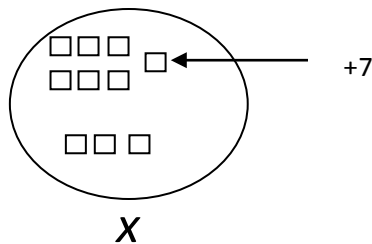

Example 4: (I Do)

Draw a Picture

$$x - 7 = 3$$

"Seven less than a quantity x is three. Let's draw a picture."

-7
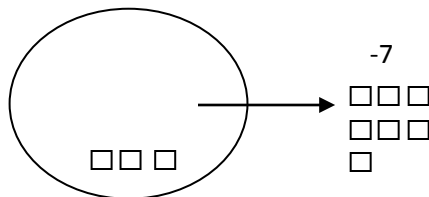
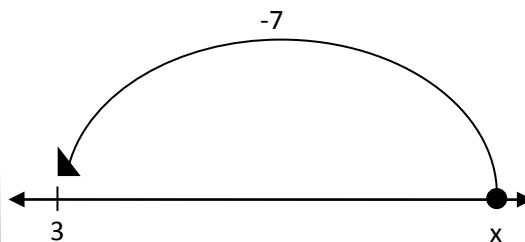"After subtracting 7 from x we had three left . Let's add the 7 back to x".

+7

X

"Now we can see that x was 10 before 7 was subtracted."
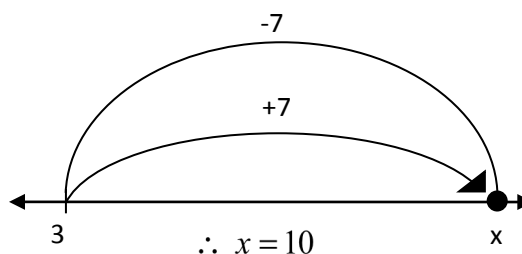
$$\therefore x = 10$$

Number Line

$$x - 7 = 3$$

"We are subtracting a positive value from x so we are decreasing in value. We move left on the number line".   (Begin on right)

-7

3   x

"Where do we end up?" (3) "How do we get back to X?" (move right 7/add 7 to 3). "Where do we land?" (10)

-7

+7

3   x

$$\therefore x = 10$$

Traditional

$$x - 7 = 3$$
$$x - 7 + 7 = 3 + 7$$
$$x + 0 = 10$$
$$x = 10$$

"Remember that we can use the additive inverse of -7 to make zero and isolate x. By adding 7 to both sides, we balance the equation and can solve for x".
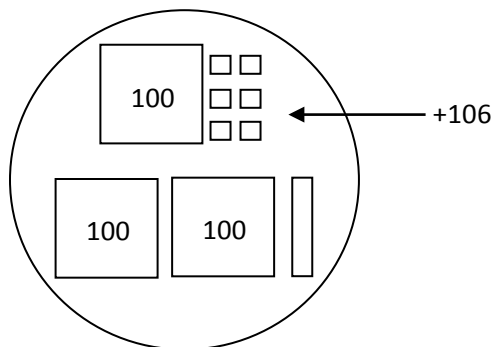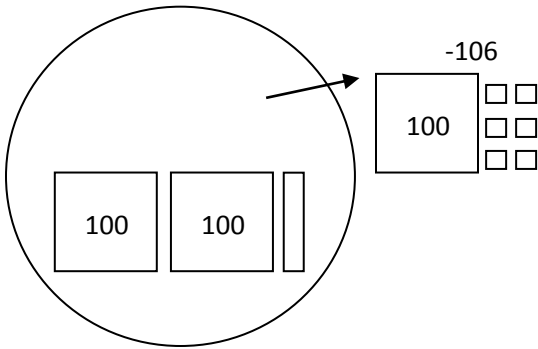
Substitute and check:

$$x - 7 = 3$$
$$(10) - 7 = 3$$

"The value x = 10 makes this statement true."

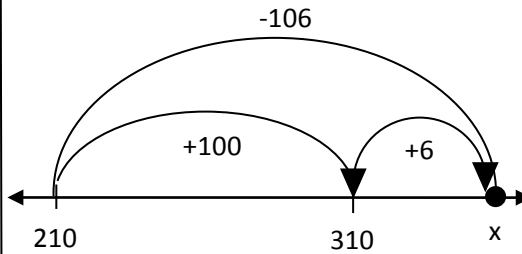**Activity/Lesson continued: Example 5 (Partner Do)**

Draw a Picture	Number Line	Traditional
$x - 106 = 210$	$x - 106 = 210$	$x - 106 = 210$

Draw a Picture:

$x - 106 = 210$

-106

100 (□□ □□ □□)

100 100

100 (□□ □□ □□) +106

100 100

X

$\therefore \ x = 316$

Number Line:

$x - 106 = 210$

-106

+100 +6

210   310   x

$\therefore \ x = 316$

*It may be necessary for students to draw more or fewer pictures/number lines at this point depending on their level of understanding.

Traditional:

$x - 106 = 210$
$x - 106 + 106 = 210 + 106$
$x + 0 = 316$
$x = 316$

Substitute and check:

$x - 106 = 210$
$(316) - 106 = 210$

"The value x = 316 makes this statement true."

Example 6 (You Do)

Draw a Picture	Number Line	Traditional

Draw a Picture:

$x - 13 = 16$

-13

+13
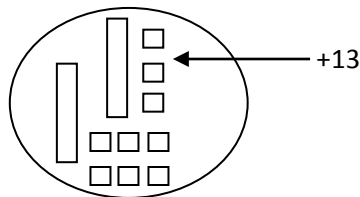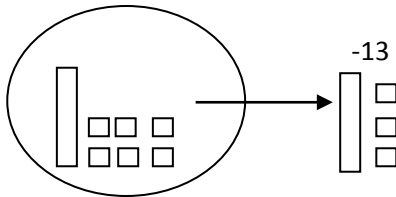
X
$\therefore \ x = 29$

Number Line:

$x - 13 = 16$

-13

+10 +3

16   26   x

$\therefore \ x = 29$
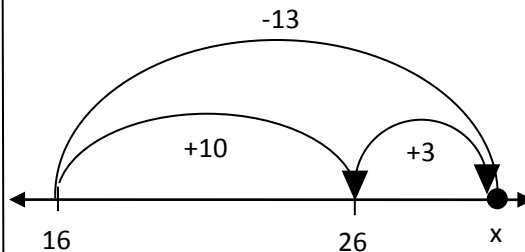
Traditional:

$x - 13 = 16$
$x - 13 + 13 = 16 + 13$
$x + 0 = 29$
$x = 29$

Substitute and check:

$x - 13 = 16$
$(29) - 13 = 16$

"The value x = 29 makes this statement true."

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Activity/Lesson continued: Example 7 (I Do)

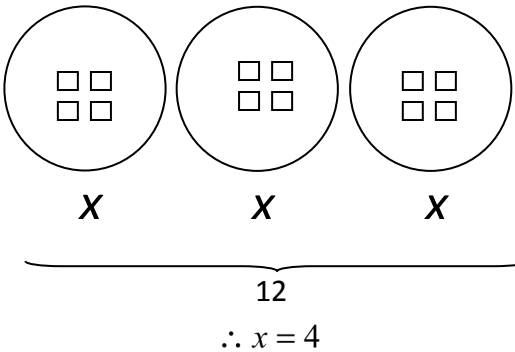| **Draw a Picture**	**Number Line**	**Traditional**
$3x = 12$	$3x = 12$	$3x = 12$

Draw a Picture

$3x = 12$

"Three times a quantity x is 12. Another way to think about it is 3 groups of x equal 12. Let's draw it."

$X \qquad X \qquad X$

12
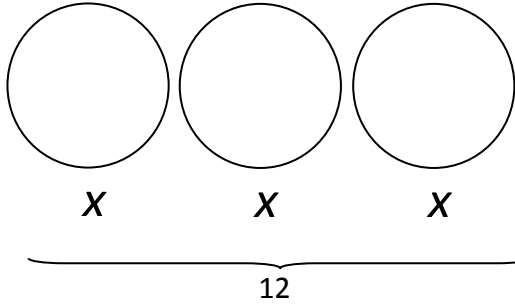
"We can see what 1 group of x is by dividing 12 evenly between all 3 groups. What does 1 groups of x equal?" (4)
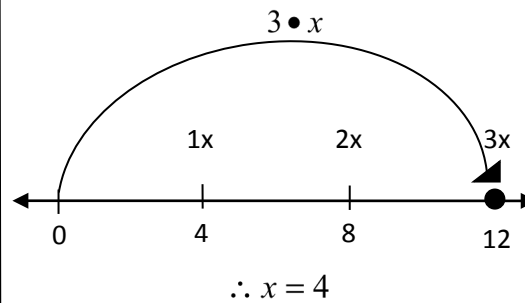

$X \qquad X \qquad X$

12

$\therefore x = 4$

Number Line

$3x = 12$

"Three times a number x is equal to 12. Since we are multiplying x by a positive value and getting a positive answer, we are increasing in value".   (Move right)

$3 \bullet x$

1x   2x   3x

12
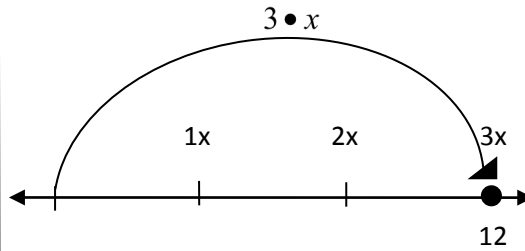
"If three times x is 12, how do we find the value of x?" (Divide 12 by 3) "What is the value of X?" (4)

$3 \bullet x$

1x   2x   3x

0   4   8   12

$\therefore x = 4$

Traditional

$3x = 12$

$$\frac{3x}{3} = \frac{12}{3}$$

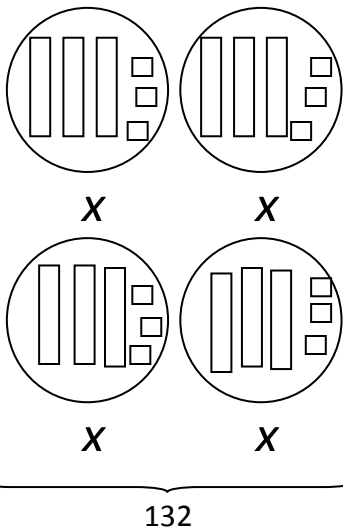$1 \bullet x = 4$

$x = 4$

Substitute and check:

$3x = 12$

$3 \bullet (4) = 12$

"The value x = 4 makes this statement true."

Example 8 (You Do)

Draw a Picture

$4x = 132$

$X \qquad X$

$X \qquad X$

132

$\therefore x = 33$

Number Line

$4x = 132$

$4 \bullet x$

1x   2x   3x   4x

0   33   66   99   132

$\therefore x = 33$
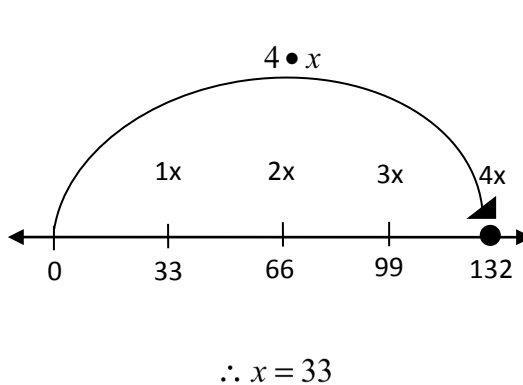
Traditional

$4x = 132$

$$\frac{4x}{4} = \frac{132}{4}$$

$1 \bullet x = 33$

$x = 33$
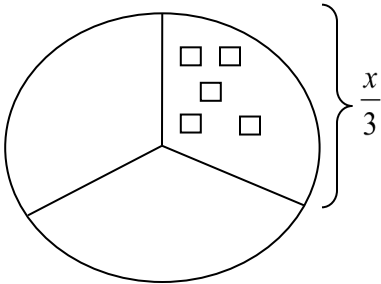
Substitute and check:

$4x = 132$

$4 \bullet (33) = 132$

"The value x = 33 makes this statement true."

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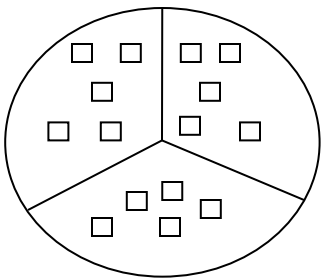
Draw a Picture

$$\frac{x}{3} = 5$$

"When x is divided by 3, each part equals 5."

$$\frac{x}{3}$$

X

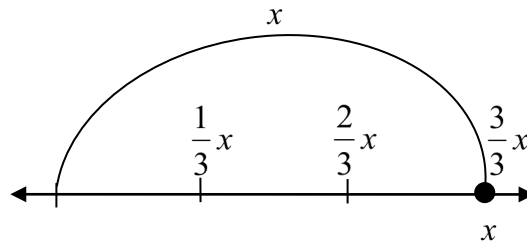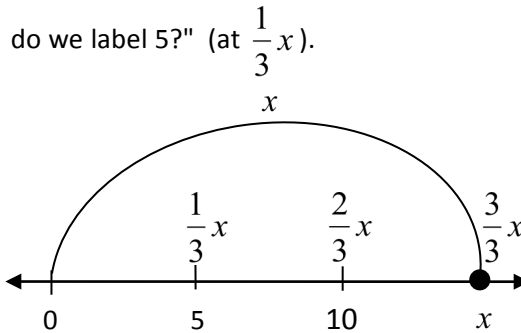"We can now recompose x. What is the value of x?"(15)

X

$$\therefore x = 15$$

Number Line

$$\frac{x}{3} = 5$$

" A number x divided by 3 is 5. Let's show x as being divided into three equal distances on the number line ".

$$\frac{1}{3}x \qquad \frac{2}{3}x \qquad \frac{3}{3}x$$

x

"The distance is 5 for each part of x. If our number line started from zero, where do we label 5?" (at $\frac{1}{3}x$).

$$\frac{1}{3}x \qquad \frac{2}{3}x \qquad \frac{3}{3}x$$

0   5   10   x

$$\therefore x = 15$$

Traditional

$$\frac{x}{3} = 5$$

$$3 \bullet \frac{x}{3} = 5 \bullet 3$$

$$\frac{x}{1} = 15$$

$$x = 15$$

Substitute and check:

$$\frac{x}{3} = 5$$

$$\frac{(15)}{3} = 5$$

"The value x = 15 makes this statement true."

Example 10 (You Do)

Draw a Picture

$$x \div 4 = 144$$

| 100 | 100 |

| 100 | 100 |
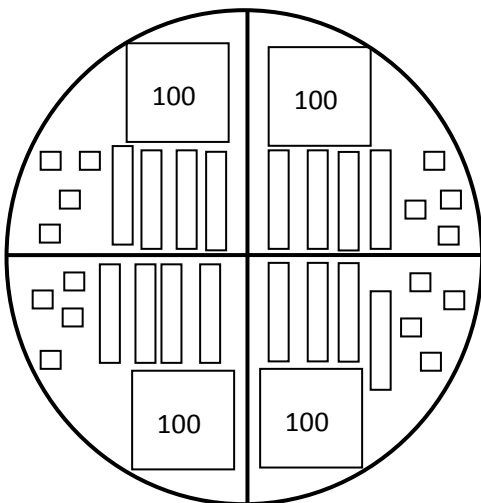
X

$$\therefore x = 576$$

Number Line

$$x \div 4 = 144$$

x

$$\frac{1}{4}x \qquad \frac{2}{4}x \qquad \frac{3}{4}x \qquad \frac{4}{4}x$$

0   144   288   432   x
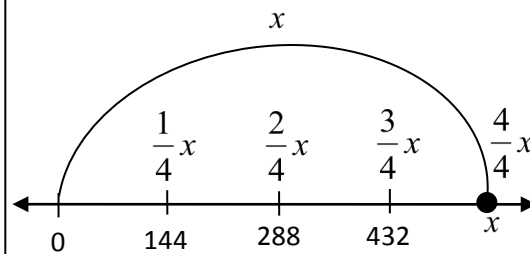
$$\therefore x = 576$$

Traditional

$$\frac{x}{4} = 144$$

$$4 \bullet \frac{x}{4} = 144 \bullet 4$$

$$\frac{x}{1} = 576$$

$$x = 576$$

Substitute and check:

$$\frac{x}{4} = 144$$

$$\frac{(576)}{4} = 144$$
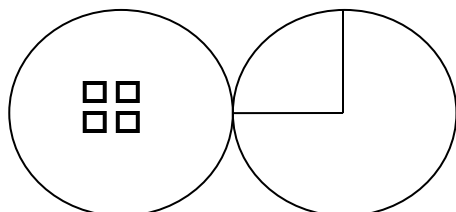
"The value x = 576 makes this statement true."

Activity/Lesson continued: Example 11 (Challenge/Extension)

Draw a Picture	Number Line	Traditional

Draw a Picture

$$x \div 1\frac{1}{4} = 4$$
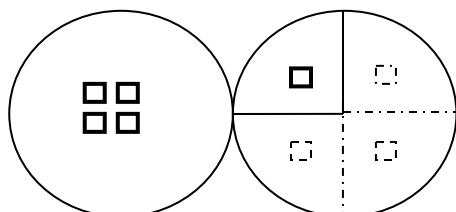
"The quotient 4 represents one whole group. Let x be the total amount before being divided."

Quotient =1 **WHOLE** group $+ \frac{1}{4}$ group $= 1\frac{1}{4}$ groups

$\underbrace{\qquad\qquad}_{X}$

"If one whole group of the divisor is 4, then one fourth group is 1. What is all of x?" (5)

Quotient = 1 **WHOLE** group $+ \frac{1}{4}$ group $= 1\frac{1}{4}$ groups
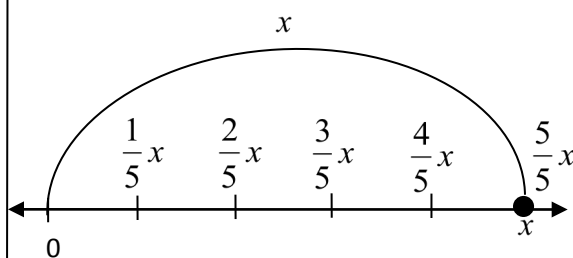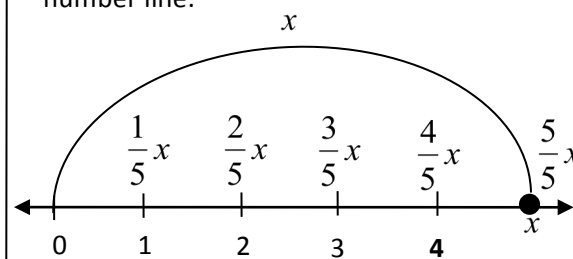
$\underbrace{\qquad\qquad}_{X}$

$$\therefore x = 5$$

Number Line

$$x \div 1\frac{1}{4} = 4$$

$$x \div \frac{5}{4} = 4$$

"Let us show the five, one fourth parts that we are dividing x into."

x

$\frac{1}{5}x \quad \frac{2}{5}x \quad \frac{3}{5}x \quad \frac{4}{5}x \quad \frac{5}{5}x$

0

"We need to place the quotient on the number line at four fifths of x because this represents one WHOLE group of our divisor. Now we can label the rest of our number line."

x

$\frac{1}{5}x \quad \frac{2}{5}x \quad \frac{3}{5}x \quad \frac{4}{5}x \quad \frac{5}{5}x$

0   1   2   3   **4**

Divisor	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\boxed{\frac{4}{4}}$	$\frac{5}{4}$
Groups					

$$\therefore x = 5$$

Traditional

$$x \div \frac{5}{4} = 4$$

$$\frac{x}{\frac{5}{4}} = 4$$

$$\frac{x}{\frac{5}{4}} \cdot \frac{5}{4} = 4 \cdot \frac{5}{4}$$

$$\frac{x}{1} = \frac{20}{4}$$

$$x = 5$$

Substitute and check:

$$\frac{x}{\frac{5}{4}} = 4$$

$$\frac{(5)}{\frac{5}{4}} = 4$$

$$5 \cdot \frac{4}{5} = 4$$

"The value x = 5 makes this statement true."

Assessment: Mark "Yes" for approaches that could be used to solve the equation $3x = 396$ and "No" for approaches that cannot.

A)

$\underbrace{X \quad X \quad X}_{396}$

○ Yes   ○ No

B) $x + x + x = 132 + 132 + 132$

○ Yes   ○ No

C) $x = \dfrac{396}{3}$

○ Yes   ○ No

D)

$3x$

0   15

○ Yes   ○ No

Grade 6 Selected Response   *6.EE.3*

For answer choices A-E, bubble in "Yes" if the choice represents the expression $2(x+4)$ and "No" if it does not.

A) $(x+4)(x+4)$ ◯Yes ◯No

$$x + 4$$

B)   2 [diagram]   ◯Yes ◯No

C) $2x+4$ ◯Yes ◯No

D)   [diagram of boxes]   ◯Yes ◯No
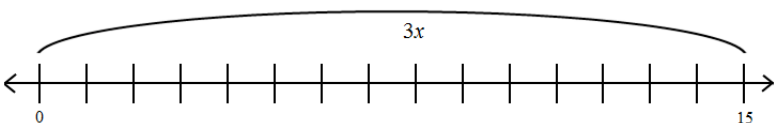
E) $2x+8$ ◯Yes ◯No

Assessment: Mark "Yes" for approaches that could be used to solve the equation $3x = 396$ and "No" for approaches that cannot.

A)   [three circles labeled X X X, bracket 396]   ◯ Yes   ◯ No

B) $x+x+x = 132+132+132$ ◯ Yes   ◯ No

C) $x = \dfrac{396}{3}$ ◯ Yes   ◯ No

D)   [number line from 0 to 15 labeled $3x$]   ◯ Yes   ◯ No