

# Variables- Discovering the Unknown

## Lesson One:

Sometimes in math we have to work with an unknown quantity or amount. When you were younger you might have had a problem like: *Three birds sat in a tree. Some more birds came. Then there were five.*

How many birds came? [2]

Your teacher probably drew a picture, and your book wrote a number sentence like this:

$$3 + \blacksquare = 5$$

You had to find the missing addend. Tell your neighbor how you found the answer.

What are some of the strategies you and your neighbor discussed?

Record student response on the board. [fact family, know my facts, draw a picture, use a model, subtract, work backwards]

When we have an unknown quantity in an expression or an equation, we can show it using a variable. A variable is a letter or symbol standing in the place of an unknown amount.

A variable is a letter or symbol that represents an unknown amount. Write this definition in your notes.

Today we are going to work with some problems that have one variable. We could rewrite our bird equation with a variable like this:

$$3 + b = 5$$

$b$  is our variable. It stands for the unknown number of birds that flew to the tree.

Show me with a thumbs-up or a thumbs-down if you can still find the missing value. Can you use the same strategies that we listed on the board? [yes]

What is the value of  $b$ ? [2]

**You Try:**  $n - 9 = 8$

Find the value for  $n$ .

Who can explain a way you found  $n$ ?

The student may have used a model or inverse operations to isolate the variable. Record as the student describes the steps of work. Use proper syntax and have the student confirm they used proper syntax as well.

### Inverse Operations

$$\begin{aligned} n - 9 &= 8 \\ n - 9 + 9 &= 8 + 9 \\ n &= 17 \end{aligned}$$

$n$	
9	8

### Bar Model

Jamal set the table for some guests. He put 3 pieces of silverware at each seat. Jamal used 24 pieces of silverware. How many people sat at the table?

We know that each person got 3 pieces of silverware. What operation can we use here? [multiplication].

How many pieces of silverware did Jamal use? [24]

So 3 pieces of silverware times the number of people is 24. Let's write that as an equation. We can use  $p$  as the variable for people.

$$3 \cdot p = 24$$

How can we find the value of  $p$ ? [divide, fact family, model ]

### Inverse Operation

$$\begin{aligned} 3 \cdot p &= 24 \\ \frac{3 \cdot p}{3} &= \frac{24}{3} \\ p &= 8 \end{aligned}$$

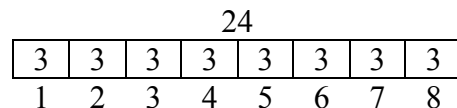
### Decomposition

$$\begin{aligned} 3 \cdot p &= 24 \\ 3 \cdot p &= 3 \cdot 8 \\ p &= 8 \end{aligned}$$

or

$$\begin{aligned} 3 \cdot p &= 24 \\ p + p + p &= 8 + 8 + 8 \\ \therefore p &= 8 \end{aligned}$$

### Bar Model



**You Try:**  $d + 29 = 54$

What is the unknown? [ $d$ ]

How did you find the value of  $d$ ? [subtract, work backward, make a model]

<b>Inverse Operation</b>	<b>Decomposition</b>	<b>Bar Model</b>		
$d + 29 = 54$	$d + 29 = 54$	<div style="display: flex; align-items: center; justify-content: center;"> <table border="1" style="border-collapse: collapse; margin-right: 10px;"> <tr> <td style="padding: 5px;"><math>d</math></td> <td style="padding: 5px;"><math>29</math></td> </tr> </table> <div style="text-align: right;"> <math>54</math>  <math>\frac{-29}{25}</math> </div> </div>	$d$	$29$
$d$	$29$			
$d + 29 - 29 = 54 - 29$	$d + 29 = 24 + 30$			
$d = 25$	$d + 29 = 25 + 29$ $d = 25$			

In this decomposition model you use the concept of equals added to equals are equal. The '+29' on the left and right sides of the equation is no longer needed, leaving  $d=25$

**You Try:**

Try this next one on your own. When you finish, share your work with a neighbor. If you both used the same strategy, solve it again using a different strategy.

$$g \div 6 = 7$$

What were some ways you found the value of  $g$ ?

<b>Inverse Operation</b>	<b>Bar Model</b>						
$g \div 6 = 7$	<div style="display: flex; align-items: center; justify-content: center;"> <table border="1" style="border-collapse: collapse; margin-right: 10px;"> <tr> <td style="padding: 5px;"><math>7</math></td> <td style="padding: 5px;"><math>7</math></td> <td style="padding: 5px;"><math>7</math></td> <td style="padding: 5px;"><math>7</math></td> <td style="padding: 5px;"><math>7</math></td> <td style="padding: 5px;"><math>7</math></td> </tr> </table> <div style="text-align: right;"> <math>6 \cdot 7 = 42</math>  <math>\therefore g = 42</math> </div> </div>	$7$	$7$	$7$	$7$	$7$	$7$
$7$		$7$	$7$	$7$	$7$	$7$	
$g \div 6 \cdot 6 = 7 \cdot 6$							
$g = 42$							

**You Try:**  $m \cdot 4 = 84$

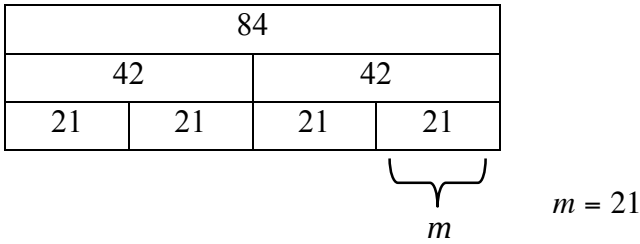
What is the inverse operation of multiplication? [division]

Raise your hand if you used division to find  $m$ .

Inverse Operation	$m \cdot 4 = 84$
	$m \cdot 4 \div 4 = 84 \div 4$
	$m = 21$

Did anyone do it another way? [model, decomposing]

### Bar Model



Here 84 is broken first into 2 groups. Then the two are broken into 4. Showing this model reminds students that doubling and halving are useful ways to deal with larger numbers. The model below is similar.

### Decomposing

$$m \cdot 4 = 84$$

$$m + m + m + m = 80 + 4$$

$$m + m + m + m = 20 + 20 + 20 + 20 + 1 + 1 + 1 + 1$$

$$m + m + m + m = (20 + 1) + (20 + 1) + (20 + 1) + (20 + 1)$$

$$m = 20 + 1$$

$$m = 21$$

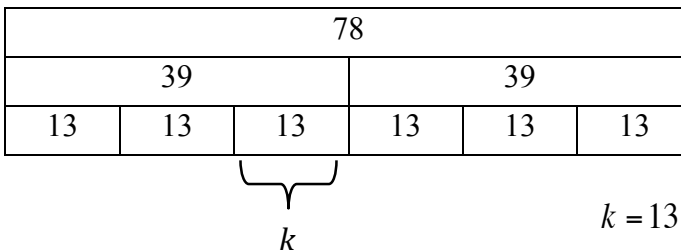
Pass out index cards.

This is your exit card. Put your name on it. Show two ways to find the value of the variable in this problem. Label your strategies. Hand your card to me at the end of the lesson.

$$k \cdot 6 = 78$$

Some Possible Solutions:

### Bar Model 1



### Bar Model 2

6	12	18	24	30	36	42	48	54	60	66	72	78
6	6	6	6	6	6	6	6	6	6	6	6	6
1	2	3	4	5	6	7	8	9	10	11	12	13

$$k = 13$$

### Inverse Operations with Decomposition

$$\begin{aligned}k \cdot 6 &= 78 \\k \cdot 6 \div 6 &= 78 \div 6 \\k &= (60 + 18) \div 6 \\k &= 60 \div 6 + 18 \div 6 \\k &= 10 + 3 \\k &= 13\end{aligned}$$

### Inverse Operations

$$\begin{aligned}k \cdot 6 &= 78 \\ \frac{k \cdot 6}{6} &= \frac{78}{6} \\ k &= \frac{2 \cdot 3 \cdot 13}{2 \cdot 3} \\ k &= 13\end{aligned}$$

### Decomposition

$$\begin{aligned}k \cdot 6 &= 78 \\k \cdot 2 \cdot 3 &= 2 \cdot 39 \\k \cdot 3 &= 39 \\k \cdot 3 &= 13 \cdot 3 \\k &= 13\end{aligned}$$

### Inverse Operations

$$\begin{aligned}k \cdot 6 &= 78 \\k \cdot 6 \div 6 &= 78 \div 6 \\k &= 13\end{aligned}$$
$$\begin{array}{r} \phantom{0}13 \\ 6 \overline{)78} \\ \underline{-6} \phantom{0} \\ 18 \\ \underline{-18} \\ 0 \end{array}$$

# Variables- Discovering the Unknown

## Lesson 2

So far we have been working with equations that have an unknown quantity. We have been able to find a value for the variable because the expressions on both sides of the equation are equal. Sometimes we use an expression with a variable as a formula to test different values. When you substitute a value for the variable you are *evaluating* the expression.

$$4n$$

What could  $4n$  mean? [students will have different ideas]

In math when we are multiplying with a variable, we do not put a multiplication symbol between the value and the variable. The value goes in front of the variable, so  $4n$  means 4 times  $n$ .

What does  $3m$  mean? [3 times  $m$ ]

What does  $12w$  mean? [12 times  $w$ ]

What does  $7r$  mean? [7 times  $r$ ]

We will evaluate  $4n$  twice. We will test for  $n = 7$  and  $n = 11$ .

$$\text{let } n = 7$$

$$\begin{aligned} 4n \\ = 4 \cdot 7 \\ = 28 \end{aligned}$$

$$\text{let } n = 11$$

$$\begin{aligned} 4n \\ = 4 \cdot 11 \\ = 44 \end{aligned}$$

When  $n = 7$ ,  $4n$  has a value of 28. When  $n = 11$ ,  $4n$  has a value of 44.

Could I have used a bar model to test these values? [yes]

$4n$				
$n$	$n$	$n$	$n$	11
11	11	11	11	11
				11
				+11
				44

I want you to test our expression two times on your own.

**You Try:**            Let  $n = 9$

Let  $n = 30$

Did you label your two tests of the expression? Did you show your work for each step? Did you line up your work using good syntax? Did anyone use a bar model to evaluate the expression?

**Let  $n = 9$**

$$\begin{aligned} 4n \\ = 4 \cdot 9 \\ = 36 \end{aligned}$$

**Bar Model**

$n$	$n$	$n$	$n$
9	9	9	9

$9 + 9 + 9 + 9 = 36$

**Let  $n = 30$**

$$\begin{aligned} 4n \\ = 4 \cdot 30 \\ = 120 \end{aligned}$$

**Bar Model**

$n$	$n$	$n$	$n$
30	30	30	30

$30 + 30 + 30 + 30 = 120$

The expression  $4n$ , that we have been testing, is an *algebraic expression*. It is not an equation. Expressions name a number. Equations describe the relationship between two expressions. So,  $4n+5$  is also expression, while  $d + 29 = 54$  is an equation.  $d+29$  is an expression; as is 54, but when the two expressions are compared by linking them with an equal sign, they form an equation. Let's add these definitions to our notes.

An algebraic expression names a number. 4,  $4n$ ,  $d+29$

An equation describes the relationship between two expressions. It shows the expressions are equal.  $d+29=54$ ;  $3k-6=21$ ;  $1+7=8$

Is  $3q$  an equation? [no]

Tell your neighbor why not. [It is standing alone. It isn't being compared to another expression.]

What is  $3q$ ? [an expression]

We'll evaluate  $3q$ .

What is  $3q$  when  $q$  equals 2? [6]

What is  $3q$  when  $q$  equals 7? [21]

What is  $3q$  when  $q$  equals 100 [300]

What is  $3q$  when  $q$  equals 4 [12]

$$9 + 6d$$

How many operations are in this expression? [2]

Which operation should we perform first? [Multiplication]

Let  $d = 3$ ,  $d = 8$ , and  $d = 11$

Let  $d = 3$

$$9 + 6d$$

$$= 9 + 6 \cdot 3$$

$$= 9 + 18$$

$$= 27$$

Let  $d = 8$

$$9 + 6d$$

$$= 9 + 6 \cdot 8$$

$$= 9 + 48$$

$$= 57$$

Let  $d = 11$

$$9 + 6d$$

$$= 9 + 6 \cdot 11$$

$$= 9 + 66$$

$$= 75$$

Think of some more expressions for us to try.

Record student examples of algebraic expressions. Discuss and clarify various examples. Clarify and prompt for phrases like “the quantity” when students have multiple operations in their examples.

Choose appropriate student samples to evaluate with different numbers. Review the order of operations.

### You Try:

Evaluate this algebraic expression.  $(6 + h) \div 3$

Let  $h = 12, 18, 27,$  and  $36$

\*Challenge: Use a bar model to test the expression with one of the values of  $h$ .

#### Bar Model

$6 + h$		
$(6 + h) \div 3$	$(6 + h) \div 3$	$(6 + h) \div 3$

$6 + 12$		
$2 + 4$	$2 + 4$	$2 + 4$

$$\underbrace{(6 + h) \div 3}_6$$

#### Simplify the Expression

$$(6 + h) \div 3$$

$$\text{Let } h = 12$$

$$(6 + h) \div 3$$

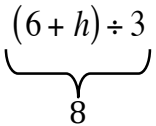
$$= (6 + 12) \div 3$$

$$= 18 \div 3$$

$$= 6$$



6+18		
2+6	2+6	2+6

$(6+h) \div 3$   


Let  $h = 18$

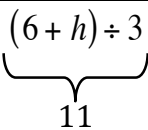
$$(6+h) \div 3$$

$$= (6+18) \div 3$$

$$= 24 \div 3$$

$$= 8$$

6+27		
2+9	2+9	2+9

$(6+h) \div 3$   


Let  $h = 27$

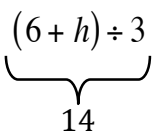
$$(6+h) \div 3$$

$$= (6+27) \div 3$$

$$= 33 \div 3$$

$$= 11$$

6+36		
2+12	2+12	2+12

$(6+h) \div 3$   


Let  $h = 36$

$$(6+h) \div 3$$

$$= (6+36) \div 3$$

$$= 42 \div 3$$

$$= (30+12) \div 3$$

$$= (30 \div 3) + (12 \div 3)$$

$$= 10 + 4$$

$$= 14$$

What is a variable? [a letter or symbol that stands for an unknown value in an expression]

How is an expression different from an equation? [It names a number. It does not describe the relationship between two expressions.]

We are going to keep working with variables in our next lesson. You are going to be learning about equations with two variables and how those equations can be used with function tables.