Re-Thinking Fact Families

<u>Objective:</u>

Teachers are given multiple strategies for teaching K-2 students the real Mathematics behind "Fact Families": the commutative property of addition and subtraction, and the inverse relationship of addition and subtraction. Students use of strategies will foster understanding and the development of automaticity that are not based on memorization of procedures, but rather on solid number sense. Strategies used include use of manipulatives, 10-frames, bar models, open number lines and equations.

<u>Standards:</u>

KNS 2.1 Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10).

1NS 1.3 Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions (to 20).

1NS 2.2 Use the inverse relationship between addition and subtraction to solve problems.

2AF 1.1 Use the commutative and associative rules to simplify mental calculations and to check results.

2NS 2.1 Understand and use the inverse relationship between addition and subtraction to solve problems and check solutions.

<u>Two Mathematical Properties:</u>

1) Commutative Property of Addition - The sum of addends is the same, no matter what order the addends are combined in.

$a + b = b + a$

2) Inverse Relationship of Addition and Subtraction - If two addends are combined to find a sum, when one of the addends is subtracted from the sum, the difference will be the other addend.

When $a + b = c$ *then* $c - a = b$ *and* $c - b = a$.

<u>Multiple Strategies for Side-by-Side comparisons of each mathematical property:</u>

* Counters with 10-frames for recording
* Open number lines
* Linking cubes
* Bar Models
* Equations

Warm-up – Rethinking Fact Families

Mutliple choice questions from 2nd grade Released Test Questions, California Standards Test

1

[Sophie did this subtraction problem. Which addition problem shows that she got the right answer?]

$$\boxed{85 - 44 = 41}$$

A $41 + 85$

B $44 + 85$

C $41 + 44$

D $44 + 44$

- Prove your answer is correct using three different methods.

2 NS 2.1

2

[Look at the two problems in the box. The same number is missing in both of them. What is the missing number?]

$$65 - \square = 60$$
$$60 + \square = 65$$

A 125

B 15

C 5

D 0

- Show how a student could show their work using an open number line. 2NS 2.1

3

[Which of these can be used to check the answer to the problem in the box?]

$$\boxed{4 + 3 = 7}$$

A $7 + 3 = 10$

B $7 - 4 = 3$

C $2 + 5 = 7$

D $10 - 3 = 7$

- Explain why you think students would choose the incorrect answers.

2 NS 2.1

4

[Which number sentence is an opposite number sentence for eight plus six equals fourteen?]

$$\boxed{8 + 6 = 14}$$

A $2 + 12 = 14$

B $7 + 7 = 14$

C $8 - 2 = 6$

D $14 - 8 = 6$

- Show two ways students could prove they are correct.

2 NS 2.1

Focus One: Commutative Property of Addition

Standards: KNS 2.1 Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10).

1NS 1.3 Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions.

2AF 1.1 Use the commutative and associative rules to simplify mental calculations and to check results.

Objective: Students use the Commutative Property of Addition to find equivalent equations with common addends and sums.

Students use counters, 10-Frames, Open Number Lines and Bar Models to show their understanding of the Commutative Property.

Materials: Counters - 20 2-sided counters for each partner pair of students and for teacher

10-Frames - blank copies for recording work.

Paper and pencils to record open number lines and bar models.

Variation: Give students blank 10-Frames, and blank number lines in sheet protectors and they show their work with whiteboard markers.

Vocabulary: add, addends, sum, commutative property

Part 1:

Introduction : "Today we're going to find out if the order we add numbers changes the sum when we add them together."

"I do it"

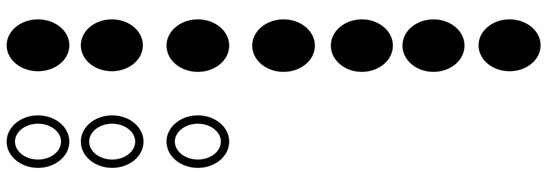
Write equation and show with counters, as you tell the math story.

"Kalil had 7 marbles. He won 3 more marbles. How many marbles does he have now?"

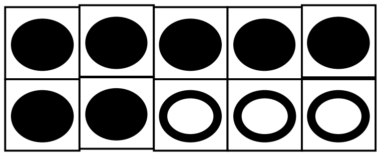
“I can represent his marbles with counters. 7 red counters for the 7 marbles he started with and 3 marbles for the 3 more marbles he won.”

Equation: $7 + 3 =$

Counters:



"We can show this using a 10-frame too" (Draw dark and open dots to show addends.)”I represent his marbles with 7 black dots for the marbles he started with , and 3 open dots for the 3 marbles he won.”



"When I put together 7 black dots and 3 white dots I have 10 dots altogether. Kalil has 10 marbles."

Write the sum.

$7+3=10$

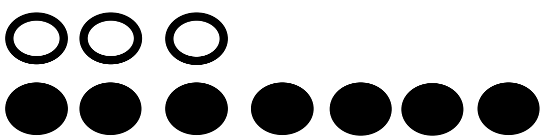
"I wonder what would happen if we changed the order of the numbers? How many marbles will Kalil have then?"

Write and show with counters, then record in a 10-frame as you tell the story.

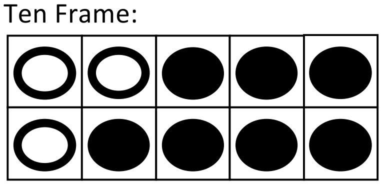
"This time let's say Kalil started with 3 marbles and he won 7 more marbles. How many marbles does he have altogether? I can represent Kalil's marbles with counters. I represent the 3 marbles he started with, with 3 yellow counters, and put together 7 red counters to represent the counters he won. I can also show what happened with a 10-frame. (Use a different 10-frame so you'll have them to compare.) I show 3 white dots and then 7 black dots.

$3+7=$

Counters:



Ten Frame:



"When I put together 3 and 7 they make 10 dots altogether. How many marbles does Kalil have altogether?"

Choral response: [10]

Write the sum.

$3+7=10$

Show 10-frames side by side for students to compare.

"When I look at these 10-frames I see the sum, the answer, is the same. It didn't matter if we started with 7 and added 3, or started with 3 and added 7, we always had 10 when we put them together. Mathematicians call the numbers we add together addends. Say that with me: Numbers we add together are addends.

"It doesn't matter which order we add numbers, or addends in, we always get the same sum. Mathematicians call this the Commutative Property of Addition. It's always true no matter which numbers you are adding."

Have students repeat definition and commutative property with you.

$3+6=9$ and $6+3=9$

"Amparo has 3 bracelets. Her friend gave her 6 more. How many bracelets does she have now?

Lead students to show with counters with you.

"Let's represent Amparo's bracelets with counters. I'm going to use red counters for the bracelets she started with. How many red counters?" Choral Response: [3]

Let's use yellow counters for the bracelets her friend gave her. "How many yellow counters?" Choral Response: [6]

"How many counters altogether? Think. Whisper to your partner. Choral Response:[9]

Lead students to record with 10-frames – either on paper on in clear sleeves with 10-frame graphic as whiteboards.

"We can also represent Amparo's bracelets with 10-frames and dots. How many black dots should we start with?" Choral Response: [3]

"How many white dots should we use?" Choral Response: [6]

"How many dots altogether?" Choral Response: [9]

Write the sum and note it is the same sum. Repeat the definition of the commutative property with students. Have them tell their partner.

You Tries: (Students use counters, 10-frames, and write the equation for each problem.) Give them the two addends, and then have them show both methods and the equation.

Formative Assessment: You can have students work both independently, or work problem 1 with a partner, and partner 2 independently. Walk the room, and/or have them show their work on whiteboard sleeves. Give feedback and more practice if needed. Note students who need additional small group support.

1) $4+1=5$ and $1+4=5$

2) $8+2=10$ and $2+8=10$

Part 2:

Show recorded work examples from previous lesson.

"We showed how the Commutative Property of Addition works with counters, and 10-frames and equations. What does the Commutative Property of Addition say is always true? Think. Whisper to your partner.

Ask non-volunteers to share definition and/or examples. Have class repeat the definition orally.

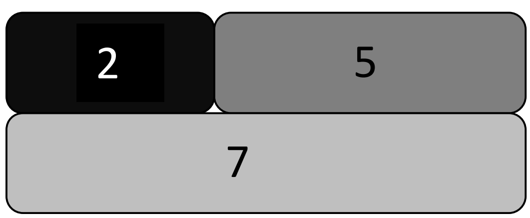
"We can show that the sum of two numbers is the same, no matter what order we add them in two more ways."

Write and show with a bar model and an open number line as you tell the story.

"Jameesha drew 2 butterflies. She drew 5 more. How many butterflies did she draw altogether?"

“I draw one bar for her 2 butterflies. I draw another bar for the 5 more butterflies. How many butterflies does she have altogether? Student Choral Response: [7]

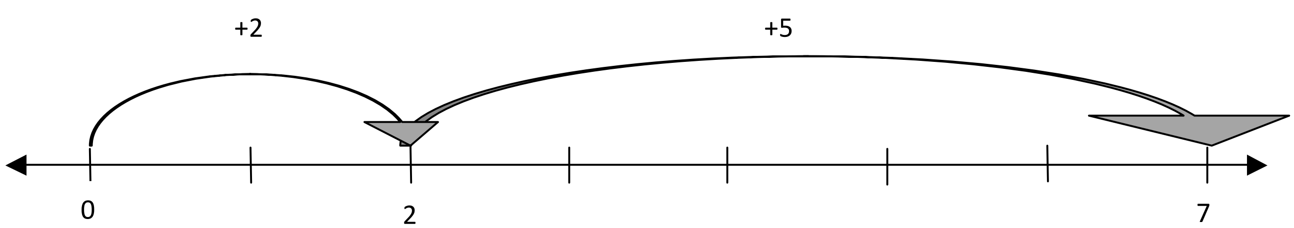
bar model:



“I can show this on an open number line too. I start at zero and jump to 2 to show her first 2 butterflies, then I can jump 5, for her 5 more butterflies, to the 7.

“How many butterflies did she draw altogether?” Choral Response: [7]

open number line:



Write: $2 + 5 = 7$

"Using the Commutative Property of Addition, what would happen if we changed the order of the numbers we’re adding, the addends? How many will she have drawn then? Think. Tell your partner how many you think and why you think that.

Students whisper to partner then T. pulls stick for non-volunteers. Check for class agreement using silent signals for yes and no. Then have students justify their answers. Connect to Commutative Property of Addition.

"Let's show it with a bar model and an open number line."

Write and show with a bar model and an open number line next to your previous examples as you tell the story.

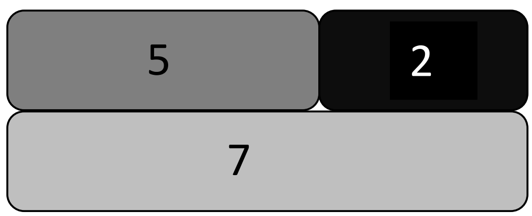
"Jameesha drew 5 butterflies. She drew 2 more. How many butterflies did she draw altogether?"

"How many should I draw for my first bar this time? Choral Response [5]

"How many should I draw for my second bar? Choral Reponse: [2]

"How many altogether?" Choral Response: [7]

bar model:

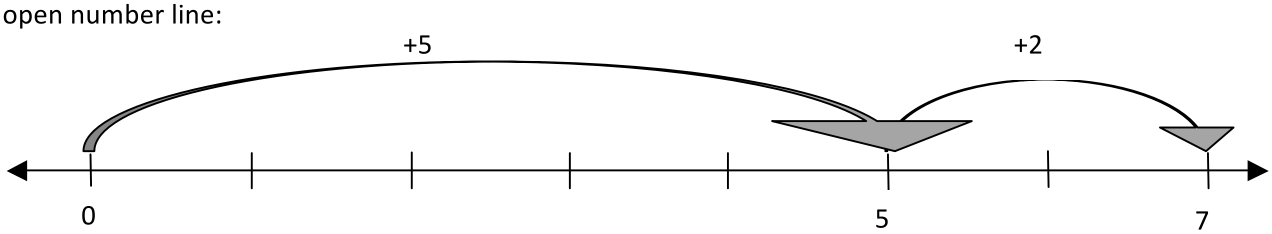


"What's my first jump this time?" Choral Response: [5]

"What's my second jump?" Choral Response: [2]

"How many altogether?" Choral Response: [7]

open number line:



$5 + 2 = 7$

"We showed that the Commutative Property of Addition is true again! Tell your partner what that Property says is always true. Call on non-volunteers for answers. [It doesn't matter which order we add the numbers, or addends, we always get the same sum. Have students repeat in Choral Response.

Students write on whiteboard with teacher - and show with bar models, open number lines, and equations. Give students 2 addends and have them show the rest of their work.

1) $4+1=5$ and $1+4=5$

2) $6+3=9$ and $3+6=9$

You Tries: Students may work the first problem with their partner. Have them work independently for second problem to check for understanding. During the next Independent Practice time below, post examples of all the methods we've used and provide coaching to those who need extra support or clarification. Note students who may need small group reteaching.

3) $2+8=10$ and $8+2=10$

4) $7+1=8$ and $1+7=8$

"Now we know 5 ways to show the Commutative Property of Addition: Counters, 10-Frames, Bar Models, Open Number Lines, and Equations. For these next problems you will choose any two ways to show your work, and write the equations.

Independent Practice:

Give students the 2 addends:

5) $3+7=10$ and $7+3=10$

6) $9+3=12$ and $3+9=12$

7) $8+6=14$ and $6+8=14$

8) $5+10=15$ and $10+5=15$

Closure:

Show a few examples of student work to note the different strategies and the Commutative Property of Addition at work.

"How did you decide which ways to show your work?"

"What did you have to do to show the work correctly?"

"What patterns do you notice?" (Be sure that students note how each pair of equations reflects the same addends and sum, but that the order of the addends is changed).

"Tomorrow we will use what we learned today with greater numbers. Remember, the Commutative Property of Addition is always true, no matter how large or small the numbers are. "

"What is the Math Property we learned about today?"

Choral Response: [Commutative Property of Addition]

"What does the Commutative Property of Addition tell us is always true?"

Choral Response: [When we add numbers, or addends, together, we always get the same sum no matter what order we add the numbers in.]

Next Step: Connect to equations with greater addends. Use bar models and Open number lines with greater numbers.

Note for K When students are adding with concrete objects bring this relationship to their attention. Have them try it out to prove it.

Note for 1st Students may use counters for more problems.

Focus Two: Inverse Relationship of Addition and Subtraction

Standard: 1NS 2.2 Use the inverse relationship between addition and subtraction to solve problems.

2NS 2.1 Understand and use the inverse relationship between addition and subtraction to solve problems and check solutions.

Objective: Students use the Inverse Relationship of Addition and Subtraction to write related addition and subtraction equations.

Students use linking cubes and Open Number Lines to show their understanding of the Inverse Relationship of Addition and Subtraction.

Materials: Linking cubes - 10 each of 2 colors of linking cubes for each partner pair of students and for teacher

Paper and pencils to record open number lines and/or whiteboards with whiteboard markers.

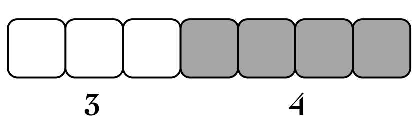
Vocabulary: add, sum, difference, inverse operation

Introduction : "Today we're going to add numbers together, find the sum, and then take the numbers apart and see what the difference is. We're looking to see if we can find a pattern as we work. "

Write equation and show with linking cubes, as you tell the math story.

"Maria had 3 carrots. Her mom gave her 4 more carrots. How many carrots does she have now?"

Linking Cubes: Put together one group of 3 cubes of one color, and one group of 4 cubes of another color first. Then put the two groups together.



“I make a group of 3 cubes of one color for Maria’s 3 carrots and a group of 4 cubes for the 4 more her mom gave her. When I put them together how many do I have?”

Choral Response: [7]

$3 + 4 = 7$

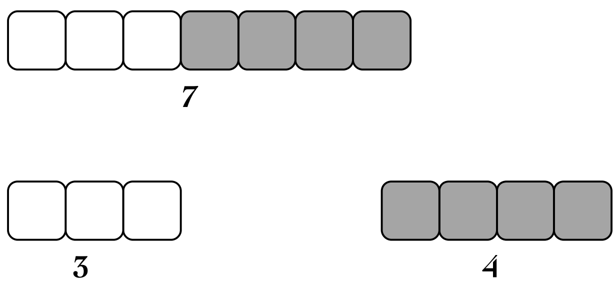
“Yes. When we put 3 and 4 together we get 7.”

"I wonder how many carrots Maria would have if her mom took those 4 carrots back from her?" "Let's try it."

Write equation and show with linking cubes as you tell the math story.

"Now Maria has 7 carrots and her mom needs her to give 4 back. How many carrots does Maria have after she gives 4 back to her mom?"

Linking Cubes:



“I take the 7 bar I made and I take the four cubes off. How many carrots does Maria have now?” Student Choral Response: [3]

$7 - 4 = 3$

“Yes she’s back to having the 3 carrots she started with . First we added and then we undid our adding when we subtracted and we had the same number we started with .

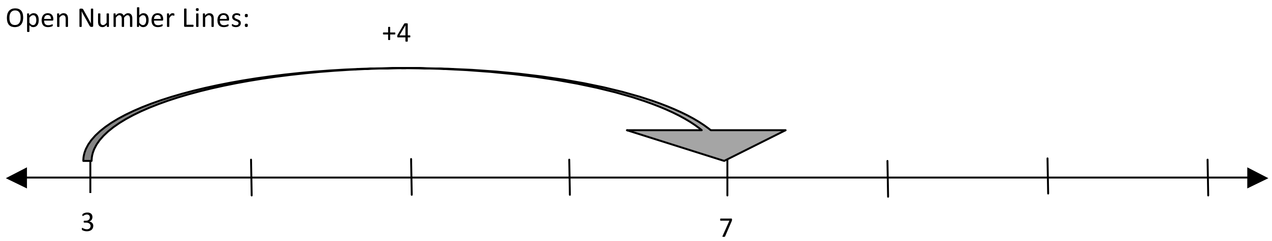
"We can show this using an open number line too."

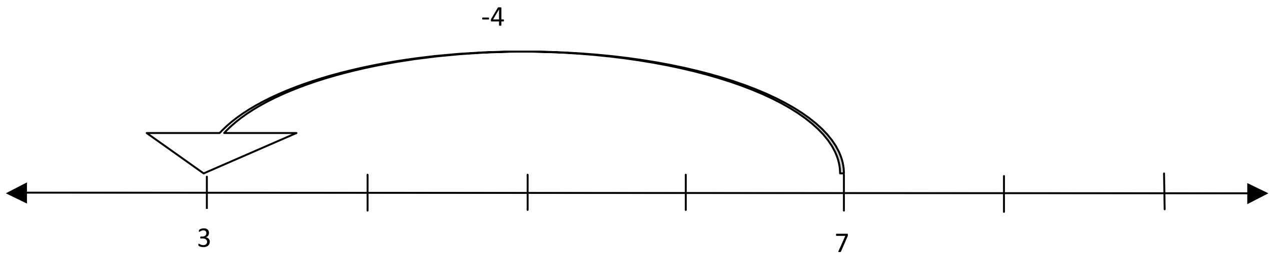
Record hops on open number line as you retell the story problem.

“We start on 3 because that’s how many carrots Maria started with. Then we jump 4 to show she got that many more. What number did we end on? Choral Response: [7]

“Yes, she had 7 carrots. But then she had to give four carrots back to her mom. I can show that on another open number line. I start on 7 and jump back 4. What number am I on now? Choral Response: [3]

Open Number Lines:





Demonstrate with both the linking cubes and the open number line as you

"So she started with 3, added 4 to have 7, and then when she gave the 4 back, she was back to 3. Here's how we would write that when writing equations."

$3 + 4 = 7$ and $7 - 4 = 3$

"When we took back the 4, we undid the addition that we did when we put together the 3 and the 4.

So, when we add we can undo subtraction, and when we subtract we can undo addition.

Mathematicians say it this way: subtraction is the inverse operation of addition.

Have students say with you chorally: Subtraction is the inverse operation of addition. We can subtract to undo addition, and we can add to undo subtraction.

"Yes, if we add 2 numbers together, and then subtract one of the numbers from the sum, the answer to the subtraction problem will be the other addend.

Write equation and show with linking cubes, as you tell the math story. Students work with partner to show with their cubes. Students draw open number lines on whiteboards.

"Frank had 5 crayons. His sister gave him 3 more crayons. How many crayons does he have now?"

Guide students to put together 5 linking cubes of one color and 3 linking cubes of another (as above).

"We have 5 cubes for his 5 crayons, and add how many more?" Choral Response: [3]

"We put the 5 and 3 together and we have how many?" Choral Response: [8]

$5 + 3 = 8$

"Let's see what happens if we take those 3 back away." Elicit student predictions.

Write equation and show with linking cubes as you tell the math story.

"Now Frank had 8 crayons. He gave 3 crayons back to his sister. How many crayons does Frank have now?"

Show with linking cubes and write the equation: $8 - 3 =$

“Now we start with 8 and take away how many?” Choral Response: [3]

"How many crayons does he have now?" Student Choral Response: [5]

Guide students to draw open number lines for the problems.

“We can show this with open number lines too. We start on 5 for the crayons he had. Then we jump how many?” Choral Response: [3] and we end up on?” Choral Response: [8].

$5 + 3 = 8$ and $8 - 3 = 5$

"We started with 5 and added 3 to get 8. Then we took back the 3, we undid the addition of putting together the 5 and the 3. 8 take away 3 is back to 5.”

So, we added and then when we subtracted we undid our addition.

Mathematicians subtraction is the inverse operation of addition. (Have students repeat.)

You Try: Formative Assessment: Note how students do with these problems and give more practice if needed, either for whole group or for a small group. Students can solve the first problem with a partner, and then the second problem on their own.

“Now I want you to show me with linking cubes and an open number line this problem:

“Dreshaun has 2 cats. His neighbor has 6 cats. How many cats do they have together?

Then:

“If his neighbor moves and takes his 6 cats away, how many cats will be left?”

Use linking cubes and open number lines to show your math thinking. Then write the equations.

Walk room to see how students show work. They should write these equations.
$2 + 6 = 8$ and $8 - 6 = 2$

Write equation and show with linking cubes, as you tell the math story.

"Here's a different example. This time we're going to subtract first, and see what we get when we add back together."

"Katie has 9 bracelets. She gives 5 bracelets to her friend. How many does she have now? This time we're taking away, not putting together first.

Show with linking cubes, (one group of 9 made of a group of 5 of one color and a group of 4 of another color). Write the equation.

$9-5=$

"How many does she have left?" Choral response: [4]

"Yes she has 4 left." Let's see what happens if her friend gives them back."

4

Write the equation and show with linking cubes, as you tell the math story.

"Katie had 4 bracelets left. Then her friend gave her the 5 bracelets back. How many does she have now?" Choral Response: [9]

$4+5=9$

"Yes, she has all 9 back again."

Show on open number lines, side by side.

"We start on 9, then jump 5 down to 4 on the number line. "

"Next we start on 4 and jump 5 up to 9."

"First we subtracted 5 from 9 and we got 4. Then when we added 5 back together with the 4, we had 9 again. We added to undo the subtraction."

"The inverse relationship of addition and subtraction is true when you start with subtraction too. If you add back a number you just subtracted, you'll get back to the number you started with.

Write the first equation in the sets for students to copy on their paper or whiteboards. Guide students to build two-color groups for the starting number. Then guide students to show on open number lines. Students show with linking cubes and open number lines and record the sum or difference and the inverse operation equation for each equation. Repeat for second equation set.

$10-6=4$ and $4+6=10$

$11-3=8$ and $8+3=11$

You Tries: Students show with linking cubes and open number lines and record on paper or whiteboards. They may work the first one with a partner. The second should be on their own providing a chance for formative assessment. Note which students are struggling. Reteach or pull a small group based on results.

$5+1=6$ and $6-1=5$

$12-7=5$ and $5+7=12$

Closure: “What do notice about these equations? What helped you to find the sum or difference when you were undoing the first operation? Do you see a pattern?

K notes: When working on adding and subtracting with manipulatives, note this relationship and have students show that it is true.

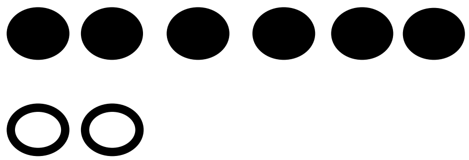
1st and 2nd: Students need multiple opportunities to practice seeing this relationship. Use as a small group or center activity.

1st and 2nd: Next steps: Connect patterns and understandings discovered in this lesson with equations with greater numbers.

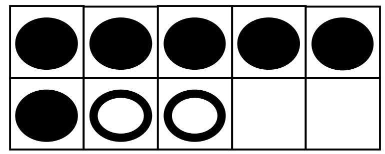
Side-By-Side Comparison for Commutative Property of Addition Strategies

$6 + 2 = 8$ and $2 + 6 = 8$

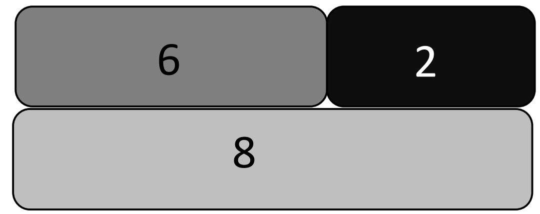
Counters

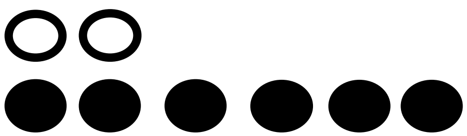


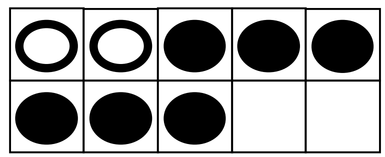
10-Frames

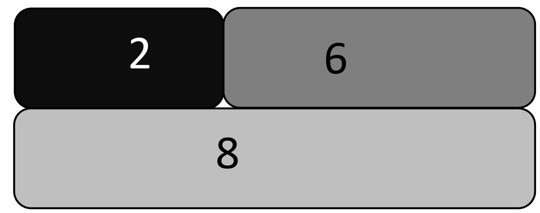


Bar Models

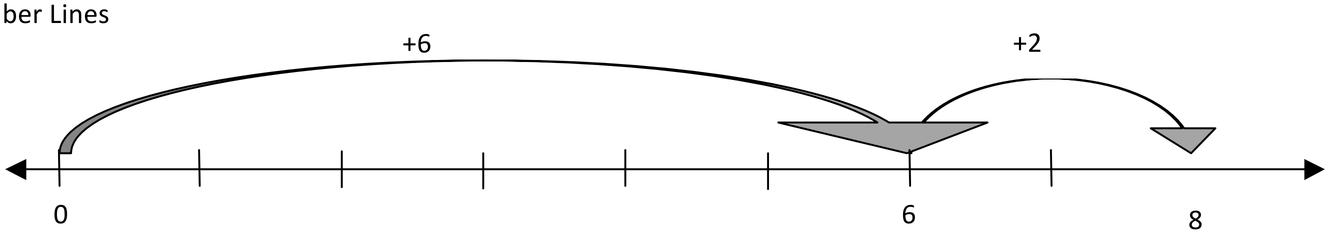


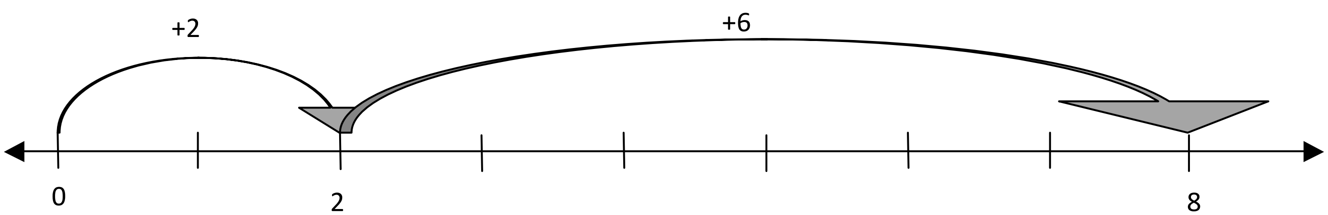






Open Number Lines





Side-By-Side Comparison for the Inverse Relationship of Addition and Subtraction $5+1=6$ and $6-1=5$

Linking Cubes

Open Number Lines