

## Improper Fractions to Mixed Numbers and Back

### Objective:

Students will be able to change Improper Fractions to Mixed Numbers, and Mixed Numbers to Improper fractions

### Standard: 5<sup>th</sup> Grade

2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

2.4 Understand the concept of multiplication and division of fractions.

2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

### Prerequisites:

Decompose Fractions, Fractional division

**Improper Fraction: the numerator is larger than the denominator**

**Mixed number: a combination of a whole number and a fraction**

Designed for fourth through sixth grade, but can be used with third graders

### Using:

Bar Models

Decomposition/ Recomposition

Fractional Division/ Common Denominator

Traditional (students discover)

**Focus on the following QCIP strategies;**  
**Student Talk**  
**Active Student Engagement**  
**Low Affective Filter**

### Part 1: Changing Improper Fractions to Mixed Numbers

#### Method #1: Bar Model

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{3}{3} = 1$			$\frac{3}{3} = 1$			$\frac{3}{3} = 1$		$\frac{1}{3}$	

$$\begin{aligned}\frac{10}{3} &= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3} \\ &= 1 + 1 + 1 + \frac{1}{3} \\ &= 3\frac{1}{3}\end{aligned}$$

**Low Affective Filter**  
Using easy to convert fractions as examples lowers anxiety for students and makes them more willing to try new methods.

Let's start by drawing a bar model to represent our improper fraction 10 thirds. How many pieces should we make?

[10]

Each piece is worth one third. How many thirds do we need to make one whole?

[3]

Draw a bar underneath to show each group of three thirds as equal to one whole. How many whole pieces do we have?

[3]

There is one third remaining. So when we add the whole pieces to the third, we get 3 wholes and one third. Now we have a mixed number. Let's show the work with an equation as well.

## Method #2: Decomposition

$$\begin{aligned} \frac{10}{3} &= \frac{1+1+1+1+1+1+1+1+1+1}{3} \\ &= \frac{(1+1+1)+(1+1+1)+(1+1+1)+1}{3} \\ &= \frac{3+3+3+1}{3} \\ &= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3} \\ &= 1 + 1 + 1 + \frac{1}{3} \\ &= 3\frac{1}{3} \end{aligned}$$

**Student Talk**  
Give students time to discuss the ways the first two methods are similar and different, and why each method is easy to use.

Let's start with an equation this time. Write the improper fraction and then we will use decompositions to break apart our fraction. I will ask you to compare to the bar model after we try this methods. I start by decomposing the numerator 10 into ones, how many ones are in 10?  
[10]

Write the numerator as an addition expression using only ones. Now we group them into groups of three because three is our denominator and we want to create equivalent forms of 1. I use parenthesis to show my groups, then I simplify my groups. How many groups of three do I have?  
[3]

I decompose my fraction and give each of my addends in the numerator, the denominator 3. What is three thirds an equivalent form of?  
[1]

Convert your fractions to whole numbers, then add them. Now we have a mixed number.

### You Try #1, part 1

#### Bar Model

$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{2}{2}=1$		$\frac{2}{2}=1$		$\frac{1}{2}$

$$\begin{aligned} \frac{5}{2} &= \frac{2}{2} + \frac{2}{2} + \frac{1}{2} \\ &= 1 + 1 + \frac{1}{2} \\ &= 2\frac{1}{2} \end{aligned}$$

#### Decomposition

$$\begin{aligned} \frac{5}{2} &= \frac{1+1+1+1+1}{2} \\ &= \frac{(1+1)+(1+1)+1}{2} \\ &= \frac{2+2+1}{2} \\ &= \frac{2}{2} + \frac{2}{2} + \frac{1}{2} \\ &= 1 + 1 + \frac{1}{2} \\ &= 2\frac{1}{2} \end{aligned}$$

### Method #3: Fractional Division

$$\begin{aligned}\frac{10}{3} &= \frac{9+1}{3} \\ &= \frac{9}{3} + \frac{1}{3} \\ &= 3 + \frac{1}{3} \\ &= 3\frac{1}{3}\end{aligned}$$

We will start by writing the improper fraction as an equation. When I decompose the numerator, I want to find a multiple of the denominator. So when I decompose 10, I break it into nine and one because nine is a multiple of three and one is the amount remaining. Then I write the numerator as an addition expression. What expression will I write?

[9+1]

Then I decompose my fraction and give both the addends in the numerator the same denominator. When I look at my new fraction I see nine divided by 3, what is the quotient?

[3]

Then I add the remaining fraction to give me the mixed number three and one-third.

### Method #4: Traditional

$$\frac{10}{3} = 3\frac{1}{3}$$

#### side work

$$\begin{array}{r} 3 \\ 3 \overline{)10} \\ \underline{-9} \\ 1 \end{array}$$

For this method I divide the numerator by the denominator using the traditional algorithm. The remainder then becomes the numerator over the divisor. So three goes into ten how many times?

[3]

And when we subtract nine from ten the remainder one becomes the numerator over three as the denominator to make the fraction one-third. We now have our mixed number.

#### **Student talk**

How do the last two methods compare? What mistakes do kids make with traditional?

### **You Try #1, part 2**

#### Fractional Division

$$\begin{aligned}\frac{5}{2} &= \frac{4+1}{2} \\ &= \frac{4}{2} + \frac{1}{2} \\ &= 2 + \frac{1}{2} \\ &= 2\frac{1}{2}\end{aligned}$$

#### Traditional

$$\frac{5}{2} = 2\frac{1}{2}$$

#### side work

$$\begin{array}{r} 2 \\ 2 \overline{)5} \\ \underline{-4} \\ 1 \end{array}$$

## Part 2: Changing Mixed Numbers to Improper Fractions

### Method #1: Bar Model

$\frac{3}{3} = 1$			$\frac{3}{3} = 1$			$\frac{3}{3} = 1$			$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Let's start by drawing a bar model to represent our mixed number three and one third. How many pieces should we make?

[three whole and one third]

$$\begin{aligned}3\frac{1}{3} &= 1 + 1 + 1 + \frac{1}{3} \\ &= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3} \\ &= \frac{10}{3}\end{aligned}$$

How many pieces do we need to break each whole into to represent thirds?

[3]

Under each whole bar, draw a bar broken into three parts that represent thirds. How many whole thirds do we have?

[10]

So we have ten thirds all together. Now we can write it as an improper fraction.

### Method #2: Decomposition

$$\begin{aligned}3\frac{1}{3} &= 1 + 1 + 1 + \frac{1}{3} \\ &= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3} \\ &= \frac{3+3+3+1}{3} \\ &= \frac{10}{3}\end{aligned}$$

Let's start with an equation this time. Write the mixed number and then we will use decomposition to break apart our number. I will ask you to compare to the bar model after we try this method. I start by decomposing the whole number 3 into ones, how many ones are in 3?

[3]

Then I will write the whole numbers as fractions with the denominator 3, to make the fraction equal to one, I have to make the numerator 3 also. Then I need to add all my numerators together. What am I adding together?

[3+3+3+1]

My numerator is 10 and my denominator is 3. Now we can write this as an improper fraction.

### Method #3: Common Denominator

$$\begin{aligned}
 3\frac{1}{3} &= \frac{3}{1} + \frac{1}{3} \\
 &= \frac{3\left(\frac{3}{3}\right) + \frac{1}{3}}{3} \\
 &= \frac{9}{3} + \frac{1}{3} \\
 &= \frac{9+1}{3} \\
 &= \frac{10}{3}
 \end{aligned}$$

All whole numbers can be written as fractions with a denominator of 1. So start by writing the whole number as a fraction. Are the denominators the same?

[no]

We want to make the denominators the same, so we multiply it by three thirds. What does the fraction change into?

$$\left[\frac{9}{3}\right]$$

Then we combine the fractions together. Now it is an improper fraction.

### Method #4: Traditional

$$\begin{aligned}
 3\frac{1}{3} &= \frac{(3 \times 3) + 1}{3} \\
 &= \frac{9+1}{3} \\
 &= \frac{10}{3}
 \end{aligned}$$

Multiply the whole number times the denominator and add it to the numerator.

What is the new numerator?

[10]

The improper fraction is ten thirds.

### **You Try #2**

#### Bar Model

$\frac{2}{2} = 1$		$\frac{2}{2} = 1$		$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

$$\begin{aligned}
 2\frac{1}{2} &= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \\
 &= \frac{5}{2}
 \end{aligned}$$

#### Decomposition

$$\begin{aligned}
 2\frac{1}{2} &= 1 + 1 + \frac{1}{2} \\
 &= \frac{2}{2} + \frac{2}{2} + \frac{1}{2} \\
 &= \frac{2+2+1}{2} \\
 &= \frac{(1+1) + (1+1) + 1}{2} \\
 &= \frac{1+1+1+1+1}{2} \\
 &= \frac{5}{2}
 \end{aligned}$$

#### Common Denominator

$$\begin{aligned}
 2\frac{1}{2} &= \frac{2}{1} + \frac{1}{2} \\
 &= \frac{2\left(\frac{2}{2}\right) + \frac{1}{2}}{2} \\
 &= \frac{4}{2} + \frac{1}{2} \\
 &= \frac{4+1}{2} \\
 &= \frac{5}{2}
 \end{aligned}$$

#### Traditional

$$\begin{aligned}
 2\frac{1}{2} &= \frac{(2 \times 2) + 1}{2} \\
 &= \frac{4+1}{2} \\
 &= \frac{5}{2}
 \end{aligned}$$

#### **Additional You Trys**

1.  $\frac{14}{5}$

2.  $1\frac{3}{7}$

3.  $\frac{17}{4}$

4.  $5\frac{1}{6}$

5.  $\frac{25}{8}$

Name: \_\_\_\_\_

## Warm-Up

CST/CAHSEE: NS 2.3	Current: 4NS 1.9
<p><b>1</b> What is <math>\frac{9}{5}</math> written as a mixed number?</p> <p>A) <math>9\frac{1}{5}</math></p> <p>B) <math>1\frac{4}{5}</math></p> <p>C) <math>\frac{5}{9}</math></p> <p>D) <math>\frac{4}{5}</math></p> <p> </p> <p>*Why would students choose answer A?</p> <p>*Show a pictorial way to solve this problem.</p>	<p><b>2</b> What is <math>1\frac{3}{9}</math> written as an improper fraction?</p> <p>A) <math>\frac{10}{3}</math></p> <p>B) <math>\frac{12}{9}</math></p> <p>C) <math>\frac{4}{9}</math></p> <p>D) <math>\frac{3}{9}</math></p> <p> </p> <p>*Will our answer be bigger or smaller than one?</p>
Preview: NS 2.4	Other: NS 2.3
<p><b>10</b> <math>\frac{3}{5} \times \frac{3}{5} =</math></p> <p>A <math>\frac{1}{25}</math></p> <p>B <math>\frac{6}{25}</math></p> <p>C <math>\frac{9}{25}</math></p> <p>D <math>\frac{9}{10}</math></p> <p> </p> <p>*How did students find the answer to each problem?</p> <p>*What is one mitigation we can use to correct this problem?</p>	<p><b>7</b> Emily jogged <math>\frac{7}{10}</math> mile and sprinted <math>\frac{1}{10}</math> mile.</p> <p><b>In simplest form, how much farther did Emily jog than sprint?</b></p> <p>A <math>\frac{6}{10}</math></p> <p>B <math>\frac{4}{5}</math></p> <p>C <math>\frac{7}{10}</math></p> <p>D <math>\frac{3}{5}</math></p> <p>*What part of the question will kids forget to do?</p>